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Mr A. C. Hartley, Member, Chairman of the Divisional Board, in the Chair

The following Lecture was delivered and, on the motion of the Chairman, the thanks of the Division were accorded to the Lecturer.

"Swedish Underground Hydro-Electric Power Stations"

by
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Introduction

In 1910 the Swedish State Power Board started construction of the Porjus power station which is situated on the River Stora Luleälv 35 miles to the north of the Arctic Circle. This power station, which represented an extensive undertaking in those days, was completed in 5 years and was placed in service at the end of 1914. In its original form, the plant comprised five machine sets with a total capacity of 55,000 kW. The machines are housed in an underground hall blasted out of the rock at a depth of 165 feet. The water conduits also are blasted out of the rock and take the form of shafts and tunnels, so that Porjus may be regarded as a typical example of an underground station and, to a great extent, served as a prototype for all underground stations built later in Sweden. The station has now been extended and has a present capacity of 135,000 kW.

It was not until 1931, however, that another power station of a similar type was built. During that year, work was begun on the first part of Krångede power station. The station lies on the Indal River in Central Norrland and its original installation comprised two machine sets totalling 60,000 kW. The station has since been extended to include six sets of a total capacity of 230,000 kW. It was followed by a long succession of underground stations; particulars of the more important of these are given in Table 1.

TABLE 1.—UNDERGROUND POWER STATIONS

Station	Years of construction	Head: feet	Capacity: kW.	Tail-race tunnels			Owner
				No.	Length: feet	Area of each tunnel: sq. ft	
1	1910–1915 1935–1950	190	135,000	2	4,200	540	S
2	1931–1936 1939–1944	190	230,000	2	4,600	1,250	P
3	1938–1942	105	90,000	2	264	1,080	S
4	1939–1943	405	55,000	1	15,500	650	S
5	1939–1945	210	100,000	1	15,000	1,130	P
6	1944–1948	115	70,000	1	3,150	1,620	S
7	1944–1949	285	168,000	1	20,500	1,450	P
8	1945–1952	355	350,000	1	9,600	2,050	S
9	1947–1952	79	26,000	1	1,320	650	PS
10	1947–1954	330	240,000	1	12,200	2,160	S
11	1948–1952	105	40,000	1	15,500	1,350	P
12	1951–1954	132	145,000	1	1,050	2,800	S

S denotes power plant belonging to the State Power Board.
P denotes power plant belonging to private companies.

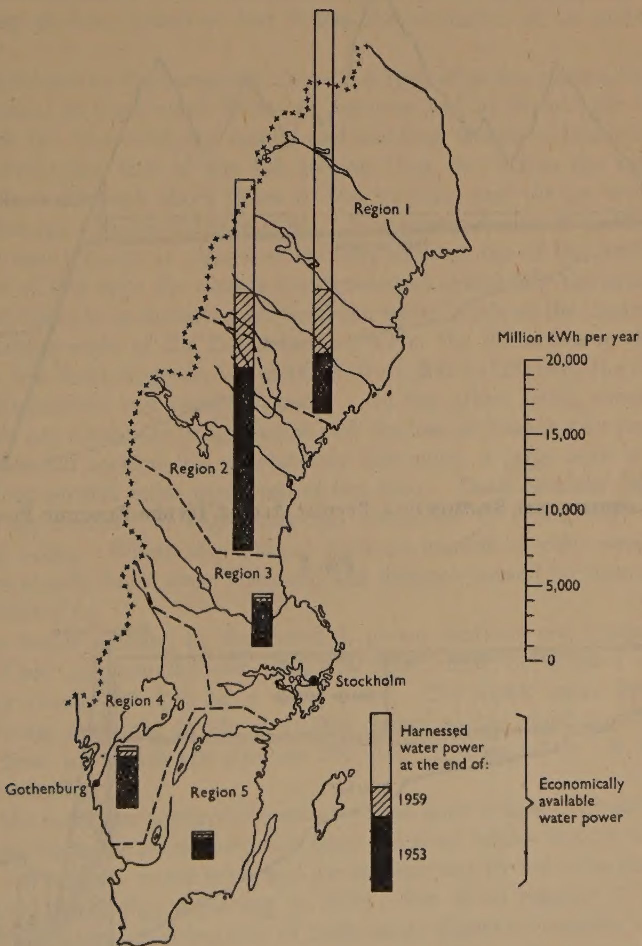
All these power stations, with one exception, are located in the northern provinces of Sweden where the greater part of the hitherto unexploited water-power resources are situated. Economically accessible water-power resources in Sweden amount to approximately 60,000 million kWh. per annum. Of this, about 50,000 million kWh. is found in the central and northern districts where only 31 per cent of the power had been harnessed by 1952. The available and exploited sources of water power in Sweden are shown in *Fig. 1.* The total machine capacity installed up to 1953 amounted to 3·8 million kW., and the total energy produced during 1952 was about 20,000 million kWh.

Comparison of Different Types of Power Plant

In all countries in which development of water power has reached an advanced stage, special types of water-power stations have been designed

to suit particular topographical and geological conditions. In Alpine countries such as Switzerland, Austria, and Italy, and also in Norway, where heads range from 600 to 3,000 feet, the dam and intake are usually placed in the vicinity of the upstream end of the harnessed fall section

Fig. 1

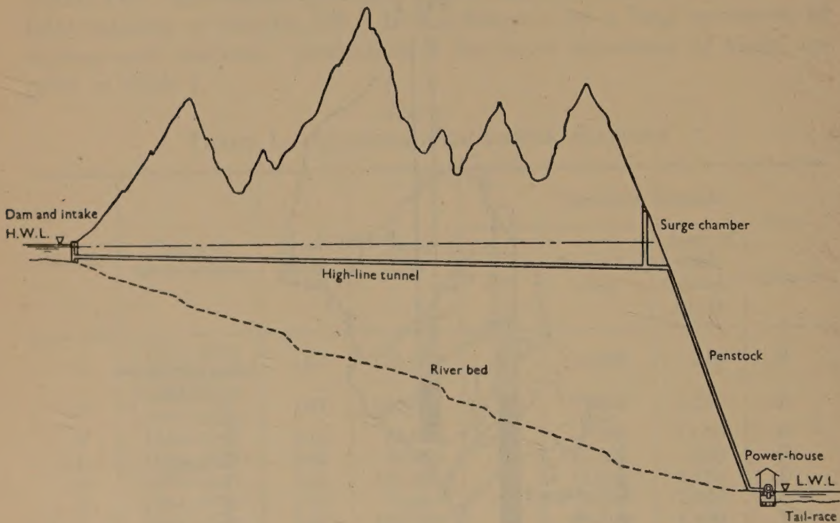


SWEDEN'S WATER-POWER RESOURCES

(*Fig. 2*). From the intake the water is led by a high-line conduit to a surge tank, and thence through penstocks to the turbines which are installed in a surface-station close to the downstream end of the fall section. From the turbines the water is passed back to the river through a short

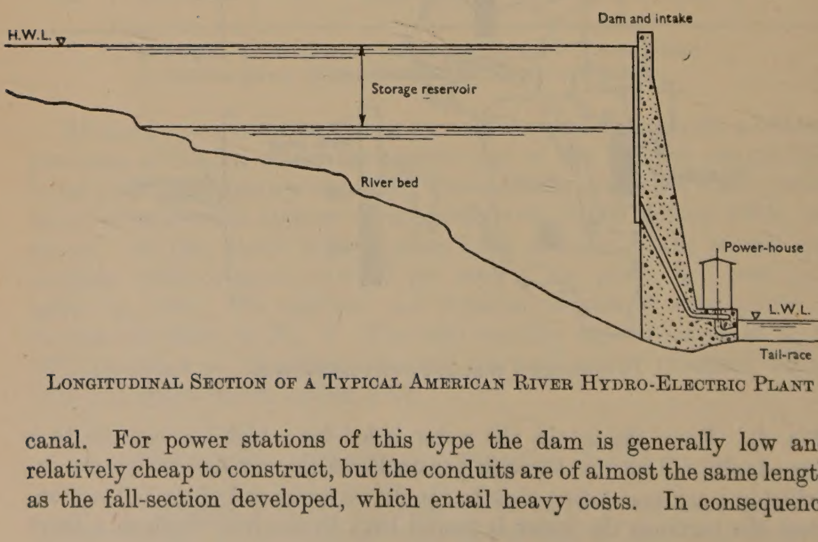
tail-race. The high-line conduit usually consists of a flat, sloping tunnel. The penstocks, which lie at a relatively steep angle, consist of steel tubes connected to the turbines, and the tail-race takes the form of a short

Fig. 2



LONGITUDINAL SECTION OF A TYPICAL ALPINE HYDRO-ELECTRIC PLANT

Fig. 3



LONGITUDINAL SECTION OF A TYPICAL AMERICAN RIVER HYDRO-ELECTRIC PLANT

canal. For power stations of this type the dam is generally low and relatively cheap to construct, but the conduits are of almost the same length as the fall-section developed, which entail heavy costs. In consequence

of the long conduits, the loss of head is also considerable; this is not of great significance, however, in view of the high heads available.

During recent decades, the penstocks and machine station have, on some occasions, been blasted out about 1,500 feet into the rock and provided with tail-race and access tunnels of corresponding length. This form of construction does not necessitate any change in principle in the type of power station, however, but it has the character of an underground station.

In contrast to the foregoing, there is a type of power station frequently encountered in the United States of America and in France, for example, in which the dam with the intake and machine station is located close to the downstream end of the fall section (*Fig. 3*). From the intake the water flows through short tubes to the turbines and thence back to the river through a short tail-race canal. The power station is on the surface and, in many cases, is placed at the downstream toe of the dam. With stations of this type the dam is constructed at practically the same height as the difference in elevation between the water levels at the upstream and downstream ends of the fall section. When the difference in elevation, that is, the head, is great, for example from 200 to 500 feet, the dam is of large dimensions and costly to build. On the other hand, costs for the conduits are relatively insignificant and the loss of head is also small.

If the fall section has a relatively flat slope, a large lake is formed, extending several miles upstream of the dam. These artificial lakes have special significance in regulating the waterflow in rivers that have no natural lakes. Where the flooded districts consist of cultivated ground and are closely built over, however, the damage caused by damming will be considerable.

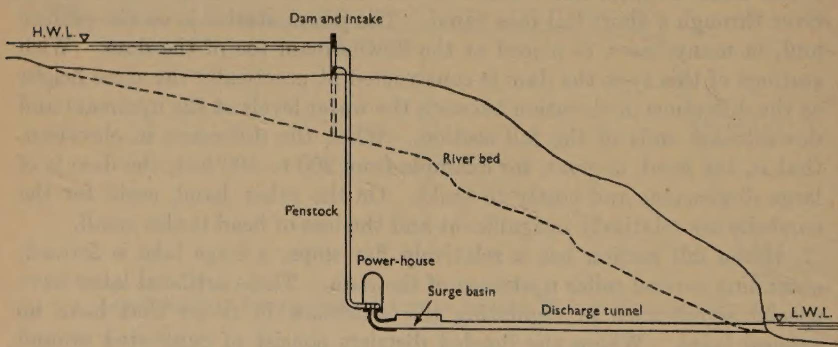
The heads utilized in the Swedish power stations are comparatively low. They are usually less than 150 feet; only in isolated instances do they reach 300 feet or a little more. The mean water flow in the large rivers amounts to about 10,000 cusecs at the river mouths. The water flow, for which the stations are built, is from 1·5 to 1·9 times the mean waterflow.

In the northern parts of the country, the most densely populated areas are situated in the river valleys. Here the river banks lie only a few feet above the natural water level and are surrounded by valuable forests and cultivated land, *Fig. 4* (facing p. 328). For these reasons it is highly desirable to avoid construction of high dams wherever possible, otherwise it is necessary to pay heavy compensation for damage caused in the flooded areas. Considerable opposition would also be encountered if large sections of the population were forced to leave the homes in which their forefathers may have lived for many generations. Difficulties of this nature do not exist in the mountainous districts but, owing to the great width of the valleys, the construction of a high dam is very costly. On the other hand, these districts contain a large number of spring-lakes, in

which the water can be suitably stored to regulate the water flow of the rivers. During recent years such lakes have been utilized to a constantly increasing extent for this purpose.

In view of these conditions, the dam is located in the central part of the fall section for the type of underground power stations that have been developed in Sweden, *Fig. 5*. The intake is generally placed at the dam itself, and the machine station, which is constructed deep in the rock by blasting, lies at a short distance downstream from the intake. From the intake, water is led to the turbines, usually through vertical penstocks, of which there is one for each turbine; the water returns to the river

Fig. 5



LONGITUDINAL SECTION OF A TYPICAL SWEDISH HYDRO-ELECTRIC PLANT

through a tail-race tunnel which is also blasted out of the rock. If the rock lies so low that it is not possible to obtain sufficient height for the roof, the tunnel is replaced by a discharge canal. For power stations of this type the dam is of moderate height and the damage from damming is relatively small. Since the penstocks are vertical, they are as short as possible which simplifies governing of the turbines, obviating installation of any special device for this purpose on the upstream side, such as a surge basin or relief valves. The length of the tail-race is also kept within moderate limits, so that loss of head is relatively small.

DESIGN OF UNDERGROUND WATER POWER STATIONS

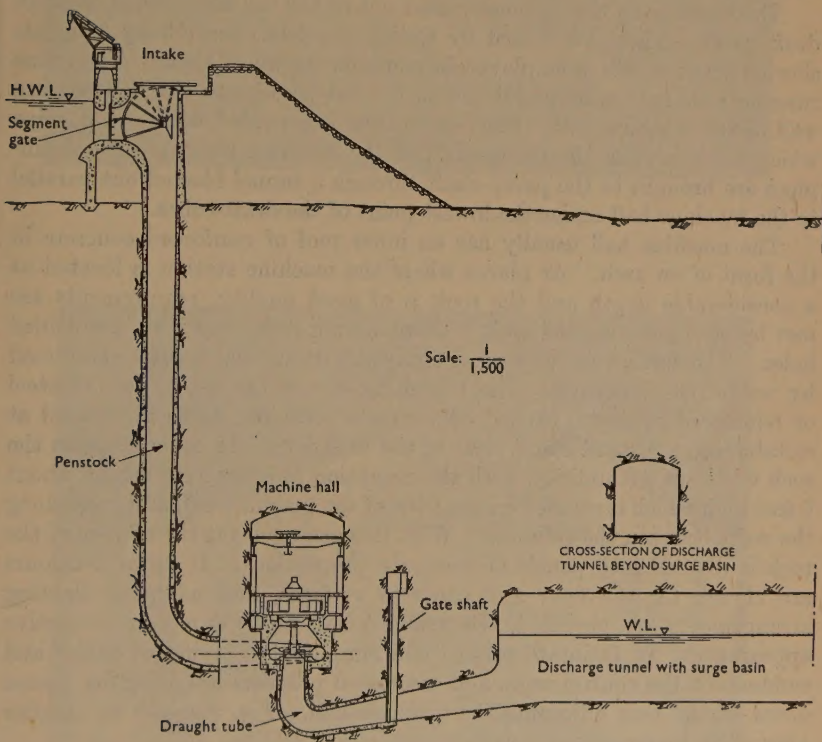
Intake and Penstocks

The intake for an underground station is largely designed in accordance with the principles adopted for a surface station. The intake is usually closed by segment gates, but flat roller gates may also be used. To prevent freezing of the gates, both the grooves and the sills are heated

electrically. The current is stepped down to a voltage of 8, 10, or 12 volts. Steps have also been taken for the electric heating of the trash racks.

As already mentioned, the shafts for the penstocks to the underground station are usually vertical. The penstocks, which are mostly constructed of steel, either stand clear of the rock or are embedded in concrete; the lower horizontal section of the penstocks, which must be

Figs 6



VERTICAL SECTION OF A TYPICAL UNDERGROUND POWER STATION

rigidly anchored, is always set in concrete, however. In particular instances where the rock is exceptionally sound, the steel pipes extend to the height of the machine-hall roof. From this point, and up to the intake, the rock is lined with reinforced concrete. A vertical section through a typical underground station is shown in *Figs 6*.

Machine Station

The depth of location of the machine station is dependent upon the height of the head and the elevation of the ground. The available quantity

of water is divided amongst a small number of sets only, from two to four. For heads not exceeding 165 feet, Kaplan turbines are used; Francis turbines are employed for greater heads, but both types have vertical shafts. The water is led to the turbines through spiral casings, constructed of concrete for heads up to 100 feet, and of steel for higher heads. The generators are three-phase alternating current generators, with shafts directly coupled to the turbines. The frequency is 50 cycles per second (Figs 7 and 8, Plate 1).

The water from the turbines passes out to the tail-race tunnel through draft tubes. These are closed by sliding shutters operated by a mobile slewing crane which is employed in common for all of them. The crane runs on rails laid in a special tunnel located parallel to the downstream wall of the machine hall. Each draft tube is provided with a drain-pipe which terminates in the pump-shaft of the machine station. The drain-pipes are brought to the pump-shaft through a tunnel blasted out parallel to the machine hall under the lowest point of the draft tubes.

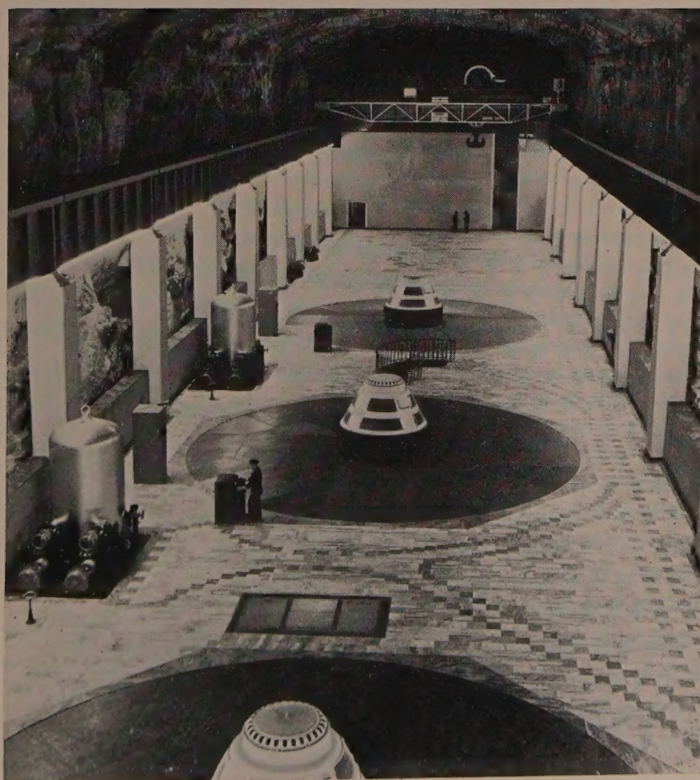
The machine hall usually has an inner roof of reinforced concrete in the form of an arch. At places where the machine station is located at a considerable depth and the rock is of good quality, requirements are met by strengthening the rock with anchoring rods cemented into drilled holes. The surface of the rock is sprayed with cement-mortar reinforced by welded steel netting. The travelling cranes run on girders of steel or reinforced concrete, carried on concrete columns; these are placed at suitable intervals and stand clear of the rock face. In other respects the rock walls are left unlined, with the exception of a concrete barrier about 6 feet high which is erected on the floor of the machine hall and runs along the walls between the columns. With this arrangement the surface of the rock is divided into panels of adequate proportions. If suitable colours are chosen for the floor and concrete surfaces, and adequate lighting arrangements are provided, the machine hall presents a very attractive appearance. To facilitate service and simplify the laying of cables and conductors, the control room and associated premises are generally placed down in the rock adjoining the machine hall. *Fig. 9* shows an interior view of an underground station.

The machine station is connected to ground level either by a vertical transport shaft, Fig. 10, Plate 1, or by an access tunnel. The transport shaft is surmounted by an unloading hall equipped with a travelling crane. Machine parts for the turbines and generators, and other material are transported by rail to the unloading hall from which they are lowered to the level of the machine-hall floor by the crane. Where an access tunnel is available, the machine parts are transported on special trailers from the nearest railway station directly to the machine-hall unloading floor. The access tunnel has a gradient of 1 in 8 and its cross-sectional area is determined by the largest of the machine parts. The tunnel is also used for the removal of rock masses excavated during blasting operations.

Fig. 4

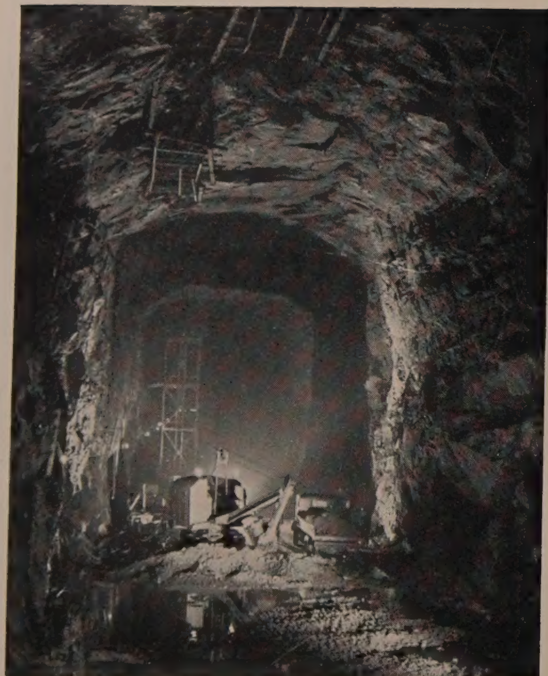
A Typical River Valley in the North of Sweden

Fig. 9



Machine Hall of an Underground Power Station

Fig. 21



Completed 2,000-square-foot Tunnel

Fig. 25

Completed Excavation for Machine Hall

Heating of underground stations is effected, as in surface stations, by surplus heat from the transformers. The quantity of heat necessary is less than needed in surface stations, but arrangements for fresh air supply are more complicated. Fresh air is conveyed to a special chamber either through a separate shaft or through a shaft or tunnel which is also used for other purposes. In the chamber the fresh air is mixed with the return air from the machine station. The mixed air is then allowed to flow through a heating plant heated by oil from the transformers, and can attain a temperature of 30 to 35° C. The air is subsequently blown into the machine station by fans. In order to remove the humidity from the fresh air, it is sometimes passed over cold water, over the surface of a surge basin, for example, before being mixed with the return air from the machine station. The quantity of air supplied is so great that in all parts of the station it is changed completely once every hour.

The generators are usually water-cooled by a closed circulating system. The transformers also are water-cooled, since the whole of the surplus heat is not used for heating purposes. In the case of low heads, the cooling water is drawn directly from the penstocks, whilst with high heads, it is pumped into the cooling system from the downstream side.

The problem of drainage has not occasioned any special difficulties in underground stations. Generally speaking, leakage through the rock is taken care of by evaporation and is carried away by the ventilating air. Any surplus leakage water, however, is led through a drainage system to the pump shaft of the machine station.

Discharge Tunnels and Surge Basins

A tunnel of circular section is the most advantageous type from the hydraulic point of view, but owing to the blasting methods customarily employed and the nature of the rock, it is necessary to adopt a rather modified form, particularly in the case of large tunnels. In sound rock the tunnels are blasted out with vertical walls and a slightly arched roof. If the rock is unsound, the roof takes the form of a semi-circle. The tunnels are usually unlined, with the exception of faulty parts in the rock that have to be reinforced with concrete. In many cases, however, the faulty parts are not so weak that they cannot be reinforced by anchoring rods drilled into the rock and embedded in cement.

Where the tail-race tunnel is short and the downstream water level varies within narrow limits the tunnel is located at such a height that a free water surface is obtained even at the maximum downstream water level. With this arrangement a greater cross-section results than if the tunnel were entirely filled with water, but the loss of head is less and a surge basin need not be constructed. The roof of the tunnel is placed only at such a height above the maximum downstream water level as will correspond to the surge occurring in an open canal on a sudden demand of the total turbine water flow.

On the other hand, where there is considerable variation in the downstream water level, the whole of the tail-race tunnel is located below the minimum water level in order to avoid a large ineffective tunnel section at the lower water levels. In such a case, however, in order to keep the fluctuation of pressure within definite limits when the turbine water flow is suddenly changed, the tunnel must be provided with a surge gallery. This can either be arranged by increasing the height of the tunnel for a sufficient length immediately downstream of the draft tubes, or by blasting out a special surge gallery which is connected to the tunnel in the vicinity of the draft-tube outlet. To reduce the size of the surge gallery the connexion between it and the tail-race tunnel is usually throttled down.

Transformer Room and Switchgear

Immunity from air bombing is regarded as a very valuable asset for the more important power stations. When the machine station is placed deep in the rock, that affords effective protection for the machine sets, the transformers, also, are severally installed in rock compartments so that all vital parts of the plant are equally well protected against air attacks. Certain measures have been taken to reduce possibilities of transformer explosions and oil fires. Every transformer and reactor therefore is separated from other units by strong concrete walls in which steel doorways with hinged doors are set (Figs 11, Plate 2). The doorways to the compartments for regulating transformers, which are regarded as more dangerous than other types from the point of view of explosions, are fitted with spring hinges. In addition, each transformer cell is provided with an aperture in the roof that opens automatically if pressure becomes excessive, as in the case of an oil conflagration, for example. Pits are provided under all transformers and reactors to collect oil flowing out if an explosion occurs. The oil has to pass through a thick layer of coarse gravel laid over a grating. Drainage pipes lead from all the oil-pits to a collecting pipe which opens to a shaft blasted in the rock.

The current from the generators flows to low-tension switchgear through bare copper bars laid in specially constructed tunnels. The current from the transformer is transmitted to ground level by cables laid in shafts and tunnels, and then passes through overhead lines to an outdoor high-tension switchgear installation. The shafts are equipped with ladders and platforms to permit inspection of the cables.

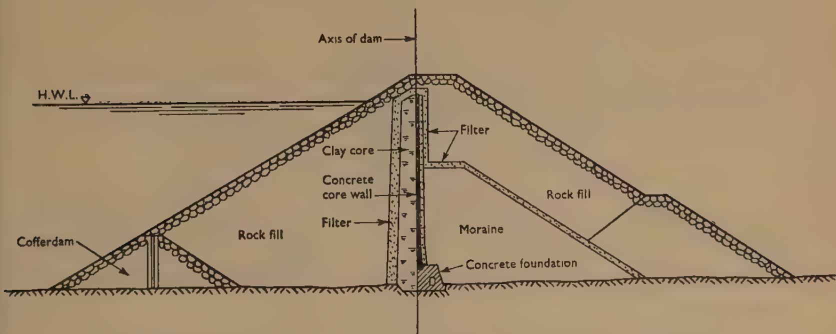
The high-tension switchgear is never installed down in the rock because the large amount of space required for the high-tension circuit-breakers would entail excessive costs for blasting.

Dams

For the majority of the underground stations it has proved economical to construct the dam as a rock-fill dam or as a combination of a rock-fill and an earth-fill dam. For building these dams, the masses of rock

excavated when blasting the tunnels and power station can be usefully employed instead of being tipped into a spoil dump. The type of dam that was most frequently adopted by the State Power Board earlier is characterized by a vertical impervious part in the central section, rounded on each side by supporting masses of earth or rock (*Fig. 12*). The impervious part consists of a core of strongly reinforced concrete which is sufficiently thin to be able to follow the horizontal movements of the dam without cracking; in front of this, a comparatively thin layer of clay or other impervious material is placed. The concrete core is considered to afford an entirely reliable tightening element during the time necessary for the consolidation of the impervious earth. Along the whole length of the dam, the core rests on a concrete foundation which, in addition to providing a support

Fig. 12



CROSS-SECTION OF A ROCK-FILL DAM WITH A CENTRAL CONCRETE CORE

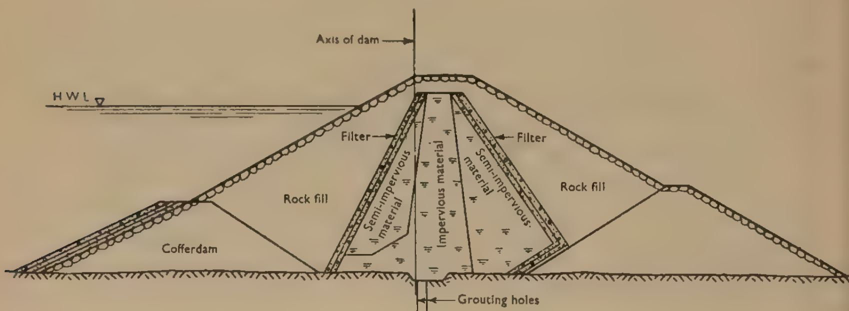
for the core, smooths out deep grooves in the river-bed. The connexion between the foundation and the core takes the form of a joint. Since compacted moraine possesses a high modulus of elasticity, which increases with the pressure, this material affords better support for the concrete core than a rock fill and has therefore been placed behind the core at the lower parts of the dam. Between the layers of fine-grained material and the rock filling, filters are inserted consisting of various layers of increasing grain size to prevent the finer material penetrating into the coarser form.

Under the prevalent climatic conditions, this type of dam offers certain advantages. Both the impervious material and other fine-grained forms of earth contain water and must therefore be placed in the dam during the summer season which lasts for only four months of the year in the northern parts of Sweden. The fine-grained masses are relatively small, however, and can therefore be laid to such a height during the summer season, that the placing of the excavated rock, which is supplied continuously from the tunnels and power station, can be carried on throughout the whole year.

Where ample supplies of impervious material are available within a reasonable distance from the working site, the dams have, during recent years, been constructed with zoned material without a concrete core in accordance with conventional American practice (*Fig. 13*). Zone A consists of impervious material, zone B of semi-pervious material, and zone C of excavated rock. The impervious material in zone A, is moraine with a flat sieve-curve and fine material chiefly coming within the range of silt. Earths of these types may have satisfactory impermeability, whilst at the same time they consolidate relatively quickly and possess good internal friction. The permeability lies at about $1{\cdot}65 \times 10^{-7}$ feet per second or 0·007 inches per hour.

For the compaction of impervious material of this type, American

Fig. 13



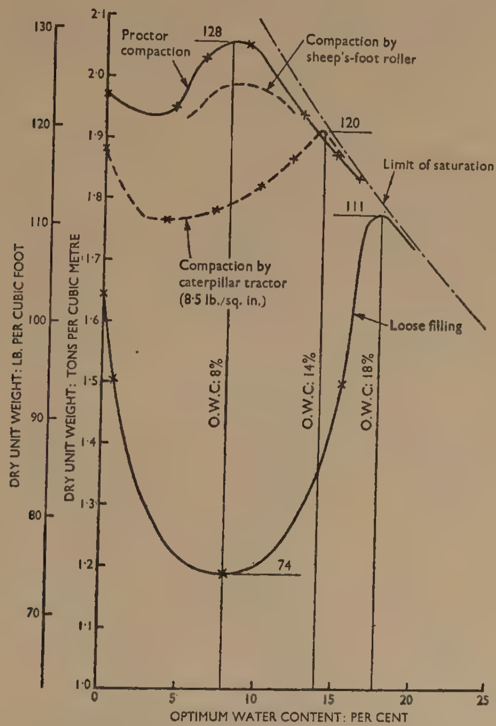
CROSS-SECTION OF A ROCK-FILL DAM WITH A CENTRAL MORAINE CORE

methods, specially devised by Proctor, are customarily employed; the purpose of these is to obtain the maximum possible weight by volume. This is achieved by compacting the earth with sheeps-foot rollers in thin layers and by maintaining a definite water-content, referred to as the optimum. In very large earth dams it has been found preferable to keep the water-content at from 1 to 2 per cent below the optimum in order to eliminate pore pressure entirely.

In Sweden, however, considerable difficulty has been encountered in maintaining the optimum water-content. The difficulty is caused partly by the fact that in many cases the material is too wet from the outset, and results partly from the rains during the summer season. Exhaustive investigations relating to the behaviour of the impervious material with various water-contents have, therefore, been undertaken by the State Power Board's building laboratory. These investigations show that the impervious material available to the Swedish engineers can be placed in the dam with a water-content considerably above the optimum, without appreciably decreasing the weight by volume. Compaction may be carried out satisfactorily by caterpillar tractors. A comparison of different

methods of compaction is illustrated in *Fig. 14.* Furthermore, the high water-content is accompanied by the result that possible variations in permeability are very small compared with those prevailing in a drier form of compaction, as is clearly shown in *Fig. 15.*

Fig. 14

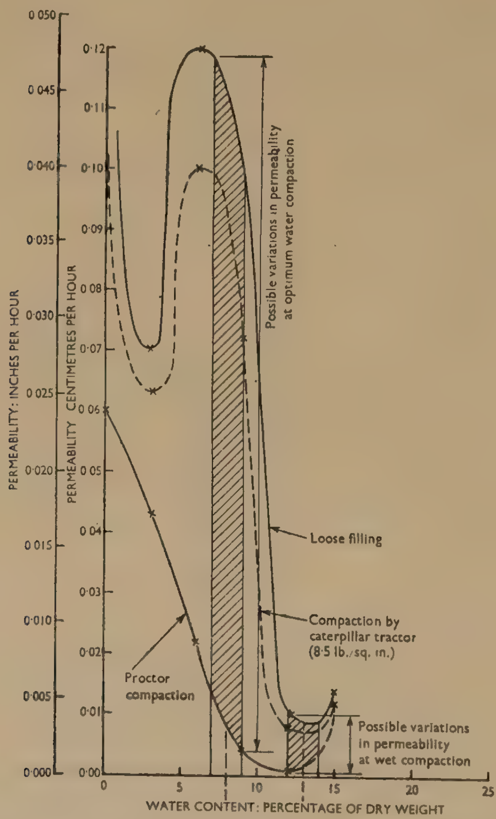


Compaction Curves for a Moraine Material

Another factor that must be taken into account is the settlement, which increases with the water-content. For an earth dam without a concrete core or similar arrangement, this is of no great significance, provided that settlement takes place uniformly. The impervious core should therefore be designed in such a form that it is not held up by the more rigid filling masses surrounding it. With the class of material employed and by constructing the impervious core of a moderate width, settlement takes place so rapidly that it is largely completed during the building period. As a rule, the impervious core is consolidated about 2 or 3 months after placing the overload. *Figs 16* shows how the pore pressure decreased with the time and filling height in a dam recently constructed in Sweden.

As regards the shearing strength of the consolidated material, this will be less with wet packing material than with dry owing to the somewhat lower weight by volume. This difference is of little importance for the

Fig. 15



PERMEABILITY WITH DIFFERENT FORMS OF COMPACTION

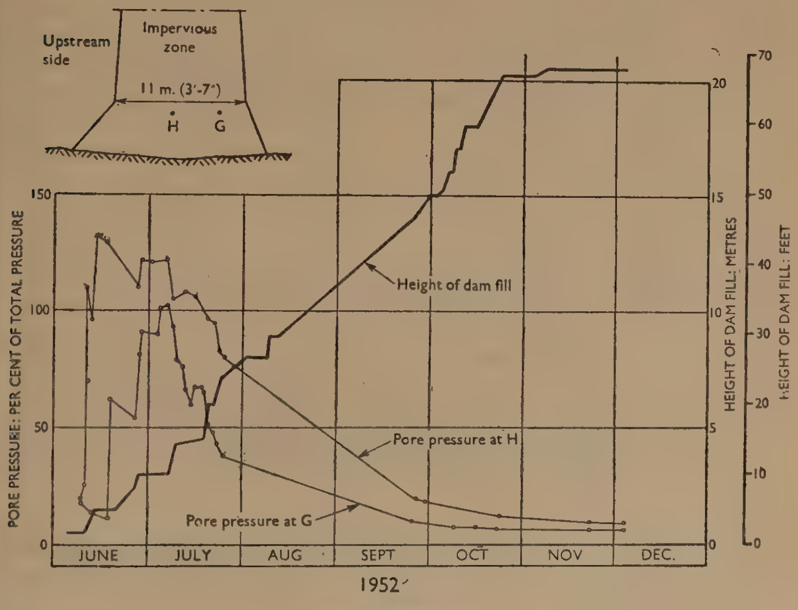
Swedish dams, because a large proportion of the supporting masses usually consists of excavated rock which imparts adequate stability to the dam.

GENERAL PLANNING OF AN UNDERGROUND POWER STATION

In power stations of the Alpine type it would not, of course, be practical to locate the dam and power station in the central part of the fall section since the access tunnel and power outlet would be very long and costly.

Nor would it be economically justifiable to blast out the machine-station premises in unsound rock which requires expensive reinforcement and lining with steel and concrete. On the other hand, for conditions prevailing in Sweden, with medium heads and rock of good quality, it has been found economical in most cases to place the machine station down in the rock. One reason is that, as previously mentioned, an underground station may be located at any desired point between the upstream and downstream end of the fall section. It is thus possible to select a site in such a position

Figs 16

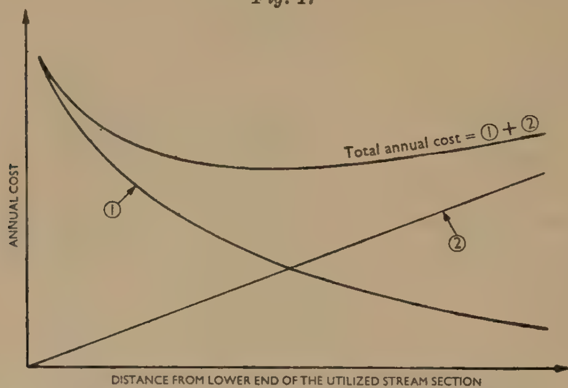


Pore Pressures

that the total annual costs for the dam, coffer dams, diversion of the river during the building period, damage from damming, value of the water storage, log-floating sluices, head-race, intake, penstocks, surge basin, tail-race tunnel, and loss of head will be reduced to a minimum. Since costs for the machine station, the mechanical, and the electrical equipment are practically independent of the power station's position, it is not as a rule necessary to include these costs in the calculation. Owing to the large number of variables, several of which cannot be expressed in mathematical form, it is very difficult to combine all these costs in an equation which by derivation will indicate the position of the cheapest station. The most reliable method of dealing with the problem consists in calculating

the costs for three or four different positions of the power station and setting out the costs in the form of curves, shown diagrammatically in *Fig. 17.* The summation curve indicates the position at which the costs of the station will be at a minimum. If, moreover, the masses of excavated rock obtained when blasting out the power station and tunnels counterbalance the masses required for the dam in this position of the power station, the minimum will be still more apparent. Since the summation curve is very flat in the

Fig. 17



DETERMINATION OF THE POSITION OF AN UNDERGROUND HYDRO-ELECTRIC PLANT

Curve 1. Annual cost of dam, cofferdam, diversion tunnel, log flumes, head-race, intake penstocks, compensation, and value of the storage.
Curve 2. Annual cost of tail-race, surge basin, and losses of head.

vicinity of the minimum point, the station may be shifted for a considerable distance upstream or downstream from this point without exercising any appreciable influence on the costs.

It is by no means certain, however, that the power station can be constructed in the manner described above, or that this type of power station will always prove the most economical. In certain instances, owing to topographical and geological conditions, only one place is available for building the dam so that its position is determined from the outset. In these circumstances it is necessary to investigate which type of dam, machine station, and conduit system will be cheapest in each particular case.

For surface stations, the penstocks have a comparatively flat slope and will therefore be of considerable length. The increased cost for these pipes compared with the vertical or steeply inclined penstocks in an underground station are usually considerably greater than the saving that can be effected by a shorter tail-race tunnel or none at all. The long penstocks

render turbine regulation more difficult and necessitate the use of surge tanks and relief valves which further increase the costs. As a rule, the costs for a machine station placed down in the rock are no greater than when it is located at the surface, since the costs for blasting out the rock are counterbalanced by the lower costs for expensive concrete work which can be appreciably reduced in an underground station by utilizing the rock. When planning an underground station, however, it is necessary to include the costs for an access tunnel to the machine station and for surge basins where long tail-race tunnels are employed.

In a comparison between an underground and a surface station, it should be remembered that maintenance costs for the tunnels and rock premises in an underground station are lower than for corresponding parts in a surface station, for which concrete must be employed in large quantities. This also applies to the costs for depreciation because rock has a far longer life than concrete work. In Sweden annual costs for maintenance and depreciation are reckoned to be 1 per cent of the building costs for rock work, and 1·7 per cent for concrete work.

No general rule exists, however, for determining whether an underground or surface design should be chosen for a station. Nevertheless, it is interesting to note that in every instance where an underground station has been built in Sweden, the choice represents the most economical solution for the station as a whole. The greater security obtained in this manner during a war is, of course, a very welcome advantage but not a determining factor.

The costs for power stations with an underground machine hall which have been built recently, or are at present under construction, amount to £45 per kW. maximum installed capacity where the capacity of the station lies between 100,000 and 300,000 kW. The costs are based on the price levels ruling in 1952 at an exchange rate of £1 = 14·5 Swedish crowns.

Blasting Tunnels and Underground Stations

During the past few decades, considerable progress has been made in the technique of blasting, in the development of machines, and in the means of transport with the help of which large masses of earth and rock can be displaced and removed rapidly and cheaply. These factors have contributed largely towards rendering the underground stations advantageous from an economic point of view.

In Sweden pneumatic pusher drilling machines manufactured by the Atlas Diesel Co., and Sandvik coromant tungsten carbide drills have been finding a constantly increasing use for blasting tunnels and rock premises. Generally speaking, $\frac{7}{8}$-inch drill steels are used, and the type of bit most commonly employed is the chisel bit with a cutting diameter ranging from $1\frac{1}{8}$-inch to $1\frac{5}{8}$-inch. The main difference between a tungsten carbide drill and an ordinary steel drill lies in the fact that, between two regrinding

periods, a 10- to 15-times greater length can be drilled with the tungsten carbide bit. For practical reasons, however, tungsten carbide drills are only made in lengths twice to four times those of steel drills. The greater length of the drills permits the use of a smaller mean diameter for the bit, in order to produce the required bottom diameter in the hole, which in turn increases the drill penetration.

A comparison of the drilling data for drilling with ordinary steel bits, or tungsten carbide bits for blasting granite tunnels of a cross-sectional area of 1,080 square feet is given in Table 2.

TABLE 2

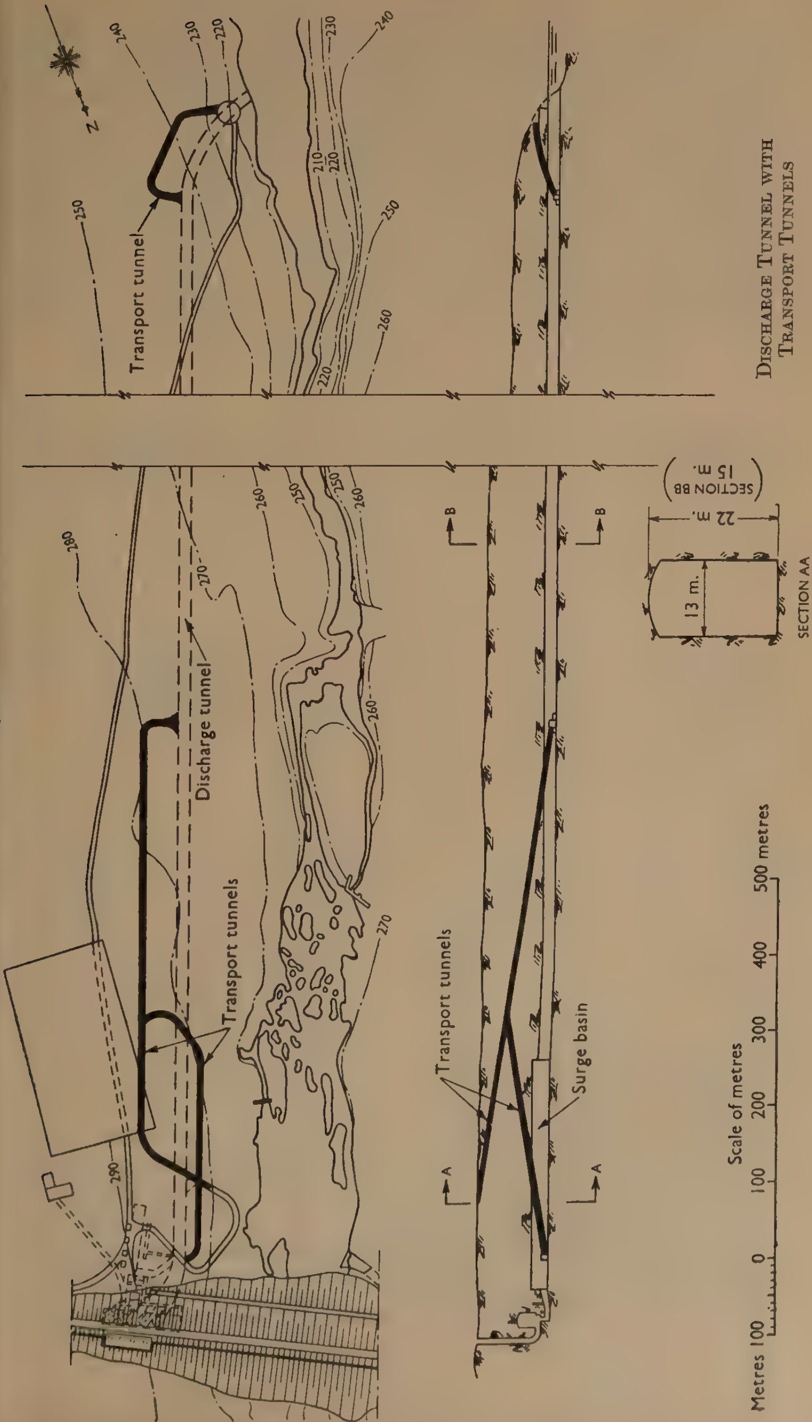
	Unit	Steel bit	Tungsten carbide bit
Area	Square feet	1,080	1,080
Number of men	—	14	11
Number of holes	—	75	75
Total length of holes drilled	Feet	1,300	1,300
Advance	Feet	16·5	16·5
Number of drill changes	—	516	290
Actual drill penetration	Inches per minute	9·2	13·8
Time for drilling a round	Minutes	4,930	3,720
Time for charging a round	Minutes	1,300	1,300
Total time for blasting a round	Minutes	6,230	5,020
Time per foot drilled	Minutes	3·8	2·85

Hoists and rail-track vehicles are now seldom used for transporting the masses of broken rock from the rock premises and tunnels. This work is chiefly carried out by Diesel-driven trucks of very large capacity which transport the rock quickly and smoothly out from the blasting sites to the dam.

When power stations and tunnels are being blasted, special transport tunnels, having a gradient from 1 in 7 to 1 in 8, are first driven from the ground level down to the machine hall and the bottom of the tunnel (*Figs 18*). The entire transport road is usually made sufficiently wide to allow two trucks to pass at any point on it. This permits as many trucks to be driven up to the blasting site as the power shovel is capable of loading. The capacity of the power shovel thus determines the time required for transport, and consequently it is very important to employ power-shovels of the most efficient type.

Tunnels with cross-sections not exceeding 1,350 square feet are blasted by the full-face method. For this purpose light frames are used for drilling and charging. For larger cross-sections, two frames placed side by side are generally employed at each face. The frames are moved up to the face on an open-sided truck. After they have been brought to the correct position they are raised from the truck by lifting jacks placed under each of the four legs of the frame, and the truck is run out.

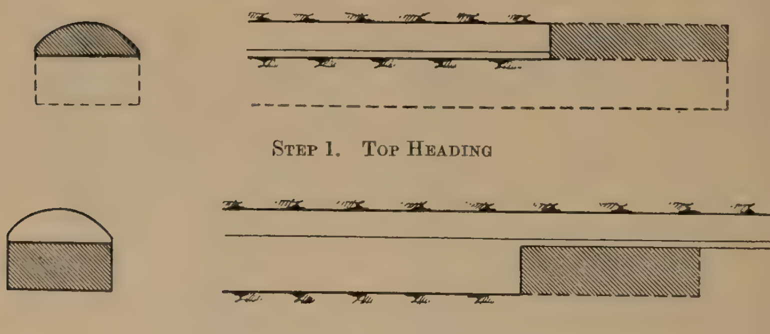
Fig. 18



DISCHARGE TUNNEL WITH TRANSPORT TUNNELS

Tunnels with a cross-section exceeding 1,350 square feet are usually blasted by one of the following methods, according to the quality of the rock. If it is very sound, the bottom heading overhand stoping method is adopted (Figs 19, Plate 2). In this method, the lower part of the tunnel is first blasted for its entire length by the full-face system. The upper part —the overhand stope—is then shot down and the muck heap is employed progressively for setting up the drilling machines. It is therefore desirable, for the bottom heading, to select a cross-section of such a size that the muck heap from the overhand stope will fill the tunnel to a height suitable for drilling and scaling down the roof. From results of experience in Sweden, the bottom heading should constitute 47 per cent of the tunnel cross-section.

Figs 20



STEP 1. TOP HEADING

STEP 2. UNDER-HAND STOPING

When the overhand stope is being blasted, removal of the broken rock need not synchronize with the drilling and blasting operations, which appreciably simplifies the work and reduces the working time. The advance per round in a tunnel of large cross-section is usually 17–20 feet for the bottom heading, and 30–33 feet for the overhand stope. The disadvantages of this method are that it necessitates careful scaling down of two roofs, and the size of the rock fragments from the overhand stope is considerable. The latter fact renders mucking more difficult; the largest boulders must be blasted into smaller fragments before they can be employed for the dam filling.

Where the quality of the rock is less satisfactory, top-heading underhand stoping is adopted (*Figs 20*). This method is characterized by blasting first and clearing the top heading throughout its full length, followed by the firing of the bottom part, the underhand stope. The dimensions of the top heading are determined by the mucking capacity or the desired height of the bench. The underhand stope is either drilled with horizontal holes, in which case the operators stand on drilling frames,

or with vertical holes, for which the drillers stand on the bench, and in this case, wagon drills may suitably be employed. The chief advantages from this method are that the size of the top heading is independent of the tunnel cross-section, and the tunnel roof may be constructed in the form suited to the rock. When the underhand stope is drilled vertically, drilling can take place independently of mucking, so that the latter can be carried on continuously. The size of the fragmentation in the underhand stope may be varied more easily, to aid in the mucking arrangements and the dam filling material. The disadvantages of the method are that the water and air lines and electric cables must be laid twice and two roads must be constructed. A tunnel completed after blasting is shown in *Fig. 21* (facing p. 329).

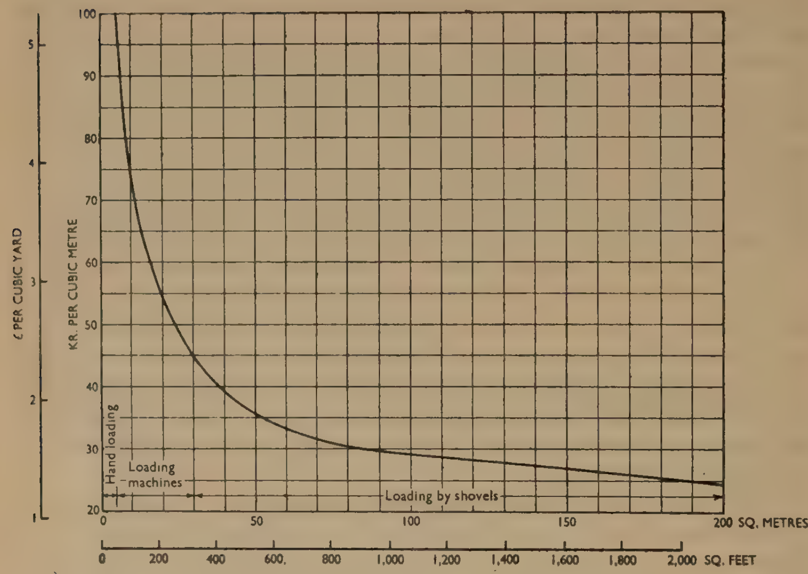
As a rule, the tunnels are blasted from two or more faces. The work proceeds in three shifts, the first for drilling and charging, the second and third for ventilation, and for mucking and transport of the broken rock. Assembling is carried out by bulldozers and transport by Euclid trucks. Loading is effected by power-shovels which, for a tunnel section of 1,080 square feet, have a capacity of 3 cubic yards. The costs for blasting tunnels of different cross-sections, according to the 1952 price levels, may be seen from *Fig. 22*.

The blasting of machine stations may be carried out in different ways, which depend entirely upon the type of the station, the nature of the ground, and the possibility of obtaining suitable points of attack. The blasting of a machine station at a considerable depth with a vertical transport shaft is illustrated in *Figs 23*. Operations are initiated by blasting out the draft tubes, beginning at the tail-race tunnel, after which the other premises are blasted in the sequence indicated by the figures in the illustration. A vertical raise is driven from each draft tube up to the roof of the machine hall. The machine hall is excavated from these raises, beginning at the roof. The rock masses are drawn by a scraper up to the raises from which they are allowed to fall to the bottom of the draft tubes. From here the rock is loaded directly on to Euclid trucks by a power-shovel and is run out to the dam. After the machine hall has been blasted out to a height providing the necessary working space, a pause is made in the blasting operations to allow the placing of the roof-arch or other reinforcement of the rock roof to be undertaken.

If the rock is of sound quality, the machine station is blasted out at two levels simultaneously, beginning at the roof and at half the height of the machine hall. In this way much time is saved, but the unsupported part of the rock between the two levels must be blasted with extreme care.

An example of a machine station blasted at a shallow depth and provided with an access tunnel from ground level is given in *Figs 24*. In this case also the blasting of the various premises takes place in the order shown by the numerals in the illustration. In order to blast out the roof portion and place the roof-arch, a special transport tunnel is driven from the ground level to the upper part of the machine hall. The rock masses in the

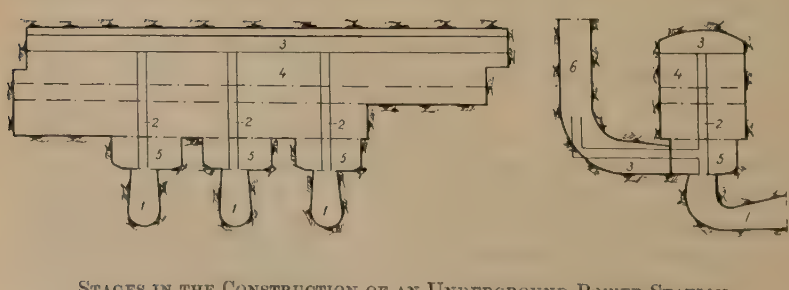
Fig. 22



TUNNELLING COST, BASED ON 1952 PRICE LEVELS, PLOTTED AGAINST AREA OF CROSS-SECTION

The costs include drilling, blasting, ventilation, mucking, transport by trucks (about 2 kilometres), dumping, drainage, road construction and maintenance, electricity supply, administration, and general costs, but not transport tunnels, ventilation shafts, and concrete lining.

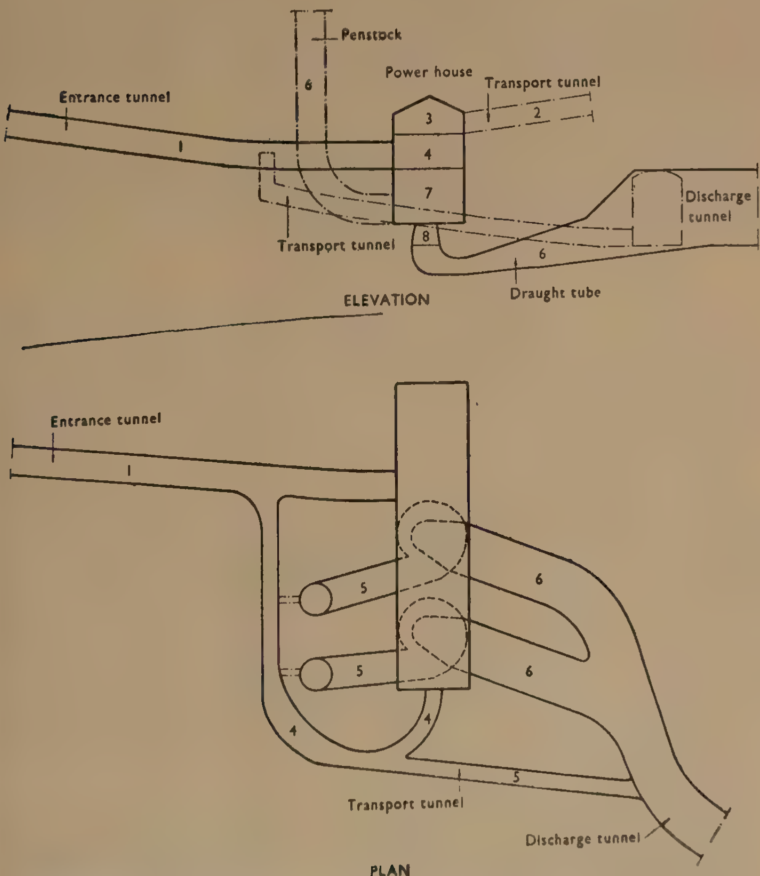
Figs 23



STAGES IN THE CONSTRUCTION OF AN UNDERGROUND POWER STATION WITH A VERTICAL TRANSPORT SHAFT

machine hall between the floor level and the roof are taken out through the permanent access tunnel down to the floor of the machine hall. The material from the bottom level of the machine hall, from the draft tubes, and from part of the tail-race tunnel, is removed through a transport tunnel

Figs 24



STAGES IN THE EXCAVATION OF AN UNDERGROUND POWER STATION WITH ENTRANCE TUNNEL

that opens into the permanent access tunnel. In addition, the penstocks can also be blasted from the transport tunnel. This blasting plan can likewise be applied with some slight modifications to a station located at a greater depth, provided that an access tunnel from the ground level is available. A machine station after blasting is shown in *Fig. 25* (facing p. 329).

When blasting the shafts for the penstocks, a vertical raise is first driven in the centre of the shaft. The shaft is then excavated to the full size. When excavating, the rock masses are allowed to fall down through the raises and are loaded on to Euclid trucks by power shovels. In one power station recently completed, the vertical raises were drilled and charged from a suspended drilling stage operated by a winch set up at ground level. The wire from the winch to the stage passed through a hole in the rock which was drilled with a percussion drill. The diameter of the raises was about 8 feet. The procedure adopted in working is shown in Figs 26, Plate 2.

Work can be carried on continuously all the year round in tunnels and machine stations irrespective of weather conditions. This is a factor of great economic significance, particularly in places where the winters are long and cold.

Operational Experience

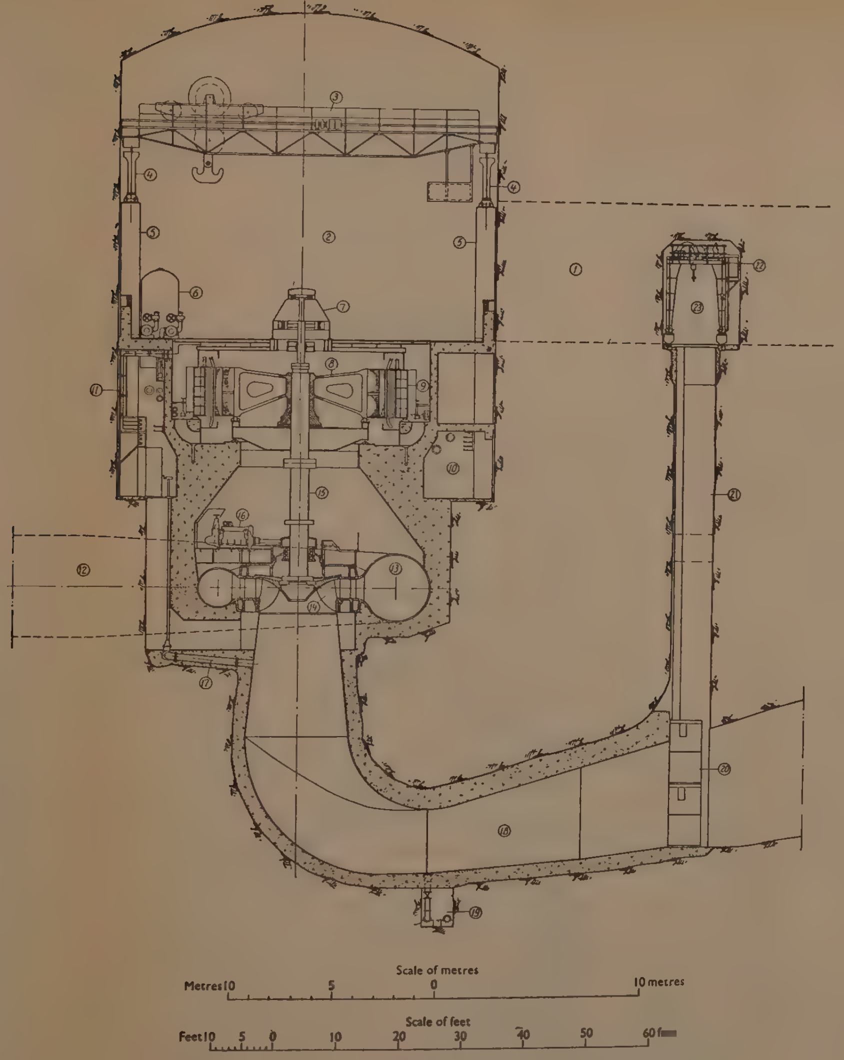
The attendance of an underground station does not differ essentially from that of a surface station. There are no direct disadvantages in the operation and maintenance of an underground station in comparison with a surface station.

In a large power station, two operators are normally on duty at each shift, irrespective of whether the station is above or below ground. One man remains continuously in the control room, whilst the other supervises the turbines, generators, auxiliary machinery, the dam, etc. In cases where the control room is located on the surface and the machine hall is below ground, the latter is not always manned continuously. If the station contains a small number of units, occasional inspections by one of the operators are considered to meet the requirements. In stations containing a large number of units, it is the normal practice to keep one man on duty in the machine hall continuously, even when the station is located above ground.

No staff troubles have been observed. Obviously, an operator working in an underground station is unable to see the sun during his shift. Apart from this, however, there appears to be no difference between working in an underground station and a surface station. One noteworthy advantage of the underground station is the greater constancy of the climatic conditions. It is conceivable that psychological factors may exercise some influence, but no such difficulties have arisen in connexion with the staff to date.

The Lecture was illustrated by thirty-one lantern slides, from some of which the half-tone page plates, folding Plates 1 and 2, and the Figures and Tables in the printed text have been prepared.

FIG. 7

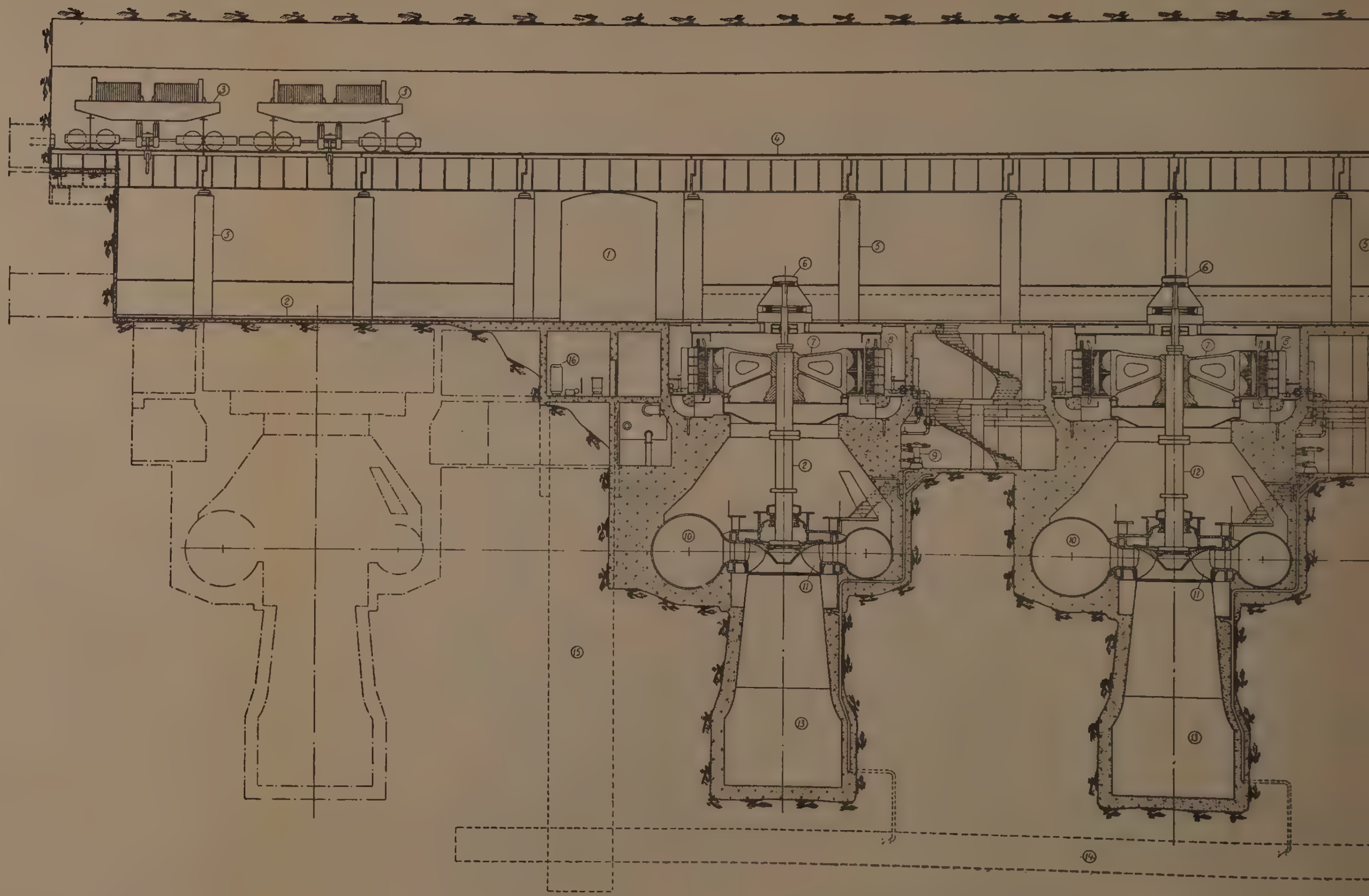


CROSS-SECTION OF AN UNDERGROUND MACHINE STATION

KEY

1. Entrance tunnel
2. Machine hall
3. Travelling crane
4. Crane girders
5. Concrete columns
6. Pressure oil device
7. Exciter
8. Alternator
9. Air coolers
10. Cables and cooling-water pipes
11. Bus-bars
12. Penstock
13. Spiral casing
14. Turbine
15. Turbine shaft
16. Servo-motor
17. Drainage tube for penstock
18. Draught tube
19. Drainage tunnel
20. Gate for closing the draught tube
21. Gate shaft
22. Movable trestle crane
23. Tunnel for the trestle crane

FIG. 8



LONGITUDINAL SECTION OF AN UNDERGROUND MA

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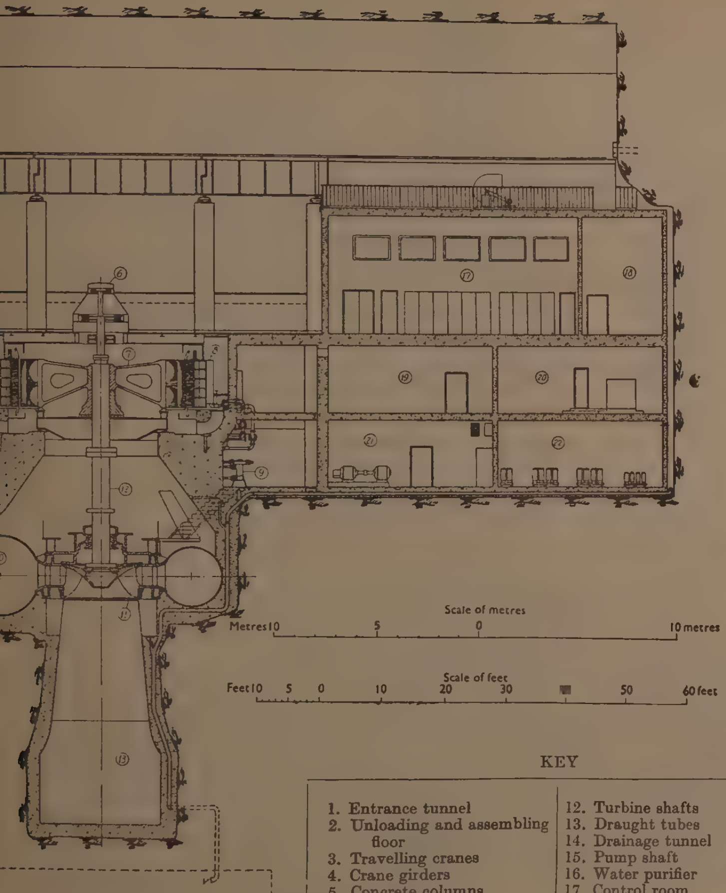
TATIONS

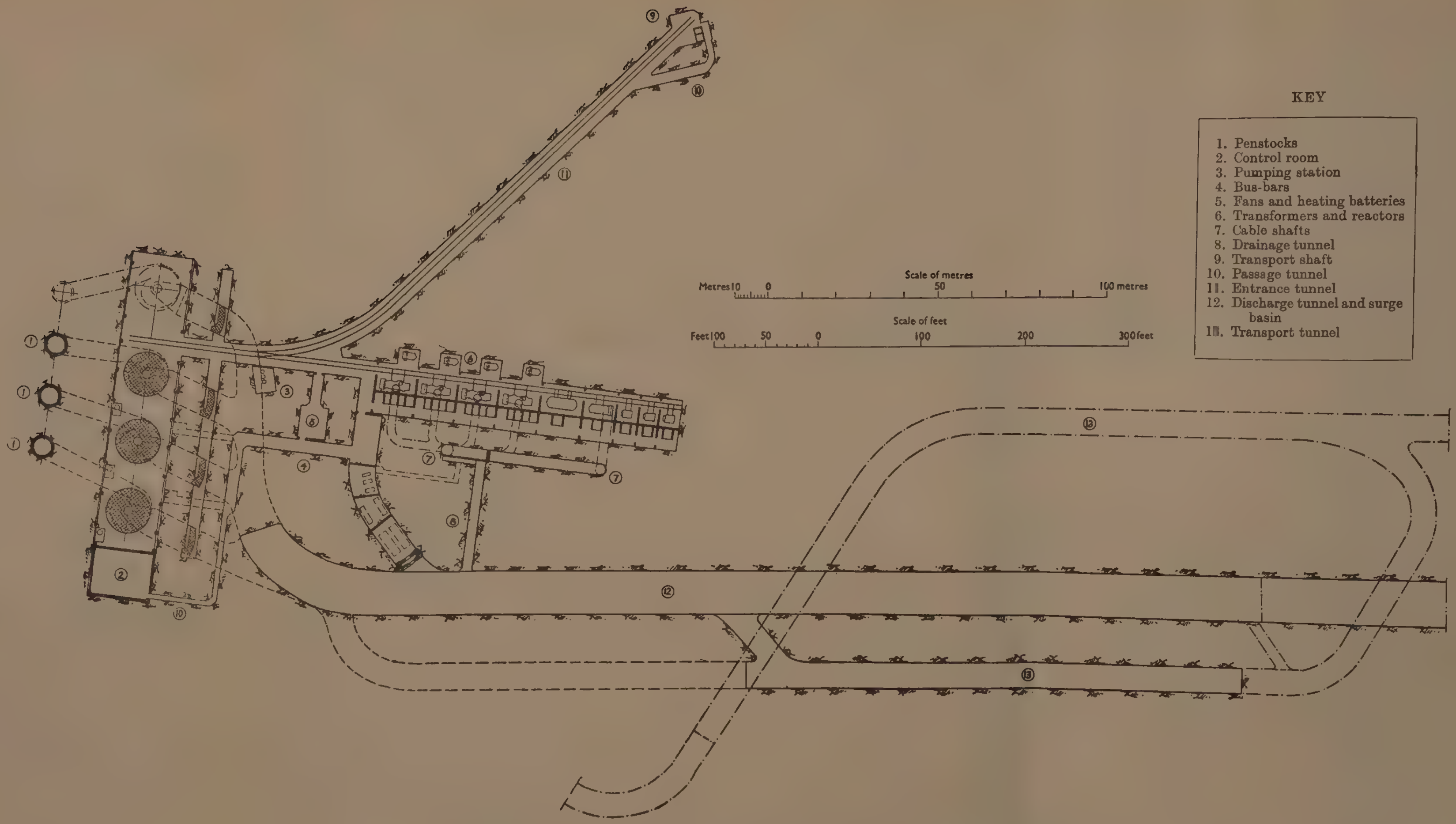


KEY

1. Entrance tunnel
2. Unloading and assembling floor
3. Travelling cranes
4. Crane girders
5. Concrete columns
6. Exciters
7. Alternators
8. Air coolers
9. Oil coolers
10. Spiral casings
11. Turbines
12. Turbine shafts
13. Draught tubes
14. Drainage tunnel
15. Pump shaft
16. Water purifier
17. Control room
18. Relay room
19. Workshop
20. Room for connexion boxes
21. Room for converters
22. Battery room

ATION

FIG. 10



KEY

1. Penstocks
2. Control room
3. Pumping station
4. Bus-bars
5. Fans and heating batteries
6. Transformers and reactors
7. Cable shafts
8. Drainage tunnel
9. Transport shaft
10. Passage tunnel
11. Entrance tunnel
12. Discharge tunnel and surge basin
13. Transport tunnel

SECTIONAL PLAN OF AN UNDERGROUND POWER STATION

J. F. HAGRUP

HYDRAULICS ENGINEERING DIVISION MEETING

26 January, 1954

Sir Claude Inglis, Member, Chairman of the Divisional Board, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Hydraulics Paper No. 3

"Rainfall, Run-off, and Storage: Elan and Claerwen Gathering Grounds"

by

Charles Arthur Risbridger, B.Sc.(Eng.), M.I.C.E., and William Henry Godfrey

SYNOPSIS

The Paper analyses certain hydrological features of a catchment area of 45,562 acres, most of the records being expressed in terms of the long-average rainfall so that they can readily be applied to other catchments.

The "rainfall" statistics are given in various forms for 63 years. The method of computing the long-average annual and monthly rainfalls, the treatment of observers' returns and the detection of faulty records are described.

Monthly records of "run-off" are given for the whole catchment for 42 years, and for about 50 per cent of the area, independently measured, for 23 years, and these are related to the corresponding rainfall. The proportion of monthly rainfall likely to run-off under normal conditions is indicated and a graph shows the probable run-off from any annual rainfall.

A curve of "reliability of run-off," evolved on lines similar to Dr Glasspoole's curve of "reliability of rainfall," shows probable extremes of run-off in any consecutive period up to 36 months' duration and demonstrates the storage required to equalize the run-off over the driest 3 consecutive years.

Records of floods and droughts are included and a mass-yield diagram shows the effect on storage of the prolonged drought of 1933–34.

A section is devoted to loss by "evaporation and absorption," whilst another assesses the reliable yield of the catchment and relates the available storage to that yield.

INTRODUCTION

With the completion of the Claerwen Reservoir, the development of the Elan and Claerwen gathering grounds is also complete.

E. L. and W. L. Mansergh [1] wrote of the design and construction of the works in the Elan valley. Morgan, Scott, Walton, and Falkiner [2] have

[1] The references are given on p. 388.

written of the design and construction of the Claerwen Dam. This Paper, dealing with the hydrology of the catchment, is submitted to complete the story.

Records of monthly values of the rainfall on the area now extend over a period of 66 years (1887–1952) and reliable figures of run-off from the whole catchment are available for 45 years (1908–52).

During the 23 years from 1927 to 1949, the run-off from about 50 per cent of the area has also been obtained by gauging independently the flow of the unreservoired River Claerwen.

Many of the records and diagrams which accompany the Paper are dealt with as percentages, in order to facilitate comparison with, and prediction of, statistics for other similar areas, and the subject matter is divided into nine main sections—details of gathering ground; rainfall; run-off; loss by evaporation and absorption; rainfall, run-off, and loss on

Fig. 1

Elan and Claerwen Catchments

23,560 acres of River Claerwen above Dol-y-Mynach Dam; comparison of records for whole area (45,562 acres) with those of River Claerwen (23,560 acres) for 23 years (1927–49); summary of discharges in millions of gallons per day for 42 years (1908–49); impounding reservoirs, reliable yield, etc; and conclusions.

Throughout the Paper, "Caban" refers to the whole drainage area of 45,562 acres of the rivers Elan and Claerwen above the Caban Coch Dam, whilst "Dol-y-Mynach" refers to the 23,560 acres of the river Claerwen above the Dol-y-Mynach Dam included in "Caban."

Gathering Ground

The gathering ground (*Fig. 1*) comprises the watersheds of the river Elan and its tributary, the Claerwen, above the Caban Coch Dam. The Elan joins the Wye about 3½ miles below the dam and about 1½ mile south of Rhayader in the County of Radnorshire. General details of the location are:—

Area.—45,562 acres (71·2 square miles) in the counties of Radnor, Brecknock, Cardigan, and Montgomery.

Altitude.—Varies from 700 feet above Ordnance Datum at the base of Caban Dam to 2,115 feet A.O.D. to the south. The average is 1,350 feet A.O.D.

Geology.—Rocks of Lower Silurian order, with slates, grits, and conglomerates. The terrain is steep with little soil cover and fairly extensive areas of peat bogs. The fetch, from the source of the River Elan to Caban Dam, is 10 miles.

Rainfall

Rainfall Stations

By a fortunate circumstance, rainfall records at Nantgwillt in the Elan valley, about 2 miles west of the site of the Caban Dam, had been kept since 1870, about 20 years before Birmingham seriously considered the area as a source of supply.

Four additional rain gauges were set up in 1891, at widely separated points and different altitudes, under the expert guidance of F. J. Symons, the originator of the British Rainfall Organization.

When the water-supply scheme was submitted to Parliament in 1892, James Mansergh, from the records of these five gauges, estimated the long-average annual rainfall of the whole catchment to be 69 inches, which is in remarkably close agreement with the figure of 69·4 inches now adopted after the establishment of many more gauges within the area, and an intensive study and analysis of the rainfall records of about two hundred gauges throughout Wales and the adjoining counties.

The number of rainfall stations within the catchment area was increased from five to sixteen in 1908, the new sites being selected by Dr H. R. Mill, then Director of the British Rainfall Organization.

Subsequent additions brought the number of recording stations up to thirty-two (eight daily and twenty-four monthly), which is closely in agreement with the standard required to secure adequate representation of rainfall on such an area as recommended in the Report " Determining the General Rainfall over any Area." [3]

The rainfall stations were inspected in 1921 by Carle Salter, then Superintendent of the British Rainfall Organization and, in 1937, the majority were visited by Dr J. Glasspoole of the Meteorological Office. All gauging sites were brought into conformity with the standard recommended by the Meteorological Office.

In the report following his inspection, Dr Glasspoole stated that " the length of the walk of certain of the Observers and the treacherous nature of the ground were comparable with the severest conditions experienced by other Observers in the British Isles."

Effect of Turf Walls

The standardization of turf walls around many of the gauges carried out in 1937 did much to eliminate occasional inconsistencies hitherto apparent, which later proved to have been due to variations in the direction and strength of the wind. The more consistent rainfall figures obtained thereafter led to the correction of the individual long-average rainfall for certain of the gauges and amply demonstrated the impoıtance of frequent inspection and overhaul of all rainfall stations.

Between 1939 and 1945, in consequence of staff difficulties and war conditions, this work was perforce somewhat neglected, particularly in so far as the remote mountain gauges were concerned. By the latter date, serious inconsistences in the monthly readings of some of these gauges were, at times, apparent and necessitated adjustments to the recorded readings.

A thorough inspection in 1946 disclosed many faults, such as settlement or disintegration of turf walls—often on the side of the prevailing winds—looseness of fit of gauge in the ground, departure of the gauge from the perpendicular, and variation of height of gauge lip above ground from 9 inches to 1 foot 3 inches, instead of the standard 1 foot.

The overhauls carried out following this inspection had immediate repercussions on the records which thereafter bore an even more consistent relationship one with another.

Inspections and overhauls of all stations have since been carried out at intervals of not more than 2 years, with the result that the necessity to adjust the recorded readings has almost disappeared, except where drifting snow has fallen or obvious interference has occurred.

Assessment of Long-Average Annual Rainfall

The long-average annual rainfall over the catchment area was computed by A. A. Barnes, the late Chief Engineer to the City of Birmingham Water Department. The standard period adopted for this purpose was the

44 years from 1887 to 1930 recommended in his Paper [4] to the Royal Meteorological Society.

The reason for the adoption of that particular period was that it had been found that, for the whole of England and Wales, the accumulated deficiency of the rainfall of the 22 dry years from 1887 to 1908 was almost exactly balanced by the accumulated excess of that of the 22 wet years from 1909 to 1930.

It will be seen later that, in the case of the Elan valley, a remarkable confirmation of this sequence resulted, the change actually occurring at the end of 1909.

The Authors realize that the choice of any particular short term of years on which to base an assessment of the long-average rainfall is open to question, for it presupposes the existence of cycles of rainfall. Obviously the true long-average rainfall is the average of the longest possible period over which rainfall records can be obtained, but, since general climatic conditions do not change suddenly, given a reasonably long period of years (44 in this case, of which half were generally "wet" and half generally "dry") the resulting average cannot be far from the true figure.

In computing the long-average annual rainfall over the catchment area, the records of about two hundred gauges in Wales and the neighbouring counties of Shropshire and Herefordshire, where readings were available during the whole or part of the 44-year period, were critically examined.

Stations where readings were available for each of the 44 years were used as a standard, the mean annual rainfall representing, of course, the long-average at that gauge. The percentage relation of each year's rainfall to that long-average was evaluated, and was also accumulated throughout the 44 years.

Sets of outline maps were prepared and isopercentals drawn for (*a*) each individual year, and (*b*) the accumulated percentage at the end of each year.

Records of shorter duration were then incorporated and, by the application of the appropriate accumulated percentage figures from the adjacent standard gauges, the history of the whole area covered by the two hundred stations was established and the long-average annual rainfall for all stations ascertained.

By this process it was possible to determine, with considerable accuracy, the percentage relation of rainfall over the catchment area for each year, and consequently to establish the long-average at each station.

From the individual long-averages a rainfall map (*Fig. 2*) was compiled and the long-average annual rainfall figure of 69·4 inches for the whole gathering ground was determined by the cartographical method.

Table 1 gives the annual rainfall for each year with the accumulated deficiency or excess which, plotted as a residual mass curve in *Fig. 3*, effectively demonstrates the existence of the 23-year dry period followed by the 21-year wet period, although it must not be inferred from that

Fig. 2

LONG-AVERAGE RAINFALL, 1887–1930, EXPRESSED IN INCHES

statement that the Authors are of the opinion that such a cycle will be repeated.

TABLE 1.—RAINFALL AND RESIDUAL MASS

Year	Rainfall: inches	Departure from long-average of 69·4 inches	Accumulated departure from long-average: inches	Year	Rainfall: inches	Departure from long-average of 69·4 inches	Accumulated departure from long-average: inches
—	—	—	0	1919	67·2	− 2·2	−75·6
1887	50·2	−19·2	− 19·2	1920	76·6	+ 7·2	−68·4
1888	65·8	− 3·6	− 22·8	1921	59·2	−10·2	−78·6
1889	60·4	− 9·0	− 31·8	1922	67·7	− 1·7	−80·3
1890	63·2	− 6·2	− 38·0	1923	90·7	+21·3	−59·0
1891	80·2	+10·8	− 27·2	1924	79·9	+10·5	−48·5
1892	56·3	−13·1	− 40·3	1925	74·7	+ 5·3	−43·2
1893	64·1	− 5·3	− 45·6	1926	69·4	0	−43·2
1894	77·8	+ 8·4	− 37·2	1927	83·2	+13·8	−29·4
1895	59·0	−10·4	− 47·6	1928	84·7	+15·3	−14·1
1896	65·8	− 3·6	− 51·2	1929	69·3	− 0·1	−14·2
1897	68·1	− 1·3	− 52·5	1930	83·6	+14·2	0
1898	61·9	− 7·5	− 60·0				
1899	67·0	− 2·4	− 62·4	1931	72·9	+ 3·5	+ 3·5
1900	67·4	− 2·0	− 64·4	1932	70·8	+ 1·4	+ 4·9
1901	60·4	− 9·0	− 73·4	1933	49·6	−19·8	−14·9
1902	55·2	−14·2	− 87·6	1934	69·6	+ 0·2	−14·7
1903	92·0	+22·6	− 65·0	1935	77·4	+ 8·0	− 6·7
1904	55·2	−14·2	− 79·2	1936	73·2	+ 3·8	− 2·9
1905	57·8	−11·6	− 90·8	1937	61·7	− 7·7	−10·6
1906	71·6	+ 2·2	− 88·6	1938	76·2	+ 6·8	− 3·8
1907	65·3	− 4·1	− 92·7	1939	77·1	+ 7·7	+ 3·9
1908	60·9	− 8·5	−101·2	1940	68·3	− 1·1	+ 2·8
1909	63·8	− 5·6	−106·8	1941	61·5	− 7·9	− 5·1
1910	82·3	+12·9	− 93·9	1942	67·8	− 1·6	− 6·7
1911	64·4	− 5·0	− 98·9	1943	70·6	+ 1·2	− 5·5
1912	78·2	+ 8·8	− 90·1	1944	69·0	− 0·4	− 5·9
1913	72·0	+ 2·6	− 87·5	1945	59·8	− 9·6	−16·5
1914	75·6	+ 6·2	− 81·3	1946	83·7	+14·3	− 2·2
1915	68·7	− 0·7	− 82·0	1947	62·3	− 7·1	− 9·3
1916	73·6	+ 4·2	− 77·8	1948	79·5	+10·1	+ 0·8
1917	64·8	− 4·6	− 82·4	1949	65·4	− 4·0	− 3·2
1918	78·4	+ 9·0	− 73·4				

Table 1 and *Fig. 3* show that by the end of 1909, the deficiency in rainfall amounted to 106·8 inches or 154 per cent of the long-average annual rainfall, equivalent to 4·65 inches or 6·7 per cent per annum. By 1930 this deficiency had been made good by the accumulated excess rainfall.

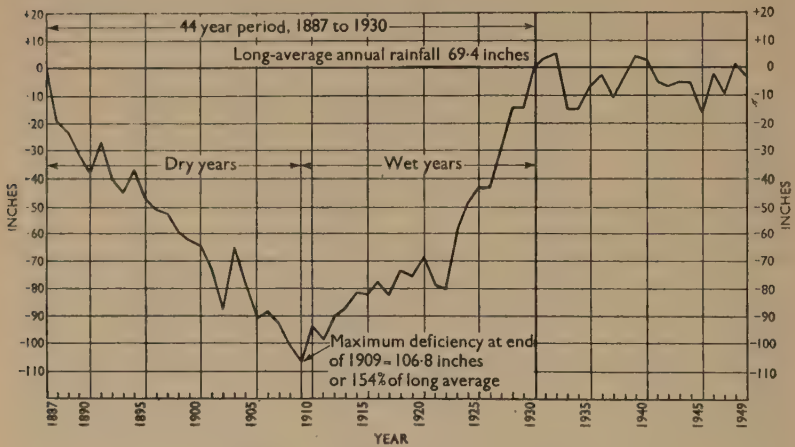
The fact that, for the 19 years from 1931 to 1949, the curve has closely approximated to the adopted long-average rainfall tends to verify the correctness of the latter.

More recently an assessment has been made of the monthly values of

rainfall on the area back to 1887, in order to determine the long-average monthly rainfall.

Table 2 shows that the figures compare satisfactorily with the standard long-average figures for the whole of Wales given by the Meteorological Office.

Fig. 3



RAINFALL—RESIDUAL MASS CURVE

TABLE 2.—MONTHLY AND YEARLY LONG-AVERAGE RAINFALL

Month	Mean of 44 years, 1887–1930. Adopted long-averages : inches	Mean of 35 years, 1891–1925. Suggested short period : inches	Mean of 63 years, 1887–1949. Whole period of records : inches	Monthly rainfall : per cent of year			
				Met. Office, all Wales	Adopted 44-year period	Short 35-year period	Whole 63-year period
Jan.	7·2	7·0	7·6	9·4	10·4	10·1	11·0
Feb.	5·3	5·6	5·4	7·8	7·6	8·1	7·8
March	5·5	5·7	5·2	7·6	7·9	8·2	7·5
April	4·3	4·5	4·3	5·9	6·2	6·5	6·2
May	3·8	3·9	3·9	5·9	5·5	5·6	5·6
June	3·8	3·8	3·9	6·1	5·5	5·5	5·6
July	5·1	4·9	5·2	7·2	7·4	7·1	7·5
Aug.	6·8	6·6	6·1	9·4	9·8	9·5	8·8
Sept.	4·9	5·0	5·0	7·0	7·0	7·2	7·2
Oct.	7·3	7·3	7·3	11·2	10·5	10·5	10·5
Nov.	7·1	6·3	7·3	10·5	10·2	9·1	10·5
Dec.	8·3	8·7	8·2	12·0	12·0	12·6	11·8
	69·4	69·3	69·4	100·0	100·0	100·0	100·0

The 35-year period (1891–1925), referred to by Barnes,[4] and recommended as a standard period in the interim report of the Hydrological Committee of the Institution of Water Engineers published in 1936, gives a long-average rainfall of 69·3 inches for this area, whilst the monthly figures for that period are also reasonably satisfactory.

TABLE 3

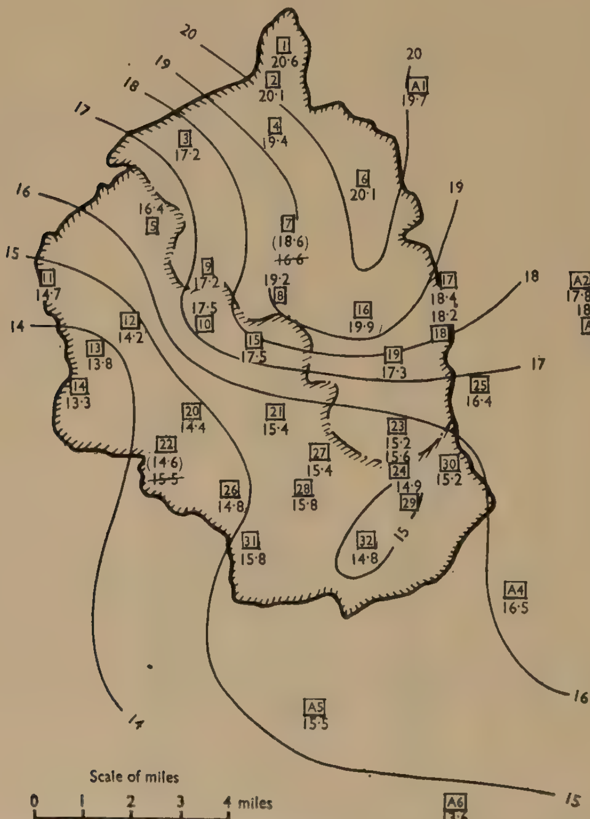
Gauge No.	Daily or monthly	Height above O.D: feet	Annual long-average: inches	Rainfall, Dec. 1949: inches	Percentage of long-average
1	M	1,375	69·87	14·39	20·6
2	D	1,250	68·37	13·74	20·1
3	M	1,490	67·62	11·65	17·2
4	D	1,150	68·91	13·38	19·4
5	M	1,700	84·00	13·80	16·4
6	M	1,070	66·15	13·30	20·1
7	M	1,490	72·10	12·00 (*13·40)	16·6 (*18·6)
8	M	1,240	68·65	13·15	19·2
9	M	1,710	76·10	13·05	17·2
10	M	1,470	76·30	13·35	17·5
11	M	1,470	72·61	10·70	14·7
12	M	1,250	68·81	9·80	14·2
13	M	1,585	69·53	9·60	13·8
14	M	1,500	69·21	9·20	13·3
15	M	1,500	68·76	12·00	17·5
16	D	1,020	63·92	12·74	19·9
17	M	1,500	56·10	10·33	18·4
18	M	1,540	56·47	10·29	18·2
19	M	1,006	64·39	11·13	17·3
20	M	1,590	72·41	10·45	14·4
21	M	1,050	64·85	10·00	15·4
22	M	1,500	72·80	11·30 (*10·60)	15·5 (*14·6)
23	D	840	60·53	9·22	15·2
24	D	840	60·50	9·44	15·6
25	D	700	56·14	9·23	16·4
26	M	1,775	77·56	11·50	14·8
27	D	1,045	63·80	9·83	15·4
28	M	1,545	74·71	11·80	15·8
29	D	832	61·10	9·11	14·9
30	M	1,320	62·62	9·50	15·2
31	M	1,775	82·38	13·00	15·8
32	M	1,560	79·22	11·75	14·8
Records supplied by Meteorological Office					
A1	M	870	50·80	10·00	19·7
A2	D	757	45·74	8·15	17·8
A3	D	700	48·16	8·85	18·4
A4	M	960	57·70	9·50	16·5
A5	M	1,000	76·68	11·90	15·5
A6	M	750	58·30	7·95	13·6

* Figures in brackets are amended totals following plotting.

Dealing with the Records as Received Each Month

The long-average annual rainfall at each station having been established, the check on the accuracy of the records each month, as received, is rendered comparatively easy by the preparation of a map showing the rainfall at each station as a percentage of the long-average figure for that station. An example taken at random is given in Table 3 relating to December 1949.

Fig. 4



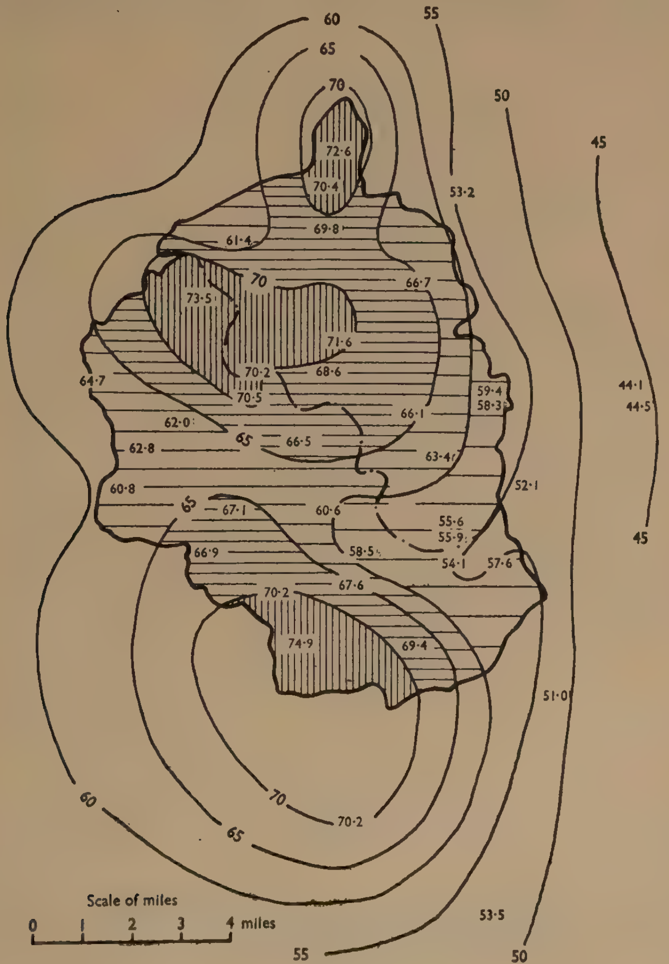
RAINFALL, DECEMBER 1949, EXPRESSED AS A PERCENTAGE OF THE LONG-AVERAGE

The percentage figures in the last column of Table 3 are plotted on an outline map and isopercentals drawn (*Fig. 4*).

It will be noted from Table 3 and *Fig. 4* that adjustments have been made to the records of two gauges (Nos 7 and 22). The need for these

adjustments is obvious from an examination of the diagram, for iso-percentals are much more regular than isohyetals and enable necessary corrections to be made with a very good assurance of accuracy.

Fig. 5



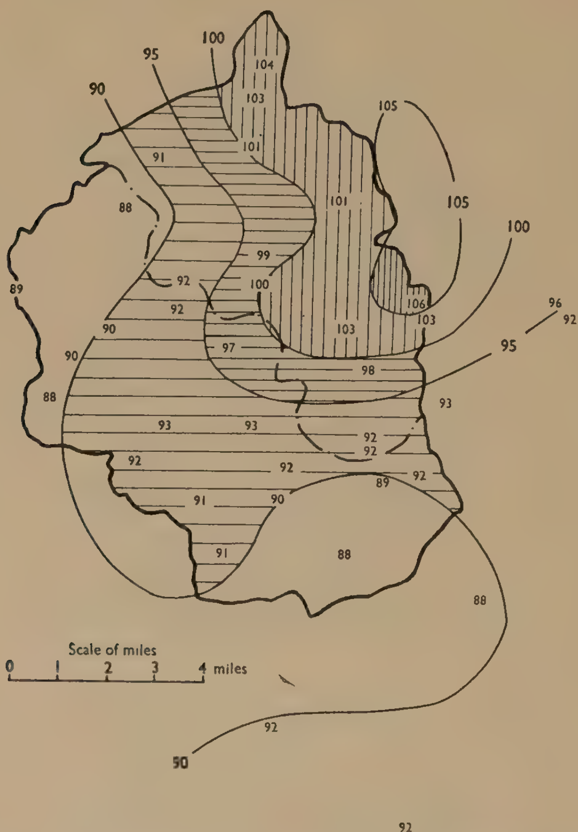
Rainfall, 1949, expressed in Inches

Frequently, when any individual record diverges seriously from the general trend indicated by the isopercentals, immediate steps taken to verify the accuracy of the record by an inspection of the station will reveal the cause of the divergence.

The records are furnished to the Meteorological Office monthly with notes regarding any unusual feature. In return, that Office supplies the readings of the stations in the vicinity of the catchment area ; this co-operation does much to ensure quick detection of inaccuracies.

Accumulated percentage maps from the beginning of the year are also prepared each month, and are occasionally instrumental in revealing a

Fig. 6



RAINFALL, 1949, EXPRESSED AS A PERCENTAGE OF THE LONG-AVERAGE

minor and persistent fault in a recording which is too small to be detected in individual monthly readings.

At the end of the year, maps of annual rainfall are prepared, expressed both in inches and percentages. Those relating to the year 1949 are shown in *Figs 5* and *6*. The weighted mean for the area from the percentage map for that year shows 94 per cent or 65·4 inches calculated on the long-average

annual rainfall of 69·4 inches, which is in exact agreement with the assessment by the cartographical method applied to the isohyetal plan.

As an instance of the value of recording rainfall as a percentage of the long-average figure, the Authors would refer to the experience in the first three months of 1947.

Owing to the severe winter conditions over the gathering ground, it was impossible for the observers to reach many of the monthly rain gauges at the end of either January or February.

Thus, the reading taken at the end of March represented in some cases 2, and in many instances 3, months' precipitation. Since all these mountain rain gauges had been seriously affected by snowdrifts, etc., the readings were of little or no value.

The readings of the eight daily gauges, however, were invaluable, although here again it was impossible, owing to the state of the roads, to reach some of the points for 2 or 3 days at a time.

With the help of these daily gauges and long experience in the general trend of rainfall values over the area under differing conditions of wind, etc., it was possible to arrive at a satisfactory estimation of the percentage value for all stations for each of the 3 months, and therefrom to determine the rainfall values from the known long-averages.

Only the invariable regularity of isopercentals made this course possible on an area where the annual average rainfall ranges from about 56 to 82 inches.

Subsequently, a careful investigation of the recorded relationship of the run-off from the gathering ground to the estimated rainfall figures for the period in question, compared with similar relationships over the previous 39 years, confirmed that the total estimated rainfall of 23·1 inches for the 3 months was likely to be correct to within $\frac{1}{2}$ inch.

Records of 63 Years

Table 4 gives the monthly and yearly rainfall for the 63 years 1887–1949.

Driest and Wettest Periods: Reliability of Rainfall

Table 5 gives the minimum and maximum rainfall of periods from 1 to 36 consecutive months, and the figures expressed as a percentage of the long-average annual rainfall have been plotted in *Fig. 7* together with Dr Glasspoole's curve of "Reliability of Rainfall," [5] which was based on the records of about forty stations in the British Isles between 1870 and 1927, and show very close conformity with it.

The figures given are very useful to an engineer in a period of prolonged drought, when reservoirs are becoming depleted; they enable him, at any time, to assess the probable worst conditions for which he must provide in the ensuing months, and thus to determine whether extraordinary measures (such as issuing appeals for economy in the use of water) should be taken. The tendency is, of course, to err on the safe side, a

TABLE 4.—ELAN AND CLAERWEN GATHERING GROUND—RAINFALL IN INCHES

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year	Per-centage of long-average
1887	5·6	2·1	4·7	2·9	4·0	0·8	3·3	4·8	5·4	3·5	5·5	7·6	50·2	72
1888	3·1	1·5	7·2	2·8	2·7	3·7	9·8	7·4	1·1	5·1	12·3	9·1	65·8	95
1889	3·2	6·3	5·4	7·1	4·6	1·1	6·1	7·1	3·6	8·1	3·0	4·8	60·4	87
1890	13·1	1·8	5·3	2·4	3·8	3·4	3·2	8·2	3·0	5·3	11·8	1·9	63·2	91
1891	7·7	0·3	4·7	4·4	4·7	4·2	5·4	10·2	5·7	11·8	7·2	13·9	80·2	116
1892	7·3	4·4	1·5	1·9	3·4	3·3	4·4	5·4	6·5	7·6	5·4	5·2	56·3	81
1893	3·8	9·4	1·6	0·3	4·2	5·0	5·2	4·9	6·5	8·1	6·3	8·8	64·1	92
1894	8·6	8·9	7·1	2·9	5·7	3·3	7·9	6·7	2·0	8·8	8·3	7·6	77·8	112
1895	5·8	0·5	7·2	3·6	1·0	1·3	7·4	6·0	1·6	7·8	9·9	6·9	59·0	85
1896	4·2	3·3	10·3	2·2	0·2	3·0	4·0	5·5	11·5	8·3	2·4	10·9	65·8	95
1897	2·8	5·4	8·1	6·4	2·0	3·7	3·7	6·8	8·1	3·3	6·7	11·1	68·1	98
1898	4·7	5·5	3·1	5·0	5·8	5·3	1·9	6·2	2·5	8·0	5·4	8·5	61·9	89
1899	11·8	7·3	2·8	7·5	5·2	2·2	3·8	2·4	7·2	5·6	6·2	5·0	67·0	97
1900	8·3	5·2	1·2	4·0	4·0	4·7	2·8	7·8	2·6	9·5	7·3	10·0	67·4	97
1901	7·3	2·1	6·3	5·8	1·2	4·4	3·1	5·8	4·0	5·4	5·6	9·4	60·4	87
1902	4·8	2·1	4·3	2·7	5·0	3·5	4·2	5·8	4·2	5·7	5·5	7·4	55·2	79
1903	9·5	6·4	12·5	3·5	4·8	3·6	4·3	9·5	7·8	15·6	8·1	6·4	92·0	133
1904	8·3	7·4	3·0	4·8	3·1	2·3	2·9	5·0	3·1	4·2	5·3	5·8	55·2	79
1905	4·9	5·0	8·7	6·8	1·1	2·8	2·6	8·9	3·0	5·7	5·9	2·4	57·8	83
1906	11·1	6·8	6·0	2·2	5·2	3·5	3·5	6·6	2·2	9·1	6·9	8·5	71·6	103
1907	4·9	3·3	4·3	3·8	4·7	7·6	6·4	6·7	1·3	8·5	4·9	8·9	65·3	94
1908	4·9	4·9	5·5	3·6	4·3	2·4	6·7	7·3	6·1	2·4	6·1	6·7	60·9	88
1909	5·1	1·9	5·1	5·7	2·6	3·2	7·7	3·8	5·7	10·9	3·2	8·9	63·8	92
1910	7·8	9·0	2·4	6·1	3·9	8·7	7·3	7·5	0·8	6·5	9·4	12·9	82·3	119

1911	2·8	5·2	3·9	3·7	2·2	5·4	1·0	5·5	5·5	6·7	8·6	13·9	64·4	93
1912	6·6	4·2	11·4	0·9	2·4	6·7	5·3	13·9	1·7	7·8	7·4	9·9	78·2	113
1913	9·2	3·2	9·8	9·0	4·1	5·0	2·2	3·9	3·7	6·4	10·1	5·4	72·0	104
1914	6·2	7·9	10·8	2·7	4·5	2·6	6·4	5·6	4·1	3·5	8·8	12·5	75·6	109
1915	9·4	10·0	2·5	3·4	4·1	1·9	6·7	6·1	1·5	3·8	5·6	13·7	68·7	99
1916	6·6	8·8	4·8	6·6	5·2	4·5	3·6	5·0	5·5	12·5	5·8	4·7	73·6	106
1917	3·0	2·0	4·1	4·2	4·6	3·6	3·1	14·7	4·3	10·9	7·6	2·7	64·8	93
1918	7·1	7·2	4·3	2·4	4·9	3·3	6·3	3·4	14·3	6·0	6·2	13·0	78·4	113
1919	7·3	4·0	9·1	5·3	1·4	5·0	2·9	5·0	4·0	3·6	5·5	14·1	67·2	97
1920	8·7	5·5	9·0	10·4	6·2	4·5	8·0	3·2	5·9	5·8	3·4	6·0	76·6	110
1921	10·2	0·8	8·1	2·6	3·2	1·1	3·8	8·8	2·8	3·5	5·4	8·9	59·2	85
1922	9·7	9·4	5·6	5·3	1·9	3·6	7·5	4·8	6·3	1·9	3·3	8·4	67·7	98
1923	9·3	15·0	3·6	4·5	6·4	1·8	6·0	7·3	7·6	13·2	7·3	8·7	90·7	131
1924	7·0	2·1	2·8	6·0	7·6	3·9	7·3	9·4	9·4	8·3	5·2	10·9	79·9	115
1925	7·6	13·0	3·2	5·8	6·1	0·2	4·4	5·7	6·3	9·9	5·1	7·4	74·7	108
1926	9·2	5·4	3·5	3·5	4·4	3·7	7·7	6·9	5·0	6·5	10·8	2·8	69·4	100
1927	10·0	6·3	7·8	4·8	2·5	8·3	6·8	12·1	8·4	6·5	6·0	3·7	83·2	120
1928	14·3	8·7	5·0	2·8	1·4	8·2	4·7	8·0	2·4	9·2	13·8	6·2	84·7	122
1929	2·6	1·6	0·8	1·5	4·5	4·9	4·9	4·3	2·6	10·1	15·8	15·7	69·3	100
1930	10·6	0·8	4·3	4·7	4·0	2·4	8·8	8·1	9·1	11·6	10·1	9·1	83·6	120
				Adopted long-average : inches										
Mean of 44 years 1887–1930 . .	7·2	5·3	5·5	4·3	3·8	3·8	5·1	6·8	4·9	7·3	7·1	8·3	69·4	
Percentage of year	10·4	7·6	7·9	6·2	5·5	5·5	7·4	9·8	7·0	10·5	10·2	12·0	100·0	
Percentage adopted by Air Ministry for all Wales .	9·4	7·8	7·6	5·9	5·9	6·1	7·2	9·4	7·0	11·2	10·5	12·0	100·0	
Mean of 35 years 1891–1925 . .	7·0	5·6	5·7	4·5	3·9	3·8	4·9	6·6	5·0	7·3	6·3	8·7	69·3	
Percentage of year	10·1	8·1	8·2	6·5	5·6	5·5	7·1	9·5	7·2	10·5	9·1	12·6	100·0	

TABLE 4 (*cont.*)

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year	Percentage of long-average
1931	9·3	8·5	1·7	6·5	5·8	4·7	6·3	6·3	3·7	3·4	11·3	5·4	72·9	105
1932	10·7	0·2	5·2	6·7	5·8	2·2	6·7	1·8	6·7	13·0	6·9	4·9	70·8	102
1933	7·4	7·6	4·8	1·1	2·3	5·4	5·0	2·4	1·5	8·5	2·1	1·5	49·6	71
1934	7·9	0·7	6·3	5·3	3·8	2·8	3·4	8·1	5·3	8·8	3·3	13·9	69·6	100
1935	4·4	9·1	2·7	7·7	2·2	6·4	2·0	2·9	11·8	11·7	10·0	6·5	77·4	112
1936	9·1	3·8	4·1	4·1	2·8	6·4	8·9	1·4	6·1	6·7	8·5	11·3	73·2	106
1937	12·0	10·6	4·8	6·6	2·5	2·6	3·6	1·3	4·2	4·7	2·3	6·5	61·7	89
1938	12·3	4·7	2·0	1·0	4·4	6·7	7·0	4·8	2·8	10·8	11·3	8·4	76·2	110
1939	11·1	8·7	4·5	5·0	1·3	3·6	10·4	3·9	1·2	4·0	17·2	6·2	77·1	111
1940	4·1	5·2	5·1	4·3	2·4	0·8	9·2	2·1	4·7	8·4	14·8	7·2	68·3	98
1941	4·3	7·6	5·4	2·6	4·9	1·8	3·7	9·5	1·5	7·6	5·8	6·8	61·5	89
1942	8·4	2·1	5·0	4·6	8·5	0·9	6·8	5·4	5·2	9·0	1·3	10·6	67·8	98
1943	10·7	6·2	2·1	2·5	6·6	5·3	4·8	6·5	7·5	6·6	7·7	4·1	70·6	102
1944	8·9	3·8	1·1	2·6	3·5	3·8	4·5	3·7	6·7	11·5	11·2	7·7	69·0	99
1945	5·6	8·4	3·2	3·3	4·7	7·0	3·4	4·5	5·1	5·7	1·1	7·8	59·8	86
1946	8·6	12·5	1·8	2·4	3·6	5·4	4·4	10·6	10·4	3·3	12·6	8·1	83·7	121
1947	6·2	3·1	13·8	5·4	4·0	3·1	4·5	0·8	4·0	1·7	8·4	7·3	62·3	90
1948	16·2	5·3	4·4	4·4	3·5	7·5	3·9	6·2	6·7	5·7	4·3	11·4	79·5	115
1949	5·0	3·2	4·5	6·1	6·4	0·8	1·9	3·9	2·7	9·3	10·1	11·5	65·4	94
Mean of 63 years 1887–1949 . .	7·6	5·4	5·2	4·3	3·9	3·9	5·2	6·1	5·0	7·3	7·3	8·2	69·4	
Percentage of year	11·0	7·8	7·5	6·2	5·6	5·6	7·5	8·8	7·2	10·5	10·5	11·8	100·0	

TABLE 5

Period : months	Driest			Wettest		
	Inches	Percentage of 69·4 inches	Date	Inches	Percentage of 69·4 inches	Date
1	0·2	0·3	May 1896 June 1925 Feb. 1932	17·2	24·8	Nov. 1939
2	1·9	2·7	Mar. 1893 *et seq.*	31·5	45·4	Nov. 1929 *et seq.*
3	3·9	5·6	Feb. 1929 "	42·1	60·7	Nov. 1929 "
4	6·5	9·4	Jan. 1929 "	52·2	75·2	Oct. 1929 "
5	11·0	15·8	Jan. 1929 "	54·8	79·0	Sept. 1929 "
6	15·9	23·9	Jan. 1929 "	57·7	83·1	Sept. 1930 "
7	20·8	30·0	Jan. 1929 "	66·1	95·2	July 1930 "
8	25·1	36·2	Jan. 1929 "	74·6	107·5	July 1930 "
9	27·7	39·9	Jan. 1929 "	77·0	111·0	June 1930 "
10	33·9	48·8	Dec. 1928 "	82·8	119·3	July 1930 "
11	38·4	55·3	Apr. 1933 "	88·6	127·7	July 1930 "
12	43·2	62·2	Mar. 1933 "	95·9	138·2	Nov. 1929 "
16	60·0	86·4	Apr. 1933 "	134·5	193·8	Oct. 1929 "
20	83·9	120·9	Feb. 1887 "	157·0	226·2	Oct. 1929 "
24	110·2	158·8	Dec. 1932 "	178·0	256·5	Oct. 1929 "
30	139·9	201·6	Feb. 1931 "	214·7	309·4	Dec. 1922 "
36	176·4	254·2	Jan. 1887 "	246·3	354·9	Dec. 1922 "

course which is the easiest to take, but not always the wisest, for consumers do not appreciate insistent appeals for economy which later prove to have been unnecessary, and a recurrence of what is regarded as a false alarm at intervals of a year or so leads to a decrease in the effectiveness of such appeals. In the critical dry months of June, July, and August of 1949, when the reservoirs were depleted to an alarming extent, the Authors were able to derive considerable comfort from the figures given in Table 5 showing that the minimum rainfall likely to occur as each successive month remained dry should be adequate (as in fact it was) to meet the anticipated demand.

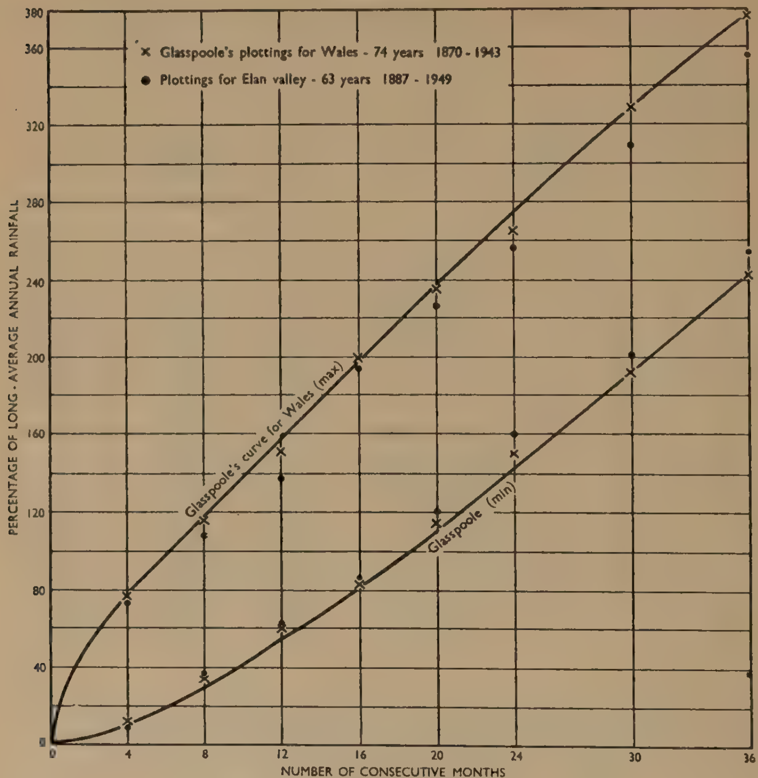
RUN-OFF

Definition

The term "run-off" throughout this section relates to the total quantity of water available as measured at the Caban Coch Dam, the lowest point of the drainage area of 45,562 acres, and comprises :—

(1) Water supplied to Birmingham.
(2) Compensation water discharged to the River Elan.
(3) Water used for works purposes.
(4) Overflow at Caban Dam.
(5) Gain or loss in reservoirs.

Fig. 7



RELIABILITY OF RAINFALL

Measuring Apparatus

These quantities are carefully measured as described below :—

(1) *Supply to Birmingham.*—By continuous recording of the discharge into the aqueduct through rectangular orifices.

(2) *Compensation water to River Elan.*—By continuous recording of the flow through submerged rectangular orifices.

(3) *Water for works purposes.*—Mainly water used for filter sand-washing, measured over a 3-foot rectangular weir.

(4) *Overflow at Caban Dam.*—The weir length of the Caban Dam is 566 feet. The depth of overflow is continuously recorded by two automatic recorders, one at each end of the dam.

The coefficient of discharge was determined by careful measurements of the flow over an exact replica of the crest of

the dam, 3 feet in length. This work has been described by Dixon and Macaulay,[6] who deduced the formula :

$$Q = 3{\cdot}043LH^{1{\cdot}484}$$

(5) *Gain or loss in storage reservoirs.*—The water level in the reservoirs is read to the nearest inch daily from fixed gauges.

Records of 42 Years (1908–49)

Daily records of the run-off have been calculated throughout the 42 years (1908–49). Monthly and yearly figures of the run-off in inches are given in Table 6, together with the mean values for the whole period.

When the detailed figures given therein are compared with the rainfall for corresponding periods, as shown in Table 4, it will be found that in some instances the run-off is in excess of the recorded rainfall for a particular month. This almost invariably occurs in the winter, when heavy rain in the last few days of a month does not completely run off until the beginning of the following month, or as a result of delay in the melting of snow.

In general, no attempt has been made to adjust these discrepancies because, over such a long period of years, they have comparatively little influence.

Relation of Run-off to Rainfall

In the period of 42 years dealt with in this section, the mean annual rainfall amounted to 71·55 inches, which is 2·15 inches or 3·1 per cent above the long-average figure of 69·4 inches.

Twenty-three were wet years with a total excess of rainfall of 188·3 inches ; one year was exactly equal to the normal ; the remaining 18 were dry years with a total deficiency of 97·8 inches. Thus, the total excess over the whole series amounted to 90·5 inches. The average run-off amounted to 50·47 inches per annum or 70·5 per cent of the mean annual rainfall during the period.

In the wettest year (1923), with rainfall 90·70 inches, the run-off was 70·69 inches or 77·9 per cent. In the driest year (1933), with rainfall 49·60 inches, the run-off was 32·99 inches or 66·5 per cent.

The relationship of run-off to rainfall (in inches) for 42 years is shown in *Fig. 8*, from which it will be seen that in a year with rainfall equal to the long-average (69·4 inches) about 47·8 inches or 69 per cent may run-off. That the run-off bears a pronounced relationship to the total precipitation is demonstrated by *Fig. 9*, which shows in each of the 42 years the relationship of both rainfall and run-off to the long-average annual rainfall.

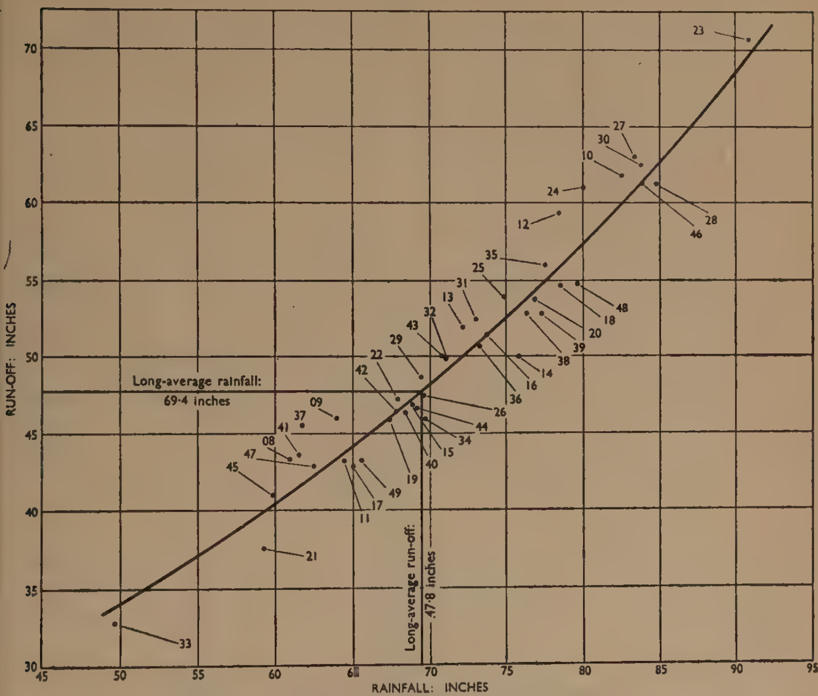
From the mean curve it would appear that in a year with 80 per cent of the long-average rainfall, only about 54 per cent of the latter figure is likely to run-off, whilst in a year with 120 per cent of the long-average rainfall, the run-off may be as much as 88 per cent of the long-average rainfall.

TABLE 6.—ELAN AND CLAERWEN GATHERING GROUNDS (45,562 ACRES)

RUN-OFF IN INCHES

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
1908	4·31	4·11	5·53	2·60	3·25	0·95	3·26	2·59	4·78	1·52	4·57	5·97	43·44
1909	4·75	1·53	3·94	3·20	1·39	1·16	3·77	1·64	3·79	9·10	1·92	9·80	45·99
1910	6·00	8·46	2·59	3·49	1·86	4·73	4·71	4·18	1·02	3·02	8·73	12·88	61·67
1911	2·99	2·76	4·07	2·05	1·65	1·65	0·43	0·71	2·09	4·76	7·02	13·11	43·29
1912	7·11	3·40	9·88	1·15	0·63	2·20	2·06	11·52	1·34	5·12	6·19	8·80	59·40
1913	8·70	2·98	6·84	7·14	3·23	2·77	0·65	1·06	1·40	4·08	8·12	4·89	51·86
1914	5·07	7·13	8·65	1·80	2·67	0·55	1·57	3·31	2·05	1·43	6·03	9·75	50·01
1915	8·13	7·69	2·28	1·86	2·40	0·32	1·86	3·93	0·40	1·30	4·52	12·15	46·84
1916	6·37	7·28	4·02	5·70	2·73	1·45	2·21	1·27	3·44	8·14	5·28	3·66	51·55
1917	3·08	1·64	3·26	3·50	1·28	1·79	0·60	9·04	3·00	7·34	6·06	2·33	42·92
1918	6·39	5·07	2·09	2·96	2·35	1·21	2·31	1·07	10·21	5·26	4·85	11·00	54·77
1919	6·14	4·32	6·56	3·50	1·44	1·61	0·75	1·32	2·23	2·25	3·86	11·92	45·90
1920	6·87	4·55	6·32	8·34	3·63	1·53	5·15	2·20	3·75	4·40	2·30	4·81	53·85
1921	9·01	1·16	5·28	1·53	0·88	0·26	0·39	4·84	1·83	1·37	4·21	6·72	37·48
1922	8·65	7·72	5·33	3·38	1·03	0·47	4·03	1·93	3·85	1·65	2·62	6·45	47·11
1923	7·71	13·32	3·85	2·40	4·53	1·02	2·07	3·95	6·19	11·20	6·36	8·09	70·69
1924	6·99	1·78	2·17	3·35	5·02	3·51	3·60	7·08	7·29	6·89	4·54	8·82	61·04
1925	6·66	11·21	3·64	3·46	2·89	0·65	0·48	1·96	4·24	8·26	4·70	5·81	53·96
1926	7·64	4·34	2·82	1·28	2·68	1·88	3·93	3·56	3·08	4·50	9·57	2·14	47·42
1927	8·64	5·12	6·63	4·00	1·16	4·38	4·37	9·47	5·55	5·12	5·21	3·44	63·09
1928	11·93	7·42	3·26	2·03	0·41	3·78	2·24	4·88	1·13	6·19	12·68	5·31	61·26
1929	2·54	2·49	0·87	0·37	2·65	2·11	1·05	2·85	0·69	7·13	13·28	12·62	48·65
1930	8·47	1·81	2·64	3·34	2·12	0·88	5·25	6·17	6·61	8·68	8·95	7·53	62·45
1931	7·95	7·26	2·04	4·40	3·13	3·16	2·74	4·38	2·31	1·94	9·27	3·81	52·39
1932	10·19	0·58	3·10	3·56	4·34	0·69	3·11	1·20	3·66	10·31	5·11	4·08	49·93
1933	6·75	5·95	5·62	0·60	0·97	2·13	2·47	0·37	0·19	4·59	2·27	1·08	32·99
1934	7·08	0·73	4·66	2·82	2·46	0·58	0·46	4·51	2·43	6·08	3·44	10·73	45·98
1935	4·67	7·33	2·62	5·42	0·92	3·12	0·73	0·37	6·88	10·26	8·03	5·55	55·90
1936	6·81	3·37	2·83	3·09	1·30	2·84	6·33	1·15	3·05	3·99	7·54	8·34	50·64
1937	10·07	8·84	6·29	5·21	1·14	0·83	1·18	0·27	1·28	2·19	1·91	6·27	45·48
1938	9·18	3·05	2·07	1·06	1·07	4·18	4·09	2·34	1·15	8·17	9·17	7·34	52·87
1939	9·49	5·92	4·21	2·64	0·68	0·53	6·21	2·13	0·53	1·25	13·15	6·11	52·85
1940	2·32	6·02	3·71	2·42	1·33	0·24	4·46	0·51	1·89	5·15	12·83	5·58	46·46
1941	1·89	8·53	4·98	2·09	2·76	1·29	0·58	4·99	1·04	5·01	4·51	5·94	43·61
1942	6·30	3·83	3·34	3·14	4·53	0·66	2·69	2·34	3·36	7·20	1·35	7·76	46·50
1943	8·96	5·58	0·89	1·59	4·04	3·03	1·84	2·87	5·97	5·15	6·42	3·64	49·98
1944	6·93	2·95	1·11	1·22	0·95	1·35	2·36	0·80	4·34	8·98	8·76	6·89	46·64
1945	4·25	7·47	2·09	2·23	2·16	3·74	2·06	1·57	3·70	4·09	1·18	6·45	40·99
1946	6·85	11·02	1·71	0·75	1·03	3·82	1·97	6·21	9·03	2·56	9·37	6·97	61·29
1947	5·54	0·58	13·92	3·49	2·09	1·13	1·83	0·40	0·64	0·66	7·13	5·55	42·96
1948	13·34	4·97	2·44	2·81	1·36	4·17	2·10	2·86	4·92	3·92	3·69	8·15	54·73
1949	5·11	1·98	3·75	3·89	3·14	0·80	0·18	0·70	0·48	6·43	7·91	8·84	43·21
Mean of 42 years	6·8	5·1	4·1	3·0	2·2	1·9	2·5	3·1	3·3	5·1	6·3	7·1	50·5

Fig. 8



RELATIONSHIP OF RAINFALL TO RUN-OFF ("CABAN") FOR EACH YEAR, 1908–1949

Long-average Run-off

The proportion of the monthly long-average precipitation which may run-off is given in Table 7 and is shown in diagrammatic form in *Fig. 10.*

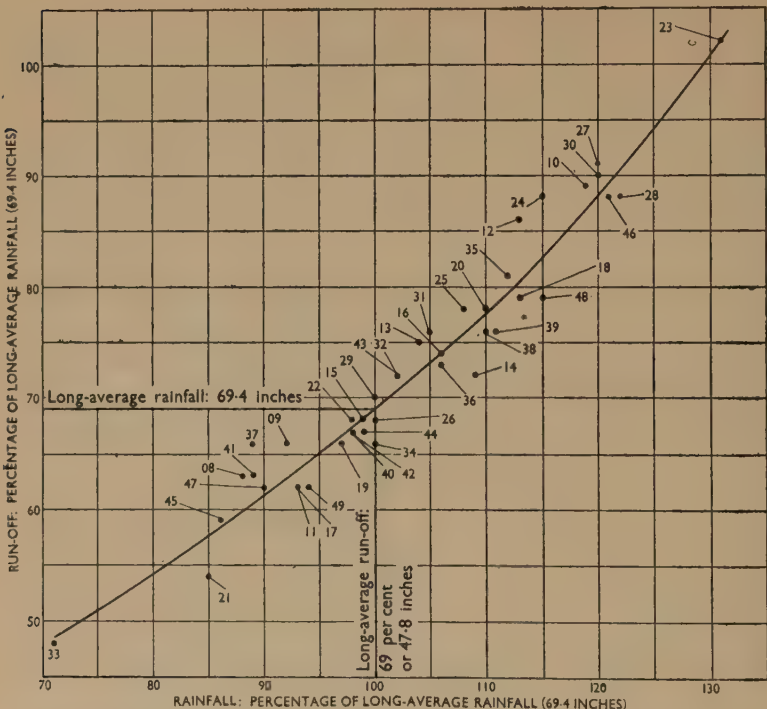
During the summer months (April to September inclusive), about 15·2 inches of the normal rainfall of 28·7 inches, or 53 per cent only, may run-off. In the winter 6 months with a mean rainfall of 40·7 inches, the run-off may amount to 32·6 inches or 80 per cent.

The figures in Table 7 are well supported by an analysis of the run-off obtained in those months when the rainfall was within ½ inch of the long-average. In that analysis, the effect of delayed run-off was taken into account, and when the run-off was unduly influenced by an exceptionally dry preceding month, or where the rainfall was concentrated into a few days only, the record was ignored.

Run-off per 1,000 Acres

The mean annual run-off from the gathering ground of 45,562 acres during the 42 years tabulated in Table 6 was 5·8 cusecs per 1,000 acres. In the wettest year (1923) with a rainfall of 131 per cent of the long-average,

Fig. 9

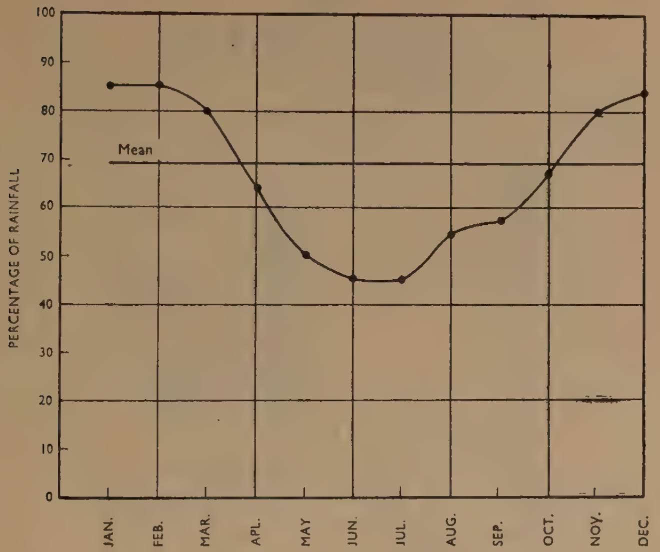


Relationship of Rainfall to Run-off ("Caban"), 1908–1949

Table 7

	Long-average rainfall: inches	Expected run-off:	
		per cent	inches
January	7·2	85	6·1
February	5·3	85	4·5
March	5·5	80	4·4
April	4·3	64	2·8
May	3·8	50	1·9
June	3·8	45	1·7
July	5·1	45	2·3
August	6·8	55	3·7
September	4·9	57	2·8
October	7·3	67	4·9
November	7·1	80	5·7
December	8·3	84	7·0
Year	69·4	69	47·8

Fig. 10



PROBABLE RUN-OFF EXPRESSED AS A PERCENTAGE OF THE MONTHLY LONG-AVERAGE RAINFALL

the average daily run-off was 8 cusecs per 1,000 acres, and in the driest year (1933), with a rainfall of 71 per cent of the long-average, 3·81 cusecs per 1,000 acres.

In a year with rainfall equal to the long-average figure, 69·4 inches, the run-off is likely to be 47·8 inches, or 5·5 cusecs per 1,000 acres, whilst in a year with 80 per cent of the long-average rainfall, the run-off is likely to be 54 per cent or 4·33 cusecs per 1,000 acres.

Periods of Maximum and Minimum Run-off

On *Fig. 11* is plotted the maximum run-off for various periods of consecutive months throughout the 42 years ; *Fig. 12* gives similar information relating to minimum run-offs.

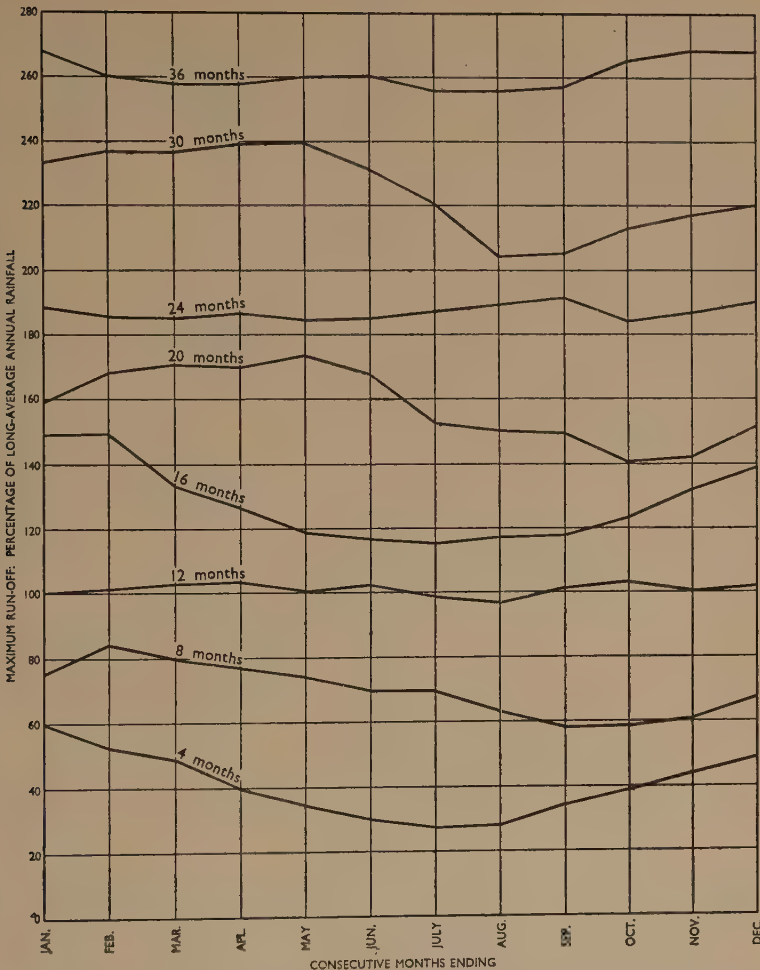
The outstanding feature in both cases is the relatively steady horizontal line for the consecutive periods of 12, 24, and 36 months ending at any month in the calendar year, indicating a tendency for the seasonal variations within any consecutive period of 12 months, or multiples of 12 months, to balance out.

There is considerable divergence in the case of all other periods, as might be expected, for, taking as an example the 16 consecutive months' period, it will be seen that both the lowest maximum run-off and the lowest minimum run-off occur when the period ends at July, and thus twice includes the frequently dry period of April to July inclusive.

TABLE 8.—MAXIMUM AND MINIMUM RUN-OFFS EXPERIENCED IN 42 YEARS FOR PERIODS SHOWN

	Maximum			Minimum		
	Run-off: inches	Period	Relation to long-average annual rain-fall (69·4 inches): per cent	Run-off: inches	Period	Relation to long-average annual rain-fall (69·4 inches): per cent
4 consecutive months	41·50	October 1929–January 1930	59·80	2·16	June–September 1949	3·11
8 „ „	58·40	July 1930–February 1931	84·15	13·08	February–September 1929	18·85
12 „ „	71·87	November 1929–October 1930	103·56	27·14	April 1933–March 1934	39·11
16 „ „	103·56	November 1929–February 1931	149·22	33·46	April 1933–July 1934	48·21
20 „ „	120·26	October 1929–May 1931	173·29	49·92	April 1933–November 1934	71·93
24 „ „	132·85	October 1929–September 1931	191·43	72·32	December 1932–November 1934	104·21
30 „ „	166·04	December 1922–May 1925	239·25	91·45	March 1933–August 1935	131·77
36 „ „	186·33	December 1922–November 1925	268·49	121·98	December 1931–November 1934	175·76

Fig. 11

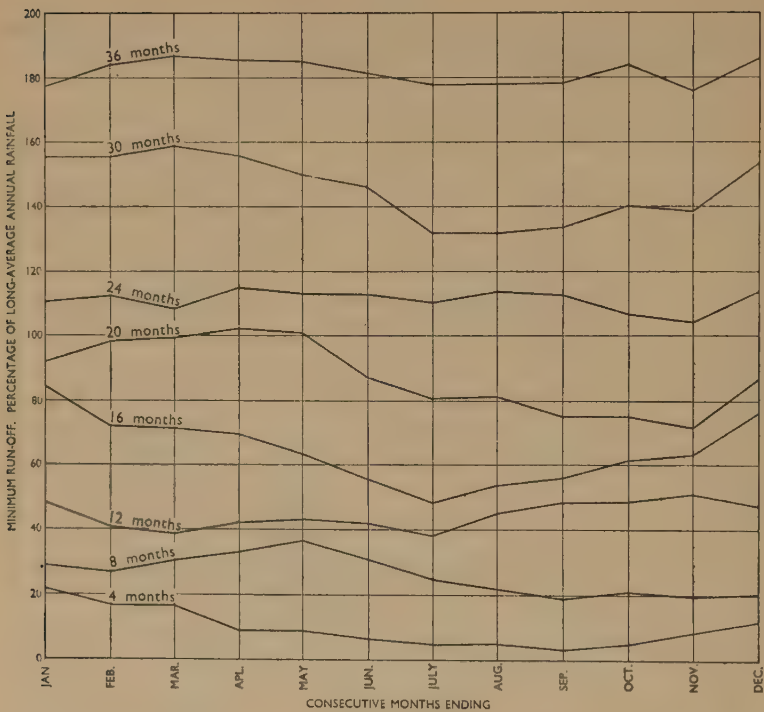


Maximum Run-off for Various Periods of Consecutive Months expressed as a Percentage of Long-Average Annual Rainfall ("Caban"), 1908–1949

Reliability of Run-off

The reliability of rainfall has already been dealt with (p. 357) on the lines adopted by Dr Glasspoole. In an attempt to deal similarly with the question of reliability of run-off, the highest maximum and lowest minimum run-offs in various periods of consecutive months are listed in Table 8 and plotted in *Fig. 13.*

Fig. 12



MINIMUM RUN-OFF FOR VARIOUS PERIODS OF CONSECUTIVE MONTHS EXPRESSED AS A PERCENTAGE OF LONG-AVERAGE ANNUAL RAINFALL ("CABAN"), 1908–1949

The curves indicate the greatest and least proportion of the long-average annual rainfall which may be expected to run off in any period up to 36 consecutive months. The curve of reliability of rainfall is also incorporated in *Fig. 13*.

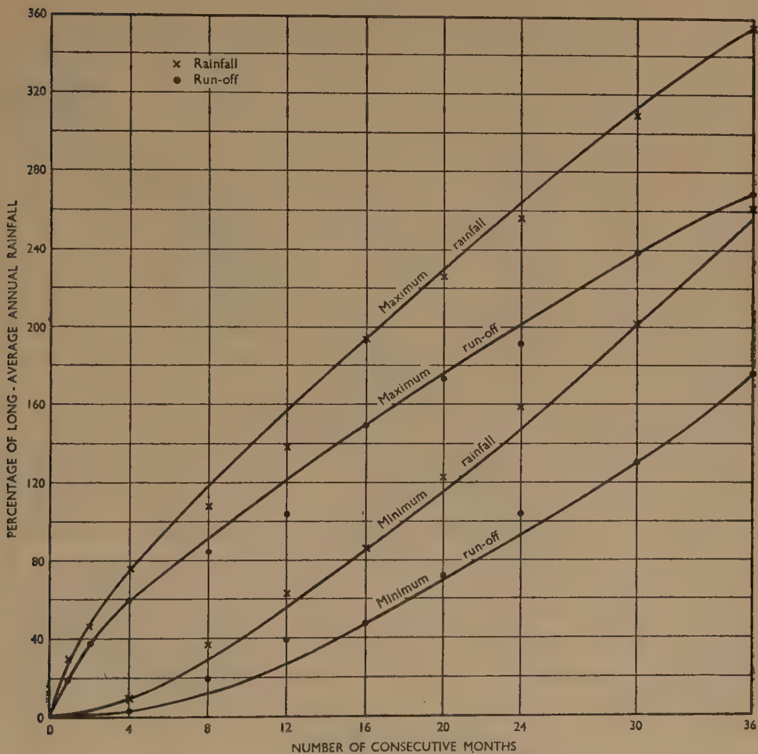
The following are two examples taken from the curves in *Fig. 13* :—

(1) In any period of 4 consecutive months the run-off may be as much as 60 per cent or as little as 3 per cent of the long-average annual rainfall.

(2) In a period of 16 consecutive months the comparable figures are 150 per cent and 50 per cent, and, for 36 consecutive months, 270 per cent and 176 per cent.

According to Dr Glasspoole's analysis, it would appear that the Elan valley may experience a greater deficiency in rainfall over a 36-month period than has occurred for such a period in the 42 years to which the records in this Paper apply. Whilst the minimum 36-month period rainfall

Fig. 13

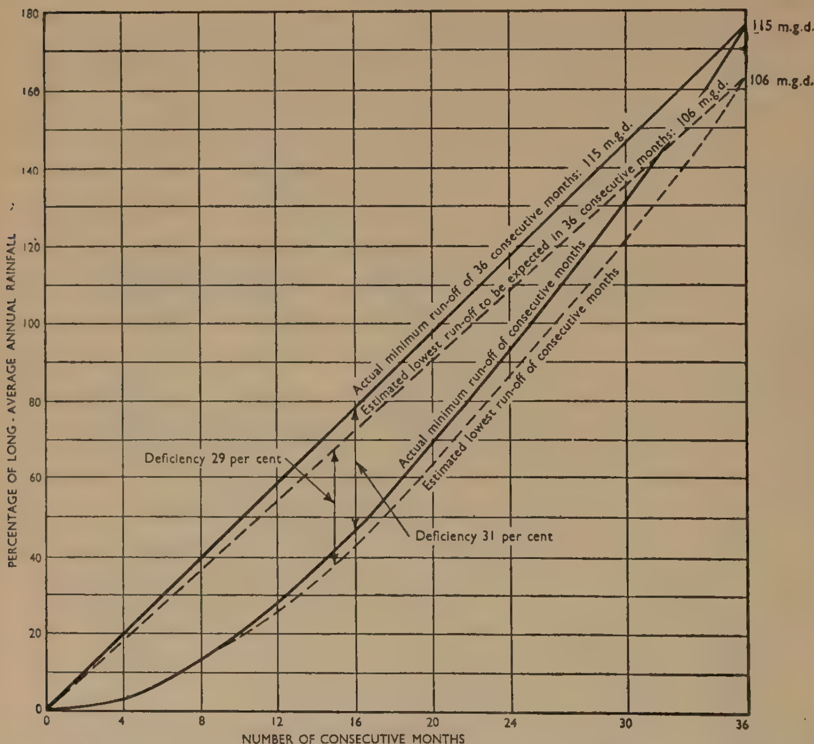


Reliability of Run-off at Caban Dam (Deduced from the Records of 42 Years, 1908–1949)

for the Elan valley within that 42-year period represents 260 per cent Dr Glasspoole gives a minimum of only 243 per cent, which agrees substantially with the generally accepted average rainfall of the 3 driest consecutive years of 80 per cent of the long-average. Assuming a similar reduction in the proportion of run-off, the probable curve of minimum run-off, indicated by the lower dotted line on *Fig. 14*, shows that the total available in the 3 driest consecutive years may be only 163 per cent of the long-average rainfall, or 54 per cent per year, representing 106 million gallons per day.

A uniform discharge of 106 m.g.d. is indicated by the straight dotted line, and the greatest vertical difference between this line and the line of estimated lowest reliable yield occurs at the end of 16 months and represents the volume of water which would have to be drawn from storage to maintain the constant supply of 106 m.g.d. The required storage capacity

Fig. 14



STORAGE REQUIRED TO EQUALIZE MINIMUM RUN-OFF OF PERIODS OF CONSECUTIVE MONTHS EXPERIENCED IN 42 YEARS (1908–1949)

would, therefore, be 29 per cent of the long-average rainfall, or 29 per cent of 69·4 inches, or 20,780 million gallons—equivalent to 196 days' supply at 106 m.g.d., compared with 150 days obtained by applying the average annual yield, Y_{3D}, of the driest 3 consecutive years to the Hawksley Rule,

$$N = \frac{500}{\sqrt[3]{Y_{3D}}}.$$

The effective storage available in the Elan and Claerwen valleys is 21,136 million gallons.

Floods

The greatest run-off during periods of from 1 to 7 consecutive days is given in Table 9.

The maximum peak flood so far recorded on the gathering ground occurred on the 20th September, 1946. This followed about 3 inches of

TABLE 9

Consecutive days	Date	Run-off:			
		m.g.	inches	cusecs	cusecs per 1,000 acres
1	11 November, 1929	2,250	3·18	4,185	91·9
2	18/19 March, 1947	3,742	3·62	3,480	76·4
3	17/19 March, 1947	5,058	4·90	3,136	68·8
4	18/21 March, 1947	5,906	5·72	2,746	60·3
5	17/21 March, 1947	7,221	7·00	2,686	59·0
6	17/22 March, 1947	8,229	7·97	2,551	56·0
7	17/23 March, 1947	9,190	8·90	2,422	53·6

rain recorded in the preceding 48 hours. Unfortunately, no recording rain gauge was at that time maintained within the catchment, but it was evident that the greater part of the rainfall was concentrated within a very few hours.

In the preceding 70 days a total of about 23 inches of rain fell and on only one day in that period was no rainfall recorded.

It is thus apparent that the rainfall of 19/20 September fell on a saturated gathering ground; the result was immediate run-off.

The maximum rate of run-off—11,800 cusecs or 259 cusecs per 1,000 acres—occurred between 1 and 2 p.m. on the 20th September and coincided almost exactly with the normal maximum flood (261 cusecs per 1,000 acres) to be expected from an upland gathering ground of equivalent area, according to the interim report of July 1933 of the Flood Committee of the Institution of Civil Engineers.

A copy of the recorder chart for the overflow at Caban Dam during the flood is given in *Fig. 15*. It will be noticed that the peak depth over the crest of the dam was almost exactly 3 feet—a figure assumed by Mansergh in designing the dam.

A flood of almost equal intensity (250 cusecs per 1,000 acres) occurred on the area in 1895 during the construction of the works.

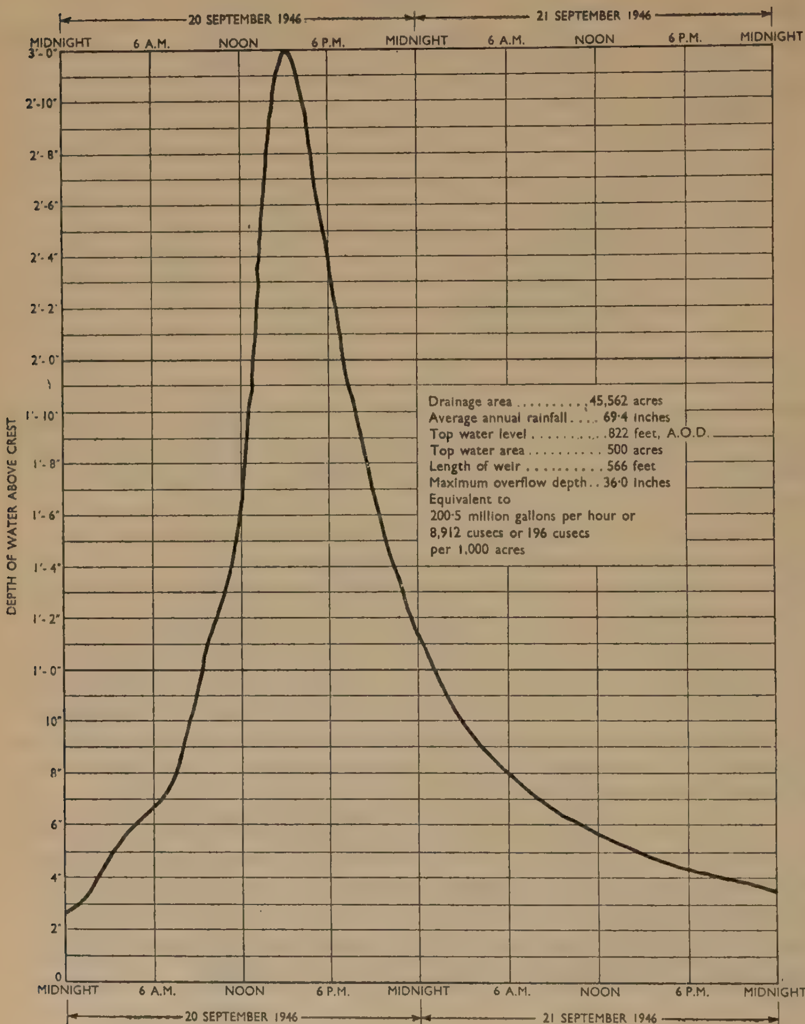
Droughts

In order to investigate the adequacy of the storage provided, a detailed study of records relating to dry periods has been made. No reference need be made to short-period droughts, other than to record that the minimum run-off occurred in 1947, as follows:—

Minimum average daily run-off during 14 consecutive days, 0·06 cusec per 1,000 acres.

Minimum average daily run-off during 21 consecutive days, 0·12 cusec per 1,000 acres.

Fig. 15



Caban Reservoir. Record Overflow

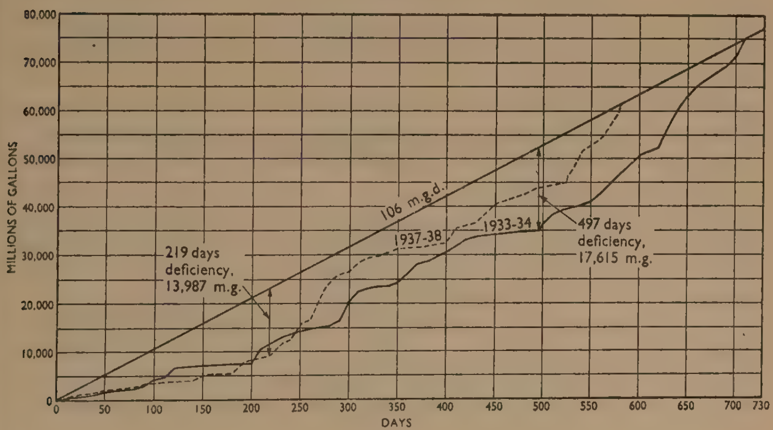
Minimum average daily run-off during 28 consecutive days, 0·28 cusec per 1,000 acres.

The estimated minimum average daily flow into the reservoirs over a period of 14 days in 1947 was 0·25 cusec per 1,000 acres, a figure generally accepted for the dry-weather flow from a gathering ground of the nature of the Elan and Claerwen valleys.

A study of the results of long periods of dry weather made in respect of the years 1911, 1921, 1929, 1933, 1937, 1947, and 1949 showed that the greatest depletion of storage would have occurred in the years 1933–34, and *Fig. 16* shows the cumulative yield diagram covering the 709 days during which the accumulated run-off at any time fell short of an assumed maximum reliable run-off of 106 million gallons per day.

It will be seen that at the end of 497 days (16½ months) the quantity required from storage would have been 17,615 million gallons, leaving only 3,520 million gallons or 33 days' supply in the now existing reservoirs.

Fig. 16



Caban Dam. Mass Yield Diagram, 1933–34 and 1937–38

When the contents of reservoirs fall to such a low level, serious concern is felt and extraordinary measures to economize in the use of water have to be taken, so that a margin of safety of 33 days' storage in excess of the figure necessary to meet circumstances which have already occurred once in the past 42 years, is not considered at all extravagant. This conclusion is supported by the fact that in the early days of the 1933–34 drought, the Elan valley derived the benefit from two considerable rainstorms of the thundery type, without which the storage would have been still more depleted.

Evaporation and Absorption

The evaporation and absorption losses for each month covering the 42 years (1908–49) are given in Table 10, the bottom line showing the monthly and yearly averages for the whole series.

The annual values of loss in inches are plotted in *Fig. 17*, and the curve

TABLE 10.—ELAN AND CLAERWEN GATHERING GROUNDS (45,562 ACRES): LOSS BY EVAPORATION AND ABSORPTION—INCHES

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Long-average rainfall . .	7·2	5·3	5·5	4·3	3·8	3·8	5·1	6·8	4·9	7·3	7·1	8·3	69·4
1908	0·59	0·79	−0·03	1·00	1·05	1·45	3·44	4·71	1·32	0·88	1·53	0·73	17·46
1909	0·35	0·37	1·16	2·50	1·21	2·04	3·93	2·16	1·91	1·80	1·28	−0·90	17·81
1910	1·80	0·54	−0·19	2·61	2·04	3·97	2·59	3·32	−0·22	3·48	0·67	0·02	20·63
1911	−0·19	2·44	−0·17	1·65	0·55	3·75	0·57	4·79	3·41	1·94	1·58	0·79	21·11
1912	−0·51	0·80	1·52	−0·25	1·77	4·50	3·24	2·38	0·36	2·68	1·21	1·10	18·80
1913	0·50	0·22	2·96	1·86	0·87	2·23	1·55	2·84	2·30	2·32	1·98	0·51	20·14
1914	1·13	0·77	2·15	0·90	1·83	2·05	4·83	2·29	2·05	2·07	2·77	2·75	25·59
1915	1·27	2·31	0·22	1·54	1·70	1·58	4·84	2·17	1·10	2·50	1·08	1·55	21·86
1916	0·23	1·52	0·78	0·90	2·47	3·05	1·39	3·73	2·06	4·36	0·52	1·04	22·05
1917	−0·08	0·36	0·84	0·70	3·32	1·81	2·50	5·66	1·30	3·56	1·54	0·37	21·88
1918	0·71	2·13	2·21	−0·56	2·55	2·09	3·99	2·33	4·09	0·74	1·35	2·00	23·63
1919	1·16	−0·32	2·54	1·80	−0·04	3·39	2·15	3·68	1·77	1·35	1·64	2·18	21·30
1920	1·83	0·95	2·68	2·06	2·57	2·97	2·85	1·00	2·15	1·40	1·10	1·19	22·75
1921	1·19	−0·36	2·82	1·07	2·32	0·84	3·41	3·96	0·97	2·13	1·19	2·18	21·72
1922	1·05	1·68	0·27	1·92	0·87	3·13	3·47	2·87	2·45	0·25	0·68	1·95	20·59
1923	1·59	1·68	−0·25	2·10	1·87	0·78	3·93	3·35	1·41	2·00	0·94	0·61	20·01
1924	0·01	0·32	0·63	2·65	2·58	0·39	3·70	2·32	2·11	1·41	0·66	2·08	18·86
1925	0·94	1·79	−0·44	2·34	3·21	−0·45	3·92	3·74	2·06	1·64	0·40	1·59	20·74
1926	1·56	1·06	0·68	2·22	1·72	1·82	3·77	3·34	1·92	2·00	1·23	0·66	21·98
1927	1·36	1·18	1·17	0·80	1·34	3·92	2·43	2·63	2·85	1·38	0·79	0·26	20·11
1928	2·37	1·28	1·74	0·77	0·99	4·42	2·46	3·12	1·27	3·01	1·12	0·89	23·44
1929	0·06	−0·89	−0·07	1·13	1·85	2·79	3·85	1·45	1·91	2·97	2·52	3·08	20·65
1930	2·13	−1·01	1·66	1·36	1·88	1·52	3·55	1·93	2·49	2·92	1·15	1·57	21·15

1931	1·35	1·24	−0·34	2·10	2·67	1·54	3·56	1·92	1·39	1·46	2·03	1·59	20·51
1932	0·51	−0·38	2·10	3·14	1·46	1·51	3·59	0·60	3·04	2·69	1·79	0·82	20·87
1933	0·65	1·65	−0·82	0·50	1·33	3·27	2·53	2·03	1·31	3·91	−0·17	0·42	16·61
1934	0·82	−0·03	1·64	2·48	1·34	2·22	2·94	3·59	2·87	2·72	−0·14	3·17	23·62
1935	−0·27	1·77	0·08	2·28	1·28	3·28	1·27	2·53	4·92	1·44	1·97	0·95	21·50
1936	2·29	0·43	1·27	1·01	1·50	3·56	2·57	0·25	3·05	2·71	0·96	2·96	22·56
1937	1·93	1·76	−1·49	1·39	1·36	1·77	2·42	1·03	2·92	2·51	0·39	0·23	16·22
1938	3·12	1·65	−0·07	−0·06	3·33	2·52	2·91	2·46	1·65	2·63	2·13	1·06	23·33
1939	1·61	2·78	0·29	2·36	0·62	3·07	4·19	1·77	0·67	2·75	4·05	0·09	24·25
1940	1·78	−0·82	1·39	1·88	1·07	0·56	4·74	1·59	2·81	3·25	1·97	1·62	21·84
1941	2·41	−0·93	0·42	0·51	2·14	0·51	3·12	4·51	0·46	2·59	1·29	0·86	17·89
1942	2·10	−1·73	1·66	1·46	3·97	0·24	4·11	3·06	1·84	1·80	−0·05	2·84	21·30
1943	1·74	0·62	1·21	0·91	2·56	2·27	2·96	3·63	1·53	1·45	1·28	0·46	20·62
1944	1·97	0·85	−0·01	1·38	2·55	2·45	2·14	2·90	2·36	2·52	2·44	0·81	22·36
1945	1·35	0·93	1·11	1·07	2·54	3·26	1·34	2·93	1·40	1·61	−0·08	1·35	18·81
1946	1·75	1·48	0·09	1·65	2·57	1·58	2·43	4·39	1·37	0·74	3·23	1·13	22·41
1947	0·66	2·52	−0·12	1·91	1·91	1·97	2·67	0·40	3·36	1·04	1·27	1·75	19·34
1948	2·86	0·33	1·96	1·59	2·14	3·33	1·80	3·34	1·78	1·78	0·61	3·25	24·77
1949	−0·11	1·22	0·75	2·21	3·26	—	1·72	3·20	2·22	2·87	2·19	2·66	22·19
Mean of 42 years . .	1·2	0·8	0·9	1·5	1·9	2·2	3·0	2·8	2·0	2·2	1·3	1·3	21·1

drawn to represent the mean loss to be expected from varying values of annual rainfall passes through the point of average loss of 21·6 inches. It will be seen that wide divergences occur between individual years of sensibly equal precipitation ; however, as could be expected, the relative loss increases with rainfall up to a total of about 80 inches. For rainfalls of more than 80 inches the relative loss decreases, for, whilst in the years of low rainfall there are generally long spells of extremely dry conditions with little or no rain to evaporate, the loss increases with rainfall, until in years of very high rainfall with long spells of dull wet conditions, the gathering ground becomes so saturated that a larger proportion runs off.

The incidence of the rainfall, not only from month to month, but from day to day, has a marked influence on the losses on a gathering ground such as the Elan valley.

The area includes some rather extensive peat bogs and many square miles of long coarse grass, known locally as "feg." In the absence of rain, and with drying winds and hot sun, the grass-lands and the bog surfaces dry quickly. After only 2 or 3 weeks of such weather conditions a rainfall of up to 2 inches, falling in a short space of time, can be absorbed by the surface soil with its vegetation before any appreciable run-off reaches the reservoirs. The worst possible condition, from the water-supply point of view, results from a succession of dry and wet spells. It is conceivable that such a succession throughout the months (say from May to September) might produce a rainfall of the order of 12 inches with no appreciable run-off.

The high figures of loss shown in Table 10 might, at first sight, occasion some doubt of their accuracy, but later in the Paper evidence will be produced to substantiate the records.

The interim report of the Hydrological Committee of the Institution of Water Engineers, published in October 1936, suggested that the loss to be expected from such an area was about 19·8 inches in a year, with a rainfall equal to the long-average figure of 69·4 inches, according to the formula :

$$\text{loss} = 4{\cdot}8C\ \sqrt[3]{R}, \text{ where } C = 1 \text{ and } R \text{ denotes the rainfall in inches}$$

With an average annual loss in the Elan catchment equal to 21·6 inches, the value of C would be 1·095.

The long-average monthly loss, ascertained by deducting the run-off from the rainfall (see Table 7), is given in Table 11.

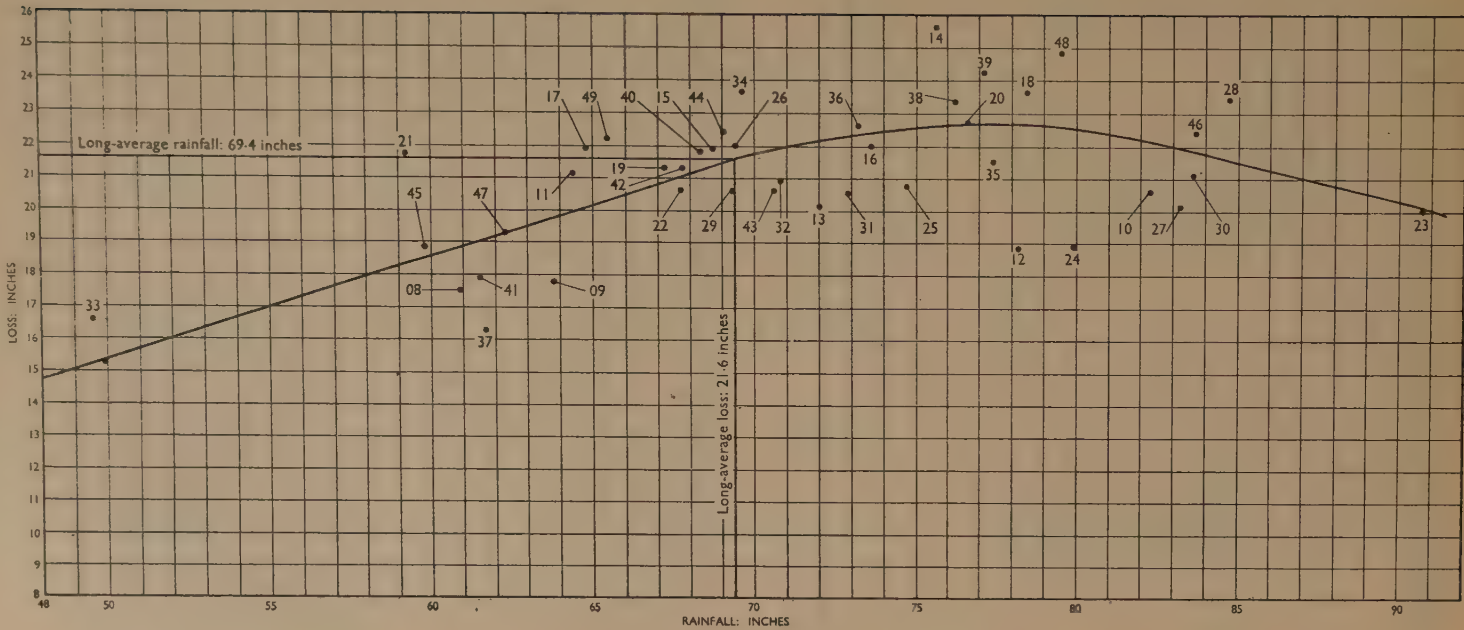
Doubt has occasionally been expressed of the wisdom of accepting records of run-off from reservoired areas, owing to the possible high loss from the water surface itself.

This loss is, in fact, insignificant in relation to reservoired gathering grounds of the nature referred to in this Paper.

Dr Penman indicates[7] that the highest figure for evaporation from water surfaces is not likely to exceed 30·0 inches per annum.

The top water area of the reservoirs in the Elan valley (omitting the recently completed Claerwen reservoir), is 840 acres or only about 2 per cent

Fig. 17



CABAN DAM. RAINFALL AND LOSS. 42 YEARS (1908–1949)

TABLE 11

	Long-average loss: inches
January	1·1
February	0·8
March	1·1
April	1·5
May	1·9
June	2·1
July	2·8
August	3·1
September	2·1
October	2·4
November	1·4
December	1·3
	21·6

of the total area of the gathering ground. If such a high figure as 30·0 inches be accepted as the loss from the water surfaces in such a terrain, then the long-average loss figure of 21·6 inches for the area is reduced to only 21·44 inches.

RAINFALL, RUN-OFF, AND LOSS ON THE UNRESERVOIRED GATHERING GROUND OF THE RIVER CLAERWEN (23,560 ACRES) AS MEASURED AT DOL-Y-MYNACH

The gathering ground of the River Claerwen lies within the area of 45,562 acres of the total catchment down to the Caban Dam. The area of the Claerwen catchment down to the weir at Dol-y-Mynach is 23,560 acres and records are available for the 23 years from 1927 to 1949 inclusive.

There are seventeen rain gauges in the area and the long-average annual rainfall is 72·4 inches. During the period of the gaugings, the stream was unreservoired, except for a small lake with a top water area of about 26 acres formed by the unfinished Dol-y-Mynach Dam. In 19 of the 23 years dealt with, all the run-off from the River Claerwen has passed over that dam into the Caban Coch reservoir. In the remaining 4 years, its waters have for short but varying periods been diverted through the Dol-y-Mynach tunnel into the Caban reservoir.

In 1926, in view of the importance of correct assessment of the run-off of this catchment area and its value in providing a check on the records for the whole gathering ground to Caban Dam, measuring and recording equipment was installed at Dol-y-Mynach Dam; this consists of two sharp-edged rectangular weirs, each 75 feet long with automatic recording apparatus.

In order to limit expenditure, the capacity of the weirs was restricted

to 1,200 million gallons per day, or 100 cusecs per 1,000 acres, which it was estimated would accommodate all but exceptional floods. In fact, this upper limit has been exceeded on an average only twice per year, and on all but one occasion for periods of less than 4 hours. It has, on these occasions, always been possible to obtain a manual measurement of the peak flood level; from such measurements and by deductions from records of flow at the other dams, a confident assessment of the discharge can be made. Any slight error in estimation can have only an insignificant bearing on the records as a whole.

In order to obtain a more accurate record of the drier-weather flows, a subsidiary weir, 16 feet long, was installed and flows of less than 24 million gallons per day (2 cusecs per 1,000 acres) are measured thereby.

Below will be found the statistics relating to the Claerwen catchment compared with corresponding records for the whole 45,562 acres above the Caban Dam.

It is, however, appropriate to refer here to two points of interest applicable to the Claerwen records themselves.

Maximum Rate of Run-off:

The highest discharge so far recorded was on the 20th September, 1946, when the flow reached about 112·6 million gallons per hour (212 cusecs per 1,000 acres).

Minimum Flow:

The minimum rate of run-off measured off this area was 2·8 m.g.d. (0·23 cusec per 1,000 acres) for 5 days in 1949. In 1947, the rate for 5 days fell to 3·0 m.g.d. (0·25 cusec per 1,000 acres).

Comparison of Records for Caban and Claerwen Catchments

Table 12 gives the basic data for the comparison of records for the whole area to Caban (45,562 acres) with those for the River Claerwen (23,560 acres) for the 23-year period from 1927 to 1949.

A comparative statement dealing with the rainfall yield and loss over each of the 23 years is given in Table 13.

In every year the rainfall above Dol-y-Mynach was greater than for the whole area to Caban. When related to their respective annual long-average rainfalls, however, the two areas show the same percentage for 7 years, whilst Caban is higher in 9 years and lower in the remaining 7 years.

Over the whole period of 23 years, the mean annual rainfall above Caban was 71·20 inches and above Dol-y-Mynach 74·10 inches, or 102·6 per cent and 102·4 per cent respectively of the corresponding long-average rainfalls; the close agreement between the last two figures tends to confirm the accuracy of the rainfall statistics as a whole. There is a similarly close agreement in respect of run-off; the mean run-off of the 23 years represents 70·1 per cent and 70·6 per cent of the rainfalls above Caban and Dol-y-Mynach respectively.

TABLE 12

	Whole area above Caban	River Claerwen above Dol-y-Mynach
Drainage area	45,562 acres	23,560 acres
Water surface area	840 acres	26 acres*
Long-average annual rainfall	69·4 inches	72·4 inches
Product of 1 inch	1,032·28 m.g.	533·75 m.g.
Mean annual rainfall, 23 years	71·2 inches	74·1 inches
Mean annual rainfall, as percentage of long-average	102·6 per cent	102·4 per cent
Mean annual run-off, 23 years	50·0 inches	52·5 inches
Mean annual run-off, as percentage of mean rainfall	70·1 per cent	70·6 per cent
Mean annual loss, 23 years	21·2 inches	21·6 inches
Mean annual loss, as percentage of mean rainfall	29·9 per cent	29·4 per cent

* Before construction of Claerwen Dam.

Fig. 18 shows the run-off plotted against the long-average annual rainfall each year for the two areas. The mean lines shown have intentionally been drawn to the same curve and indicate the mutual consistency of the records for the two areas.

The mean annual losses for the two catchments were 21·2 inches at Caban and 21·6 inches at Dol-y-Mynach.

The greater loss which occurs on the unreservoired catchment of the Claerwen is in conformity with the observations already made in the section dealing with losses by evaporation and absorption.

A further comparison of the mean rainfall and run-off each month at the two points is given in Table 14.

The close relationship between the rainfall of the two catchments is shown by the rainfall at the end of each month, expressed as the accumulated percentage of the mean yearly rainfall; the greatest deviation is 0·4 per cent only at the end of April and May, whilst at the end of August they are in exact agreement, 58·9 per cent of the mean yearly rainfall being recorded.

The figures of accumulated run-off are even more consistent month by month; the greatest difference is 0·2 per cent at the end of February and November, whilst at the end of March, July, and September they are in exact agreement.

SUMMARY OF DISCHARGES AT CABAN DAM DURING THE 42-YEAR PERIOD FROM 1908 TO 1949

Table 15 gives a summary of the water drawn off each year for supply, compensation, and operational purposes, and also overflow at Caban Dam.

It will be noted that the quantity drawn off for supply has increased during the 42 years from 6·69 inches (18·6 m.g.d.) to about 17·00 inches

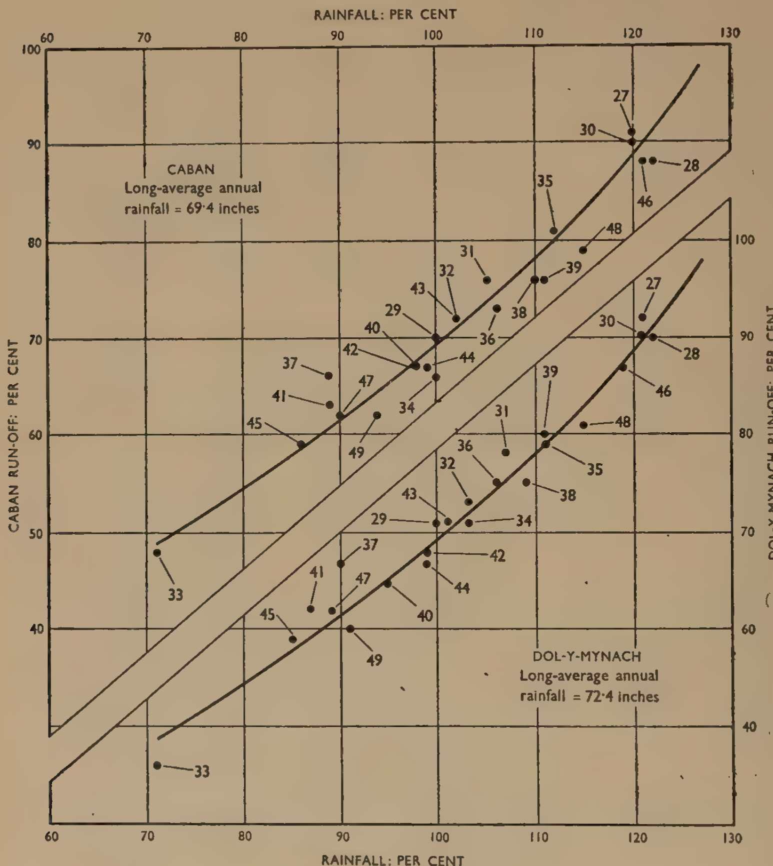
Accum. percentage	Dol-y-Mynach: percentage of long-average rainfall of 72·4 inches	Caban: percentage of long-average rainfall of 69·4 inches	Accum. percentage	Dol-y-Mynach: percentage of long-average rainfall of 72·4 inches	Accum. percentage
91	92	29	29	29	29
179	90	34	63	32	61
249	71	30	93	29	90
339	90	30	123	31	121
414	78	30	153	29	150
486	73	30	183	30	180
534	46	23	206	25	205
600	71	34	240	32	237
681	79	31	271	32	269
754	75	33	304	31	300
820	67	23	327	23	323
896	75	34	361	34	357
972	80	35	396	31	388
1,039	65	31	427	30	418
1,102	62	26	453	25	443
1,169	63	31	484	31	474
1,241	71	30	514	30	504
1,308	67	32	546	32	536
1,367	59	27	573	26	562
1,456	87	32	605	32	594
1,518	62	28	633	27	621
1,597	81	36	669	34	655
1,659	6[illegible]	32	701	31	686
	7[illegible]	30·5		29·8	

TABLE 14.—CABAN AND DOL-Y-MYNACH : COMPARISON OF MEAN MONTHLY RAINFALL AND RUN-OFF FOR THE 23 YEARS FROM 1927 TO 1949

	Caban						Dol-y-Mynach						Percentage of rain run-off	
	Rainfall			Run-off			Rainfall			Run-off			Caban	Dol-y-Mynach
	Mean of 23 years : inches	Per-centage of year	Per-centage accum.	Mean of 23 years : inches	Per-centage of year	Per-centage accum.	Mean of 23 years inches	Per-centage of year	Per-centage accum.	Mean of 23 years : inches	Per-centage of year	Per-centage accum.		
January .	8·7	12·2	12·2	7·2	14·4	14·4	9·0	12·1	12·1	7·6	14·5	14·5	83	84
February .	5·6	7·9	20·1	4·9	9·8	24·2	5·7	7·7	19·8	5·2	9·9	24·4	88	91
March . .	4·4	6·2	26·3	3·7	7·4	**31·6**	4·6	6·2	26·0	3·8	7·2	**31·6**	84	83
April . .	4·2	5·9	32·2	2·7	5·4	37·0	4·3	5·8	31·8	2·9	5·5	37·1	64	67
May . .	4·0	5·6	37·8	2·0	4·0	41·0	4·1	5·6	37·4	2·0	3·8	40·9	50	49
June . .	4·4	6·2	44·0	2·2	4·4	45·4	4·7	6·4	43·8	2·3	4·4	45·3	50	49
July . .	5·5	7·7	51·7	2·6	5·2	**50·6**	5·8	7·8	51·6	2·8	5·3	**50·6**	47	48
August . .	5·1	7·2	**58·9**	2·7	5·4	56·0	5·4	7·3	**58·9**	2·9	5·5	56·1	53	54
September .	5·2	7·3	66·2	3·1	6·2	**62·2**	5·5	7·4	66·3	3·2	6·1	**62·2**	60	58
October .	7·7	10·8	77·0	5·4	10·8	73·0	8·1	10·9	77·2	5·7	10·9	73·1	70	70
November .	8·5	11·9	88·9	7·1	14·2	87·2	8·8	11·9	89·1	7·3	13·9	87·0	84	83
December .	7·9	11·1	100·0	6·4	12·8	100·0	8·1	10·9	100·0	6·8	13·0	100·0	81	84
	71·2	100·0		50·0	100·0		74·1	100·0		52·5	100·0		70	71

NOTE.—Identical figures are shown in bold type.

Fig. 18



Caban and Dol-y-Mynach.

Rainfall and Run-off Each Year expressed as a Percentage of Long-Average Annual Rainfall (1927–1949)

(48·0 m.g.d.). The statutory rate of discharge of compensation water as laid down in the Act of 1892, namely, 27 m.g.d., has been maintained throughout, with the exception of the years 1944 and 1949.

In 1944, about 300 million gallons of compensation water was held back in order to provide a potential emergency supply for the Coventry Corporation, whose resources were being taxed to their utmost under war conditions. As Birmingham could not at that time guarantee, from its existing works, a supply to Coventry, powers were obtained by the Birmingham Water Order, 1944, to retain in the Elan reservoirs a part of the compensation

water at such times as the flow of the River Wye, from its gathering ground above Rhayader, exceeded a given minimum.

Again, in 1949, the compensation water was reduced by nearly 2,000 million gallons, in consequence of the operation of the Birmingham (River Elan Compensation Water) Order, 1948, made for the same purpose of enabling Birmingham to assist Coventry to meet the increasing demand, until such time as that city's River Severn supply could be brought into operation, or until the construction of the Claerwen reservoir sufficiently increased Birmingham's resources. Provision was made for the retention in the reservoirs of a proportion of the compensation water as a potential supply to Coventry, should the need arise. The authorized rate of discharge of compensation water varied inversely as a function of flow in the River Wye below its junction with the River Elan, but subject also to a minimum flow in the Elan sufficient to keep that river in condition. The need for augmentation of the Coventry resources did not in fact arise, and the retained compensation water eventually reached the river as overflow.

Table 15 shows that the maximum quantity lost by overflow at Caban Dam in the very wet year, 1923, was 52·02 inches, or an average of 144·4 m.g.d., whilst the minimum loss, 13·17 inches or an average of 37·2 m.g.d., occurred in 1933.

The average total run-off of the gathering ground during the 42 years amounted to 50·48 inches per year, or 141·3 m.g.d. The maximum was 70·69 inches (196·3 m.g.d.) in 1923 (140 per cent of the mean) and the minimum 32·99 inches (93·3 m.g.d.) in 1933 (66 per cent of the mean).

Impounding Reservoirs : Reliable Yield, etc.

The total capacity of the impounding reservoirs included in the original works was as follows :

	Million gallons
Craig Goch	2,028
Pen-y-Gareg	1,332
Caban Coch	7,815
	11,175

Of the 7,815 million gallons capacity provided in the Caban Coch Reservoir, a quantity of 664 million gallons has to be retained upstream of the submerged dam in order to keep the inlet to the aqueduct submerged, thereby reducing the effective storage to 10,511 million gallons, which, together with 10,625 million gallons, the capacity of the Claerwen reservoir, gives a total available storage of 21,136 million gallons.

From the analysis of the records given earlier in the Paper, it appears that, with the storage now provided, the reliable yield of the gathering ground may be as much as 106 million gallons per day. By the usual

computation, the 3-dry-year rainfall would be 80 per cent of the long-average rainfall of 69·4 inches, namely, 55·5 inches.

From *Figs 8* and *9* it is evident that in an 80-per-cent-rainfall year the loss by evaporation, etc., can be expected to be 18·0 inches. Deducting this 18·0 inches from the 3-dry-year rainfall of 55·5 inches, the resulting available yield is 37·5 inches, or 106 million gallons per day.

Mansergh's estimate of the safe yield at the time of the Parliamentary Bill in 1892 was 99 m.g.d. It was based, however, on the slightly smaller catchment area of 44,000 acres. Calculated on the present area of 45,562 acres this would be equivalent to 102 m.g.d. In evidence he also stated (4th April, 1892) that " it might be anything up to 105 m.g.d."

During the proceedings leading to the 1940 Act, the assessment was 104 m.g.d., but as a result of the investigations leading to this Paper, it would appear that 106 m.g.d. may possibly be obtainable, although it must be borne in mind that this possibility is based upon the assumption that the average rainfall of the three driest consecutive years will be not less than 80 per cent of the long-average rainfall.

It was previously considered that storage of 180 days of the required daily quantity would be adequate to provide for the longest drought likely to occur in the British Isles.

Investigations carried out by the Hydrological Committee of the Institution of Water Engineers revealed, however, that in the 1933–34 drought the maximum depletion of storage occurred at the end of 18 months (547 days) from the commencement of the drought and reservoirs which had provided the equivalent of 180 days' supply were practically empty and curtailed supplies became necessary before the 18 months had expired.

It is the Authors' opinion that, as a reasonable factor of safety, a further month's supply should be reserved, that is, the equivalent of 7 months, or about 210 days' supply.

Fig. 14 shows that to meet a daily demand of 106 m.g.d. a storage of 20,780 million gallons, or 196 days' supply, is required.

The actual effective capacity of the reservoirs at the Elan Works is 21,136 million gallons, representing 199 days' supply at 106 m.g.d.

Conclusions

Summarizing the analysis of the records covered by the Paper, the Authors submit the following conclusions.

Rainfall

The assessment of the long-average annual rainfall on a large area is not an exact science, but obviously the longer the period for which records can be obtained, the greater will be the accuracy, and in order to establish the figure of 69·4 inches in this case, a careful study was made of the records for many miles around the gathering ground as far back as 1887.

This figure was also confirmed by a report of the Meteorological Office made in 1940, which stated that "A value of 70·0 inches is given as the best estimate for the standard period 1881–1915, but there is hardly sufficient information to define the value within an accuracy of an inch."

So far as the rainfall for the individual years dealt with in the Paper is concerned, the Authors have confidence in their accuracy, for the following reasons :—

(1) The concentration of gauges is high.
(2) The siting of the instruments was, in almost all cases, determined by recognized experts—Messrs Symonds, Mill, and Carle Salter, and Dr Glasspoole.
(3) All gauges have been frequently inspected and well maintained.
(4) The results have been plotted and scrutinized monthly in collaboration with the Meteorological Office.
(5) The records of the two areas dealt with show a very close relationship.
(6) The results have been analysed annually in comparison with records of surrounding areas.

Run-off

The discharges to supply and compensation are accurately measured hrough submerged orifices.

The largest individual constituent of the run-off in most years has een the quantity discharged over the crest of the Caban Dam. The ccuracy of its recording is supported by the following facts :—

(1) The coefficient of discharge was determined by direct measurement of the flow over a full-size replica of a 3-foot length of the crest of the dam.
(2) The close similarity of the relationship of rainfall to run-off in periods of no overflow and in periods of high overflow.
(3) The confirmation of the records at Caban by those at Dol-y-Mynach where the gauging station has knife-edged rectangular weirs.

oss by Evaporation, etc.

The magnitude of the losses by evaporation and absorption is of conderable importance and was discussed by G. N. Croker,[8] who dealt with e run-off at a number of measuring points on the River Wye and the sociated rainfall. He showed that, whilst the losses for the Elan and aerwen were in accord with those for the area to the west, namely, the iver Irfon down to Abernant, they were much greater than those for the ea to the east, namely, the River Wye down to Rhayader. Dr Glasspoole s since pointed out that additional rainfall records, which have been tained for this Upper Wye area, indicate that the area above Rhayader

may well be about 4 inches wetter than the previous assessment and this would result in a loss for that area comparable with that for the Elan and Claerwen.

Reliable Yield and Storage

The reliable yield of the catchment area may prove to be as much as 106 million gallons per day. For full economical utilization of the yield of a gathering ground such as that of the Elan and Claerwen valleys, storage capacity equivalent to about 7 months of the reliable average daily yield should be provided.

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They would also make known their indebtedness to previous Chief Engineers of the Water Department—Messrs F. W. Macaulay, J. W. Wilkinson, and A. A. Barnes, MM.I.C.E.—under whose direction the work which has led to the collection of the hydrological data used to develop this Paper was carried out. Their enthusiasm and skill inspired, in all those members of the staff concerned with the field work and statistics, that interest without which the information collected might well have remained little more than a set of statistical Tables.

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The Paper is accompanied by seventeen sheets of diagrams from which the Figures in the text have been prepared.

Discussion

Mr R. C. S. Walters said that in Table 5 the Authors gave the average rainfall for various periods, and taking the three driest consecutive years, an average was obtained of 84·7 per cent, compared with the usually assumed 80 per cent. Mr Risbridger had said that it would be a bold man who would rely on 84·7 per cent, and Mr Walters said that he, and perhaps others, would like to keep to 80 per cent, notwithstanding the fact that the Authors had proved over the 42-year period that the average annual rainfall for 36 months was 84·7 per cent.

On p. 373 floods were referred to, and the Authors confirmed that the maximum flood had been 259 cusecs per 1,000 acres, and on the Claerwen part of the gathering ground 212 cusecs. That agreed with the Flood Committee's report for a gathering ground of 45,000 acres. He had seen figures for American gathering grounds where the floods had been double, and others half, but it was noteworthy that, from the information given in the Paper, during 44 years no suspicion of a flood greater than that referred to in the report of the Floods Committee had been recorded. There was nothing in the nature of a catastrophic flood of double the quantity; the Floods Committee had implied that their figures might be doubled for designing an overflow weir which might have to be made twice as long because the flood might be twice as heavy, but there was no suspicion that any flood had occurred on the gathering ground in question greater than the normal maximum flood; that was an interesting point to emphasize. It would be interesting if the Authors could give the computation for the figure 259 from 195 on *Fig. 15*.

On p. 374 the Authors computed a drought at 0·25 cusec per 1,000 acres for the lower gathering ground, and on p. 381, for the Claerwen gathering ground, a figure of 0·23 cusec per 1,000 acres. If he did not know (through lack of data) what the dry-weather flow was, he assumed 0·2 cusec per 1,000 acres for large gathering grounds, and it appealed to him very much that that figure was sustained by the Authors. Would they say how the figure of 0·06 cusec per 1,000 acres on p. 373, had been computed? That was the flow, he assumed, below the dam, and it would be interesting if the computation between that and 0·25 cusec per 1,000 acres was given; it would probably take into consideration the evaporation from the water surface of the reservoir, and other factors.

Mr Walters said that his remarks did not mean that, simply because certain facts seemed to be confirmed, all that information was known already. For instance, *Fig. 17* was a very interesting statement of the case that evaporation and other losses, when there was low rainfall, were less than they would otherwise be, because, as the Authors had stated on p. 378, there was not enough rain to evaporate, and therefore in years of low rainfall the evaporation was less. Similarly, in years of very high

rainfall, on the right of the curve in *Fig. 17*, the run-off was so quick that there was no time for it to evaporate, and therefore the evaporation loss was less than it might be for normal average conditions about the middle of the curve. It was true that the spots were dotted about, but probably the curve gave a fair average.

Another very interesting point was that in Table 10 the average loss for the three driest consecutive years was 18·63 inches, but the average annual loss was 21·1 inches. He would compare that with the Ministry of Health's Compensation Report,[9] reprinted in 1946, for reservoir " C." The rainfall there was very similar to that at Elan, about 71 inches. In that report the average annual loss was given as 18·13 inches, and the average loss for the three driest consecutive years was 14·96 inches, so that it seemed necessary to add about 3 inches to both the figures in the report; that at any rate gave the sense of it.

Storage and yield was a big question, but he would say that if the actual storage, the actual proved evaporation given by the Authors, and the actual yield were considered, the average annual rainfall, less the evaporation, agreed on the Deacon diagram and more or less on the Lapworth chart, which was pessimistic.

In conclusion, he wished to draw attention to a sentence on p. 388 of the Paper, where the Authors said " For full economical utilization of the yield of a gathering ground . . . " In using the word " economical " there, he thought that the Authors meant with regard to the three consecutive driest years, because it so happened that the economical dam and reservoirs might coincide with the three driest consecutive years. It might, however, be that some more information was needed about what was meant by the word " economical " in that context.

Mr R. W. S. Thompson said that he had compared the Authors' figures with the figures which had been collected by his predecessors and himself for the catchment area of the River Derwent in Derbyshire down to Lady Bower Dam. Some data obtained there had been published a few years previously by the Institution of Water Engineers.[10] He had looked at the rainfall figures as well as the run-off figures up to date, and they tallied in a remarkable manner with those presented in the Paper. For instance, the average of the past 20 years was extraordinarily close to the estimated true long-average for the gathering ground.

He thought that the rainfall figures given by the Authors were probably as accurate as any which were available, and the way in which they had been obtained and dealt with was almost beyond criticism. There were difficulties, of course, in obtaining accurate rainfall data, particularly in

[9] Ministry of Health Report of Technical Sub-committee on the Assessment of Compensation Water. H.M.S.O., 1930.

[10] R. W. S. Thompson, " The Application of Statistical Methods in the Determination of the Yield of a Catchment from Run-off Data." J. Instn Wat. Engrs, vol. 4, p. 394 (Aug. 1950).

hilly country, but everything possible had been done in the case in question to obtain figures of extraordinary accuracy.

The run-off data were also, no doubt, very accurate, particularly as so many of them had been measured by gauges on the aqueduct and the compensation gauge, but he felt that the figures of discharge over the Caban Dam might be slightly under-estimated. Recently he had been at Claerwen, when there had been an exceptionally high wind, and tremendous waves had been coming down the Caban Reservoir and causing a very exciting-looking overflow over the spillway. That might have been an extreme case, but he knew that on a similar rock-faced dam it was possible to get quite a good discharge over the crest with the water-level 6 inches below the crest, and there was bound to be an under-estimate of the discharge when measured in that way. He would like to suggest to the Authors and to the Birmingham Corporation that it might be worth while to construct a gauge below the Caban Dam and a little way down the river, perhaps 200 yards, of the Venturi-flume type, to measure those big overflows independently of the crest of the dam and compare the results with the crest discharges for a few years in order to see whether, in fact, the Birmingham Corporation had a little more water in the catchment area than they seemed justified at present to think they had. He thought that that might be found to be so.

The evaporation loss of 21 inches seemed high. It might be that the explanation which had been given was correct, but if the run-off had been slightly under-recorded, the evaporation loss would be overestimated. In the Paper it was suggested that the accuracy of the Caban Dam measurements was borne out by the gaugings at Dol-y-Mynach, but Mr Thompson wondered whether wave action might not come in at Dol-y-Mynach also. He did not suggest that there was anything wrong with the calibration of the overflows, the sharp edge at Dol-y-Mynach, or the rock edge at Caban, but they might be affected by wind action and he would like the Authors to consider that point. He was sure that they would always get their 106 million gallons per day, and he thought that they had a much bigger margin than many water undertakings in Great Britain.

He had a slight criticism on a point of detail in the Paper. Taking Table 4, if the figures were added up it would be found that the 6 summer months produced 41 per cent of the rainfall; and in Table 6 the 6 summer months, April to September added up represented only 32 per cent of the mean annual run-off. The large difference between the 6 summer and the 6 winter months had an effect on the shape of the curves shown in *Figs 7* and *13*, and, with due respect to the Authors, he would like to suggest that possibly the smooth curves shown ought to be slightly corrugated. Dr Glasspoole was present and might contradict that, but the point which Mr Thompson wanted to make was that the average run-off in the 6 summer months was so much less, and had a slightly greater variability, so that one was almost bound to get the lowest 6 months

including the summer months, whilst the lowest 12-month period would include both winter and summer and be disproportionately greater. In the case of rainfall the effect was not so pronounced, because the difference between summer and winter rainfall was not great, but with run-off, the summer accounted for only 32 per cent of the total and it had a bigger effect. The position of the spots on the graph rather suggested that. It was true that on that gathering ground the worst conditions had not been experienced, but if a line were drawn through the points for the Elan valley, it would be found that they suggested corrugated curves. In the case of a catchment area with as much storage as at Claerwen, he did not think that the smoothed curve introduced a great deal of error. But in dealing with an under-reservoired catchment, where it was necessary to get more water, and where an additional reservoir could not be built, but more water could be brought in, the question of the shape of the lower part of the curve would be of great importance, and the Authors' smooth curve, which was a sort of envelope, was not, perhaps, quite the limit of accuracy which was possible.

The lowest yield would be given by run-off in periods which were approximately multiples of 6 months—6, 18, or 30 months, according to whether there was little, normal, or excessive storage. It seemed to be an accident that the lowest run-off experienced in Birmingham had been in 16 months, because in the next 2 months in 1934, August and September, they had had 13 inches of rainfall which produced about 8 inches of run-off. That had been fortunate, because it just made the 18-month drought rather less severe than it might have been. The proper thing to do was to take 18 months as the period which in that sort of climate was likely to produce the most testing circumstances.

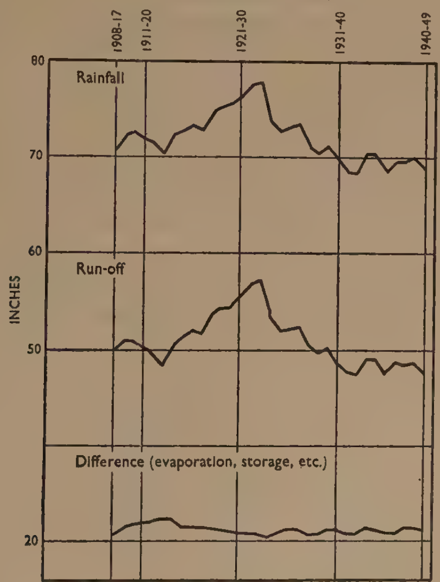
Dr J. Glasspoole observed that the residual mass curve, which was based on the cumulative excess or deficiency, depended on the starting point. If records covering earlier years had been available, *Fig. 3* might not have looked so convincing. The arguments for taking 1881 to 1915 as a standard for reference had been based on records covering 200 years, and the report on "Determining the General Rainfall over any Area," [3] drew attention to the period 1887 to 1930, but recommended that 1881 to 1915 should be adopted for the present as a standard for reference. Mr Barnes had been a member of that Committee, but in Dr Glasspoole's view the Authors' policy of mentioning both periods was a reasonable one. The procedure of inspecting rainfall stations at least every 2 years, and of plotting rainfall maps monthly, was especially welcome, and it was to be hoped that other water engineers would adopt the procedure, as indeed a good many of them did, and that the Meteorological Office would be able to resume visits and inspections.

He had plotted the values of rainfall, run-off, and loss as running 10-year means, and the shape of those curves was quite different from that of the residual mass curve of rainfall, although plotted from exactly the

same data. The trends of the rainfall and run-off curves appeared identical, but the loss curve was much more uniform and had the maximum and minimum at different times from the maximum and minimum of the rainfall and run-off. (See *Fig. 19*.)

Fig. 19

RUNNING 10-YEAR MEANS



ELAN AND CLAERWEN AREAS: RAINFALL AND RUN-OFF

When wet years set in, the ground storage and stream storage made their demands, but with further wet years their capacity to absorb became smaller. The Paper showed that a series of wet years started in 1910, when the storage was made up, but after the wet years, the losses eventually decreased. He pointed out that the minimum in the loss curve was at about the time of the maximum of the rainfall and the run-off curves. He suggested, for the consideration of the Authors, that that arose from the fact that there had been dry years before that period, and the loss met that demand, but with the wet years the ground became saturated and satisfied, and the loss then became a minimum. It was interesting to note that the general trend of the loss in recent years had been to an increase where there was an indication that the run-off had decreased. That was very similar to what had happened in the Thames valley. In the Thames valley the range of loss in the 10-year means was about 3 inches for the period 1908–1950. The range in his curve of loss for the Elan and Claerwen area was 1·7 inches, which was the difference between the means of 10

years and therefore corresponded to 17 inches, which was of the order of the loss for any individual year.

He knew of no variation in temperature or sunshine which corresponded to that trend of loss. Similar running 10-year means of temperature and sunshine for the seasons of summer, autumn, winter, and spring had been prepared for England and Wales. None of the trends was similar to that trend of loss, nor was there any similarity in the trends of the winter rainfall as distinct from the summer rainfall. It was quite a distinct trend, and that suggested that the trend was a storage factor rather than a meteorological or evaporation factor.

Figs 8 and *9* showed that in some years the annual values of rainfall and run-off did not fit the curve precisely. It was noticeable that those years which were plotted above the curve—the years which had more run-off than would be expected from the rainfall—followed a wet October to December, whilst the years plotted below the line—the years which had less run-off than would be expected from the rainfall—followed a dry October to December. Taking the 6 years with the greatest departures above the line and the 6 with the greatest departures below the line, the difference in the mean rainfall during the preceding October to December had been of the order of 7 inches, so that storage again came into the picture. He thought that those two factors might be of interest in interpreting *Fig. 17*.

The Authors had dealt with three variables, namely, rainfall, run-off, and loss. The next step seemed to be to break down the loss into evaporation, absorption, and any other variables. It seemed from what he had already said that the storage was more variable year by year than the evaporation. Dr. Glasspoole was unable, as yet, to offer any suggestion why the loss was as much as 21 inches when 17 inches might have been expected, but it was clear that the figures did hang together extremely well.

Mr J. C. Dickinson stated that, recently, he had had occasion to deal with two catchments of comparable size, which he would call Catchments A and B. Catchment A was about 40,000 acres, rather long and narrow, being about 20 miles in length, largely cultivated, with limited surface soil over impervious rock. It was steep, and its climate was insular equatorial. The precipitation at the outlet of the valley was about 214 inches per annum, but such was the maldistribution of rainfall that the average over the catchment was only 124 inches, and there were records of intensities of 20 inches in 24 hours. It was interesting to compare that maldistribution with the comparatively regular distribution over the Caban catchment. Catchment B was about 90,000 acres, slightly elongated, generally covered with heavy jungle, again with underlying impervious rock, and the climate was insular tropical. The average rainfall was 106 inches. Catchment C (the one dealt with in the Paper) was 45,600 acres, with an average rainfall of 69½ inches.

It seemed to him that under the extremes of rainfall and sun found in

the tropics any abnormalities would be emphasized, and it was therefore necessary to submit to critical examination anything which appeared to be a general rule in a catchment such as Caban. So far as long-term run-off was concerned, the figure for Catchment A was 71 per cent, and for Catchment B 63½ per cent, whilst he had taken the figure for Caban at 70 per cent. He did not place much value on that long-term average run-off as a yardstick to apply to shorter periods; all that he would suggest was that those figures showed that heavy tropical jungle was a very greedy consumer of water. The average annual loss in the three cases was 36 inches, 38½ inches, and 21 inches respectively, and he fancied that it was the heavy vegetation which consumed the larger quantity of water.

The minimum recorded flows for the three catchments were: Catchment A, 1·1 cusec per 1,000 acres; Catchment B, 2·16 cusecs per 1,000 acres; and Catchment C, 0·06 cusec per 1,000 acres. Taking the relative sizes of those compared with the average daily precipitation gave figures of 70, 180, and 7½, so that he would suggest that the heavy jungle was compensating for its greed by smoothing out the low-flow periods, whereas the Authors were probably right in suggesting that the bog areas in their catchment had made the condition distinctly unfavourable. In the matter of maximum recorded flows, Catchment A showed a figure of 750 cusecs per 1,000 acres, Catchment B a figure of 131½, and Catchment C a figure of 259 cusecs per 1,000 acres. Having regard to the intensities of rainfall, the relationship between A and C was not unreasonable, and he thought that B was the odd one, because the heavy jungle had smoothed off the tops of the peaks.

Mr Dickinson said that, with certain general characteristics in mind, he thought that it would be interesting to investigate *Fig. 8*. The result was far from a grouping of points above a mean line, and he would suggest that *Fig. 8* should really be represented by a lenticular space bounded on the top by a straight line representing such conditions as gave the minimum losses, and on the lower part by a curve showing the conditions which gave rise to the highest losses. Under tropical conditions the width of that lenticular space between the two curves was so great that it was almost impossible to plot anything resembling *Fig. 9*.

The Authors made a plea for continuously-recording gauges. In the comparatively equable climate of Wales they had collected a mass of thoroughly reliable data by daily and monthly readings of rain gauges, but even there they failed completely in the case of their maximum flood, because their 3 inches of rain fell in two different days, when obviously it had occurred, as they said, in a short period which bracketed the division between the two days.

Further to illustrate that point, he would like to give a few figures giving the history of two floods which occurred in Catchment A. The precipitation had been recorded on more than 60 rain gauges scattered throughout the area. Both floods followed steady rain, so the catchment might be

assumed to be thoroughly wet. The figures were for 5 days, and he would call the two floods (a) and (b).

TABLE 16.

Day	Flood (a) : thousands of acre-feet	Flood (b) : thousands of acre-feet
1	8·15	13·6
2	50·08	36·4
3	43·28	20·3
4	19·33	12·7
5	11·91	10·77
	132·75	93·8

It would be observed that in the case of the flood (a) there had been a total of 93,000 acre-feet precipitation in a period of 2 days, whereas in the case of the flood (b) there had been only about 56,000 acre-feet. It would be thought, therefore, that the flood in case (a) would be very much greater than in case (b), but as a matter of fact the peak of (a) had been, if anything, slightly less than that of (b), the reason being that the comparatively level top of the flood hydrolograph in (a) lasted for about 54 hours, and in (b) for only about 18. In the absence of continuously-recording gauges that point would not be made manifest. He made no apology for some degree of inaccuracy in observations of those big floods, because the physical conditions were so unpleasant that the attention of the observer was a little distracted, and he might be forgiven for not observing very closely, whereas a continuously-recording gauge, unless physical damage occurred, would record and provide the necessary data.

Mr A. G. McLellan asked the Authors whether he would be right in assuming that the observers in the Elan and Claerwen valleys, almost since the inception of the records, had been employees of the Corporation. He was at present concerned with the hydrological investigation of a gathering ground on which nineteen rain gauges had been established, and because his own undertaking had no organization in that area, and in the absence of volunteers, paid observers had to be used. It would probably be agreed that the enthusiastic amateur was the best rain-gauge observer, but they had had to use local farmers, etc., and he would not say that on the whole the result was always satisfactory.

Like Mr Thompson, he had imagined that the wind carry-over might have been appreciable, but assumed that the Authors would have considered that and decided that, like the evaporation from the water area, it could be neglected.

Mr McLellan would have liked the Paper to have dealt a little more with the readings of what might be called the " ratio station " at Nant-

gwillt which existed before the present gauges had been installed at Elan. He was trying, on a new gathering ground, to arrive as quickly as possible at a long-period average figure, and it would have been interesting to know how long it took to get a reasonable approximation to the average figure by the application of the readings of that ratio station to the readings from the four additional rain gauges installed by Symons in 1891. It would be reassuring to know that in a short time before formulating a scheme it was possible to arrive at an average figure with a certain degree of accuracy, so that it could be adopted with some degree of confidence.

For those who were starting to investigate a gathering ground, in his own case of some 30,000 acres, it was an object-lesson and an encouragement to learn the results of what Birmingham had done, but at the same time he wondered whether some of the practical results, apart from the academic interest, might not be more apparent than real. At the end of the Paper, for instance, the Authors advocated that 7 months' storage should be provided. That was after they had made the refinement in calculating the yield to be 106 rather than 104 million gallons a day. He took it that the 7 months' storage, which they suggested, in effect amounted to over-reservoiring the gathering ground—in fact, reservoiring it on the basis of more than 3 dry years. They had not got that storage, but had a condition of reservoiring for 3 dry years, and should therefore regard the safe yield as less than their calculated figure.

Mr McLellan thought that the paper was one of the most detailed hydrological analyses to have been published and he expressed the hope that other such analyses might be made available to those interested in the subject.

Mr C. F. Lapworth expressed his admiration for the very useful factual material contained in the Paper. Mr Thompson, he said, had made a point on *Fig. 14* which he wished to underline, regarding the use of a smooth curve for the minimum run-off diagram. He had himself investigated a very large number of these minimum run-off diagrams of various catchment areas, and he had invariably found that, in areas of high rainfall such as the Elan, there was a distinct cusp at 12 months, 24 months, and 36 months, and not, as shown in *Fig. 14*, a smooth curve. That was borne out by the fact that, if the actual figures for the Elan catchment were taken and put on that diagram, the period for 8 months was nearly 20 per cent, and the period for 12 months was nearly 40 per cent, of the long-average annual rainfall; if those actual figures were plotted, a different sort of curve was obtained from that shown on the diagram. The practical effect in the case in question would have been negligible, because it so happened that the storage required for 16 months would be almost the same as that required for 14 months, but in other cases, where the catchment was not so fully reservoired, it would make a difference. From that point of view he thought that the method was not to be recommended.

On p. 386, the Authors gave Mr Mansergh's estimate of the yield of the

catchment area, which, taking the present area, would be equivalent to 102 million gallons per day, which was very close to the figure of 106 million gallons per day given by the Authors. It is probable, however, that in those days, the figure which would have been used for storage would have been much less than the amount of storage which was now provided.

On the same page, reference was made to the fact that 180 days' storage had been, at one time at any rate, considered adequate to provide for the longest drought likely to occur in the British Isles. The amount of storage required for any particular catchment area depended, of course, on a number of factors. It depended on whether the rainfall was low, on the proportion of the average run-off which was required, and on whether the stream was flashy or not. In areas of low rainfall much more than 180 days was often provided. He knew of two cases of storage reservoirs which had amounts of storage of between 350 and 400 days.

Mr N. A. F. Rowntree thought that the term "3 driest years" was an incorrect assumption. It had been interesting to note that the Authors, in introducing the Paper, had not used that expression but had said "3 dry years," which was more accurate, and which had been used by Thomas Hawksley nearly 100 years ago.

Many of the calculations in the Paper were on a calendar year basis, notably Tables 5 and 7 and *Figs 7* and *10*. He wondered whether the Authors had considered using annual moving averages instead of taking arbitrarily January to December in each year. Was there any reason for not taking from 1 October to 30 September, and any or all of the other 12-month periods ? For example, taking the extremely dry period referred to in the Paper from March 1933 to February 1934, the rainfall had been only 43·2 inches, the run-off 28·1 inches, and the losses 15·1 inches. Inserting those figures in *Fig. 8*, another point was obtained much lower down than the 33 marked as the lowest ; it was, in fact, just off the chart. By taking many more of those "moving averages" it was possible to get quite a large number of points at that end of the curve, which might help to establish its position more accurately.

The same comment applied to *Fig. 17*, to which other speakers had referred. There was a very considerable scarcity of points at both ends of that curve. So far as the high rainfall end of the curve was concerned, looking at the scatter of the points marked, he would say that if they represented the information on which that line had been based, it would be just as logical to draw a horizontal line on the 22-inch loss line, or thereabouts. That did not mean, however, that the shape of the Authors' curve was not well reasoned, and one would tend to agree, but it would be good to see it confirmed by more data.

Another point which had occurred to him, and which had to some extent been raised by other speakers, was that instead of taking yearly figures for the purpose of computation it might be possible to take 6-monthly periods,

or in other words to consider all the summers and all the winters and then match them together to produce years, not necessarily taking the lowest of each, but matching the figures to give a year or period of years corresponding to what might be expected in 3 dry years. In that way much of the scatter which was obtained in *Fig. 17* and other diagrams might disappear.

Now that such an excellent long period record had been published, it should be possible to establish the probability of occurrence of periods of drought, for example, the number of times in 100 years when a drought of 1933–34 dimensions might occur.

Mr W. N. McClean thought that the Paper was the most complete record yet produced of the rainfall over a definite area of about 70 square miles and of the run-off or draw-off over a period of 42 years, but there was no measure of loss.

Whilst monthly records could give useful information regarding water resources and the relation of storage to supply, they were insufficient for the water survey and record of the River Wye and for problems of control, usage, and flood. With regard to *Fig. 2* (rainfall distribution), Mr McClean said that in his opinion the Paper would gain by having a similar map showing the height contours, giving the positions of the rainfall gauges, and showing whether they were continuous recorders, or daily, or monthly gauges. That would be very informative, as the rainfall of a rain-period was admittedly orographical ; and it would not alter the isopercental distribution of the month.

Taking the 42-year period 1908 to 1949, it had been pointed out that the annual rainfall average was 71·6, the run-off 50·5, and the loss 21·1. In Table 4 the Authors gave the assessment of " area " rainfall, and in Table 6 the volumetric measure of run-off, whilst in Table 10 they computed the loss by evaporation and absorption, which was rainfall less run-off. He felt that there was perhaps something missing there, in that the rainfall table, Table 4, did not give the summary for those 42 years. It might go very well at the bottom of Table 6, giving the resultant facts of run-off, rainfall, and remaining loss.

With regard to the dry periods, percentages were given in Table 5 which were percentages of the year's average rainfall. It seemed to him that those percentages should give the relative rates of rainfall. When it was said that the 3 dry years gave a figure of 254 per cent, that did not give the rate. The figure for the rate was 84·7 per cent. The same applied to the monthly periods ; he thought that the percentages given were not the best which could be used there. Knowledge of the average rate of those different dry periods was needed, and it was a little disappointing that the periods in the rainfall table were all the months up to 12, and then 16, 20, 24, 30, and 36, giving quite a number of values ; but in Table 8, which dealt with the run-off, the lowest run-off, under 12 months, was only given for 4 and 8 consecutive months. He missed having run-off

values for all the driest periods and had extracted them from Table 6. It was remarkable to see how the period of 9 months in 1929 corresponded to the longer period in 1933–34. Those two dry periods seemed to be outstanding in the 42 years, and from certain mass diagrams which he had made, it seemed as though such extreme dry periods did not extend to much more than 18 months—that was a critical point.

Mr R. H. MacDonald, commenting on the residual mass curve shown in *Fig. 3* of the Paper, stated that the residual mass curve for the Nile for the same years was very similar to the Caban curve except that it was upside down, with the wet period at the beginning and the dry period at the end. It did not mean very much, he thought, except that those periods did occur, but whether they really occurred in cycles of that kind or not it was still very difficult to say.

On p. 349, the Authors said " Obviously the true long-average rainfall is the average of the longest possible period over which rainfall records can be obtained. . . ." Would the Authors qualify that ? If one had short rainfall records the true long-average would be the mathematical truth only for the time being.

In Table 10 the mean monthly evaporation figures were given. Those were slightly different from Dr Penman's figures for the British Isles. A few years previously Mr MacDonald had worked out, on a much shorter number of years, the figures for the River Shin, in Scotland. Those figures were very similar to those found at Caban. He was glad to see his figures for mean monthly losses thus justified.

Would the Authors state that where they dealt with the gathering ground, on p. 347, that there was no seepage into or out of their areas which affected the water-supply at all. It was well known that the Welsh hills were like that, but perhaps it should be mentioned for the record.

*** **Mr H. F. Wilmot** remarked that the use of isopercentals was particularly worthy of note, and the manner in which they were used to check suspect readings.

The residual mass curve of rainfall (*Fig. 3*) conformed very closely with the similar curve relating to the discharge for the River Thames, for which the periods of shortage and excess above the normal were 1883–1909 and 1910–41 respectively. That would indicate that the weather over the areas concerned was, on the broad outlook, relatively similar; since the records for the Thames could be considered very reliable Mr Wilmot was hesitant in raising the question of whether the earlier recordings for the Severn were accurate ; for instance, where heavy intense rain occurred, had the older rain gauges in the past been of adequate capacity, when recordings were taken at considerable intervals of time ?

The percentage errors in the two rain gauging records shown were + 6 and − 11, and some explanation how that might have occurred would

*** This and the following contribution were submitted in writing upon the closure of the oral discussion.—SEC. I.C.E.

be reassuring for the accuracy of the other readings; an excess should not be possible without interference by an unauthorized person. Were further errors confined in large measure to the same sites or were they widespread?

The presentation of the facts was so clear and straightforward that it tended to minimize the difficulties of the hydraulic engineer in assessing potentialities for water supplies where the data available were less adequate. It should be fully stressed that conclusions based on a period of less than 1887–1930 (48 years) in the case mentioned might be very badly out, for example, 1910–30 (20 years), 1887–1910 (24 years), or 1930–49 (19 years).

The assessment by Mr Mansergh in the first instance was really remarkable when taken over a long term. In 1910, however, there might well have been serious misgivings that he was too optimistic without wishing to belittle his sound conclusions; it had so happened that the years of data available to him were helpful in arriving at the conclusions he formed.

Mr Wilmot had had occasion to investigate flow conditions of the River Severn at Bewdley (1921–49), and had tried unsuccessfully to correlate the levels and discharges at Worcester (where levels alone were available from as far back as 1862) to extend the results and obtain greater validity for them. The period of build-up existed from 1921–32 and recession lasted until 1944; the average annual discharge between October 1921 and October 1949 was 2,190 cusecs. The trend to 1932 closely followed that of *Fig. 3*, but there was a marked difference for the succeeding years.

From the water-supply view point, *Fig. 14* giving the maximum deficiency which had, and which might, occur over varying periods of consecutive months was the all-important result which justified the long analysis preceding it.

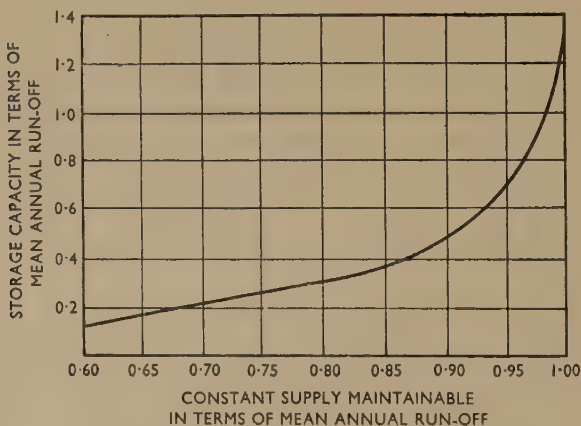
Mr G. Bransby-Williams shared the Authors' regret that there had been no recording gauge on the catchment in 1946. Autographic records from an earlier date would have furnished useful information about the frequency and intensity of heavy falls in short periods and data for the construction of a basic hydrograph for the catchment.

The 42 years' record of run-offs seemed to be a more satisfactory basis for estimates of storage capacities and run-offs than the rainfall figures, which introduced an extra element of uncertainty into the calculations. There were factors affecting run-offs that did not influence rainfall. The extent to which they did so might be judged by the discrepancies in the proportional ratios between them in individual years. It might be added that, whereas the frequency distribution in a long series of records of annual rainfall in Britain was nearly symmetrical, and conformed closely to the normal curve of probability, such records of run-offs as it had been possible to construct from the data available showed distributions that were definitely askew.

The method of calculating storage by means of empirical formulae based on the assumed run-offs in a hypothetical series of dry years, which

had been inherited from the Victorian engineers, ignored the economic aspect of the problem and also the relationship between the capacity of the reservoir and the quantity that could be supplied from it. The Figures in the Paper supplied data for a different form of calculation. A mass depletion curve could be plotted from the run-off records which would show the storage required to maintain a constant supply (for all purposes including losses from the reservoir) equal to 90 per cent, 80 per cent, 75 per cent, and so on, of the mean annual run-off throughout the 42-year period. That storage would be what Allen Hazen had called the "annual" storage; that was to say, the storage required to equalize the annual run-offs over a term of years. To that should be added what he had termed the "seasonal" storage that was needed to balance the excess run-offs in the wet seasons of the year with the deficiency in the dry seasons. That was obtained from records of monthly rainfall.

Fig. 20



Curve Showing Storage/Supply Ratios

The curve in *Fig. 20* had been plotted to show the relationship between storage and supply on the Elan and Claerwen catchments, in terms of the mean annual run-off, obtained in the manner described. The total existing reservoir capacity was 21,136 million gallons. That was about 0·432 of the mean annual run-off, taken as 48,864 million gallons, which was sufficient for a constant supply of 0·88 of the mean run-off or about 116 million gallons per day. The figure of 106 million gallons per day in the Paper thus seemed to allow a substantial margin.

The form of analysis which Mr Bransby-Williams had briefly described should be applied to actual records of run-offs extending over a period of years. But the storage/supply-ratio curves obtained for a number of

catchments, with varying characteristics, showed sufficient consistency to allow an appropriate curve to be selected for catchments on which no stream gaugings had been made, which would be sufficiently reliable, at all events for the initial stages of a storage-reservoir project.

Mr Risbridger, in reply, said that Mr Walters had stated that it was usually assumed that the average rainfall of the three driest years would be 80 per cent. The Authors had stated in the Paper that, although they had found hitherto that their average rainfall for each of the three driest years had been 84 per cent, they had based their deductions of the yield and the storage required on less than 84 per cent; in fact they had based them on 80 per cent, according to the rainfall curve developed by Dr Glasspoole from observations for the whole of Wales over a period of 74 years.

Mr Walters had referred also to the question of floods. Their maximum flood had been, of course, only the normal maximum flood, and they knew that they were as liable as any other upland gathering ground to be subjected one day to the "catastrophic flood" referred to in the report of the Flood Committee. If such a "catastrophic flood" did occur, however, Mr Risbridger was confident that a visitor to the district 2 days later would find the dams—both the old ones and the one recently built—still in position.

With regard to the reason for the difference between 0·06 cusec per 1,000 acres recorded at the Caban Dam and the figure which the Authors gave of 0·25 cusec per 1,000 acres for the lowest run-off, the difference arose because the 0·06 was measured at Caban after evaporation from the surfaces of the reservoirs had been deducted.

Mr Walters had asked for the computation of the figure, 259 cusecs per 1,000 acres, given as the maximum rate of run-off on p. 373, from the figure of 196 cusecs per 1,000 acres given in *Fig. 15*.

The latter figure represented only the maximum rate of flow over the crest of the Caban Dam. The maximum rate of run-off should include the maximum rate of increase of water stored in the reservoir itself, the rate of supply to Birmingham, and the rate of discharge of compensation water.

The rate of increase of storage was calculated by taking the greatest rate of increase in water level between 1 p.m. and 2 p.m. on the 20th September, which represented 4,200 cusecs. The rate of discharge to the aqueduct and to compensation was, together, 130 cusecs. Those figures, added to the rate of discharge over Caban Dam at 2 p.m., 7,450 cusecs, produced a total of nearly 11,800 cusecs or 259 cusecs per 1,000 acres. The peak rate of overflow at Caban Dam at about 3 p.m. on the same day was only 196 cusecs per 1,000 acres.

Mr Walters had requested some clarification of the term "full economical utilization of the yield." The Authors had had in mind, when using that expression, the storage capacity necessary to make the maximum use of the run-off of the three driest consecutive years, assuming that the rainfall of that period would be 240 per cent of the long-average annual rainfall.

Hence in *Fig. 14* the curve of actual minimum run-off in 36 consecutive months experienced in a period of 42 years had been amended as shown by the dotted line to represent the lower 3-year run-off which might be expected as a result of the 3-dry-year rainfall actually experienced in the same 42 years being more than the 3 driest years' rainfall in Wales during a period of 74 years as given by Glasspoole.

Whilst dealing with that point it might be an advantage to refer to the suggestion made by one or two other speakers that the storage provided erred on the side of safety. Mr Risbridger was of the opinion that, if such was the case, the margin was by no means extravagant when the supply to an important industrial city like Birmingham was at stake. *Fig. 16* showed that had the demand from the gathering ground in 1933–34 been 106 million gallons per day, 17,615 million gallons would have been drawn from storage, leaving only 3,520 million gallons in the reservoirs, or 33 days' supply at 106 million gallons per day. It should be remembered, however, that the capacity of the tunnel and cut-and-cover sections of the aqueduct was 75 million gallons per day which, with 29 million gallons per day compensation, produced 104 million gallons per day as the anticipated maximum demand from the gathering ground. Even so, with that demand, the reservoirs would have been depleted in 1933–34 by 16,621 million gallons, leaving only 4,514 million gallons or 43 days' supply.

When reservoirs, upon which the supply to a vast industrial area depended, contained only enough for a few weeks the position was already serious, and necessitated precautionary measures entailing great inconvenience to consumers and supplier alike. Complete failure of the supply would result in huge financial loss to industrial consumers. It was, therefore, considered a justifiable insurance to provide storage with a reasonable margin of safety.

The additional cost spread over the loan period was not of a serious order when weighed against the assurance provided and the total cost of the scheme. It would seem a little illogical for engineers—who, quite rightly and unashamedly, adopted factors of safety of 3 or 4 when dealing with structural materials—to endeavour to provide storage capacity to a factor of safety of unity, when the storage required depended upon a relatively unpredictable factor such as the incidence as well as the quantity of rainfall.

Mr Thompson had drawn a diagram corresponding to *Fig. 13* but with curves shown by "wobbling" instead of smooth lines. That would be correct if the curve represented run-off starting from January, but that was not the case; the Authors' curve represented the lowest run-off in 4 months starting in any month of the year, and the run-off in 8 months starting from any point in the year. If January were used as a starting point, "wobbles" would gradually diminish until after 3 or 4 years they disappeared altogether.

In reply to Mr Thompson's suggestion that a Venturi-flume gauge

might be constructed below the Caban Dam, from the point of view of its practical usefulness, it would not now be worth while, since the gathering ground was now fully reservoired and the necessary expenditure could therefore hardly be justified.

There was the question of unmeasured loss over Caban Dam from wind action. It was true that some water was blown out but, in proportion to the total run-off from the gathering ground, it was almost negligible. Mr Risbridger had seen it blowing over and, with a gale in the right direction, a considerable amount did go over. From observations of river levels below the dam, however, and consideration of the aggregate duration of conditions leading to blow-out, the Authors had come to the conclusion that the total loss from that cause was not of such a magnitude as to affect appreciably the total-annual-run-off computations.

The only point in Dr Glasspoole's remarks to which Mr Risbridger felt able to reply without further consideration was the tendency to disagree with the aptness of choice of the particular 44-year period on which to base the assessment of the long average. Dr Glasspoole had expressed similar disagreement before, but the long-average rainfall deduced from the 35-year period, with which Dr Glasspoole agreed as a standard, differed from the Authors' 44-year period computation by only 0·1 inch in 69·4 inches, so that the averages of the particular periods were not far apart in any case.

Mr Risbridger thanked Dr Glasspoole for his very interesting curves in *Fig. 19*, which certainly demonstrated the trends of rainfall, run-off, and losses better than a plotting of those figures for individual years, but he did not attach the same importance to the effects of ground storage as did Dr Glasspoole because the comparatively close compact nature of the geological formation of the gathering ground did not provide much storage. It was true that there were considerable areas of absorbent peat bogs, but observations showed that, even when the preceding weather conditions had been wet, with the onset of dry conditions the run-off falls to dry-weather flow in a surprisingly short period.

Mr Dickinson had spoken of rainfall in tropical countries. Mr Risbridger knew from experience that rainfall in tropical countries could be great in a very short time, but he was not competent to discuss it from the point of view of hydrological statistics. He agreed, however, that the curve in *Fig. 8*, where run-off was plotted against rainfall, could be shown as a lenticular space, an observation which might apply equally to any curve derived from a series of plots having a degree of scatter. The line the Authors had used was merely a line through what they thought was the middle of that lenticular space.

Mr Dickinson had expressed regret that there had been no continuously recording gauge to amplify the information concerning the maximum flood. The Authors had experienced similar disappointment at the time the flood occurred and had subsequently installed an automatic gauge.

Mr McLellan had asked about the observers of their gauges. Observers were not employees of the Birmingham Corporation, except the observers of daily gauges which were reasonably accessible and were read by a man with a van. Gauges on tops of mountains, where access was difficult and, at times, dangerous, were read monthly, in some cases by local farmers, who had been instructed and took a very intelligent interest in the readings. It was possible, if the isopercental method of plotting was adopted, to find out very quickly if some observer had rung up a neighbour and substituted a reading.

On the question of length of time for establishment of rain gauges in order to make an assessment of the rainfall, Mr Risbridger said that when Mr Mansergh went to the area he had had the advantage of 20 years' records from one gauge and 1 year's record from four gauges, and he had arrived at what was now the accepted long average within 0·4 inch, which was remarkably accurate.

Mr Risbridger recommended reference to the Report of the Joint Committee on the Determination of the General Rainfall Over Any Area,[3] which included an extensive illustration of the method of deducing the long-average rainfall from the recordings of few gauges for a short period.

Regarding storage capacityof 210 days, Mr Risbridger said that from the mass curve of run-off which had actually occurred on their gathering ground, corrected to allow for the difference between their rainfall and the lowest rainfall which they might expect, the storage required was 20,780 million gallons ; they had only 21,136 million gallons, and he still did not feel that they were at all extravagant. On the same point, raised by Mr Lapworth, the storage which Mr Mansergh intended to have was very little less than the Authors themselves had. Mr Mansergh had provided for three reservoirs in the Claerwen and three in the Elan ; of those, he had constructed the three in the Elan, and their capacity, together with the capacity of the three which he had intended to have in the Claerwen valley, would have given very little less storage than was now available.

Mr Lapworth had pointed out that the curve of minimum run-off shown in *Fig. 14* should not be as smooth as was shown, but that there should be cusps at 12, 24, and 36 months. Mr Risbridger agreed that that was so, as could readily be appreciated if the plottings of minimum run-off on *Fig. 13* were actually connected instead of being enclosed in a smooth curve. The practical effect of doing so in *Fig. 14* was, however, not very marked because the smooth curve passed through the actual run-off plots for 16, 20, 30, and 36 months, and the point of actual greatest deficiency still remained at 16 months. The anticipated deficiency at 14 months would, however, be slightly less than the 29 per cent shown.

Mr Risbridger regretted that the definition of the true long-average was at fault, as pointed out by Mr McDonald. The intention was to point out that, given a sufficiently long period of annual records, then since that period extended, the effect of a departure of any 1 ensuing year's record

from the previously computed long average would diminish with the number of years; namely, if the average of 49 years' recordings produced a long-average figure of 50 inches, the rainfall of the 50th year would have to depart from the computed long average by 50 inches or 100 per cent in order to produce a variation of 1 inch in that long average.

The adjustments to rain-gauging records referred to by Mr Wilmot were made to two monthly gauges and were the only adjustments made for that month.

It was not always possible to trace the cause of an isolated discrepancy in the reading of any gauge—particularly a monthly gauge. It might arise from incorrect measurement or recording by the observer, interference by an unauthorized person, drifting snow, or other reasons. The particular adjustments in question might have resulted from any of those causes, but since some snowfall occurred earlier in the month, that might have been a contributing factor to the apparent discrepancies brought out by the percentage map.

In the case of persistent discrepancies, however, an inspection was called for, particular attention being given in cases of undermeasurement, to ensure that the gauge was not leaking.

It might also be pointed out that, in the first place, adjustments were made provisionally and referred to the Meteorological Office for consideration and approval before final acceptance.

Mr Risbridger thanked Mr Bransby Williams for his interesting curve in *Fig. 20*.

Mr Godfrey, in reply,* dealt with the question of water blowing out, and said that that was a factor which they had always borne in mind, but he did not think that it had ever been very serious. One year, 1923, he believed, when there had been a rather heavier loss than usual, and they had been a little worried about it, they had gone through the records and found that during that year there had hardly been an occasion when the reservoir was near enough to top water level to permit water to be blown clear of the dam. When the reservoir was 6 to 18 inches below overflow, and a high wind was blowing down the valley, some water would be lost, but in that particular year there had hardly been a period when the water in the reservoir had been at that level. After that, they had taken some measurements in the valley below the dam during times of wind action, but wind action made so little difference that it had not been worth bothering about. Mr Godfrey did not think that the same thing had happened at Dol-y-Mynach, where there was a small lake of 26 acres continuously full behind it, and the condition of the water level below the sill did not occur.

With regard to what Mr McLellan had said about observers, some of the men in the Authors' area were mountain farmers, and the best way to keep

* Mr Godfrey has died since making this brief oral reply and has had no opportunity of approving it.

them up to the job was to pay regular visits and get them interested; then they would do the job equally well as the Corporation's men.

Mr Rowntree had raised the question of moving the average 6-monthly periods, taking the year from October to September, and so on. That involved a number of complications. The Authors had found during many years, and particularly in the drought years, that the maximum depletion of storage had occurred about the end of September, and they had not got out of their troubles then. There might be a very long dry spell, and the gathering ground would dry up, so that if the year were closed at the end of September the next year would start with depleted reservoirs. Commencing with the calendar year, the ground was normally saturated by then, and that appeared to be the more appropriate time to open the books for the ensuing year. That was a very strong point, and after giving the matter a great deal of consideration they had decided to adhere to that practice.

Mr McClean had suggested that other periods should be included in the maximum and minimum figures in Table 8. Such figures had been included in the original draft but the length of the Paper had had to be reduced considerably. The figures were available.

Mr MacDonald had raised the question of seepage in or out of the gathering ground. The Authors were convinced that it did not occur because of the characteristics of the geological formation of the gathering ground.

Correspondence on the foregoing Paper is now closed and no contributions other than those already received at the Institution will be accepted.—SEC. I.C.E.

STRUCTURAL AND BUILDING ENGINEERING DIVISION MEETING

Professor A. J. S. Pippard, Member, Chairman of the Divisional Board, in the Chair

9 March, 1954

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Structural Paper No. 37

"Design and Construction of a Prestressed Concrete Framed Transit Shed for the Port of London Authority"

by

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and

Ivan Sydney Stroulger Greeves, A.M.I.C.E.

SYNOPSIS

The project described is part of the post-war reconstruction programme being undertaken by the Port of London Authority. It represents the first use by the Authority of prestressed concrete and illustrates certain aspects of current practice in the planning of transit berths. An account is given of the site peculiarities and the means of overcoming them to meet operating requirements.

The design of the prestressed concrete portal-framed superstructure is outlined together with remarks on the influence of construction methods on the design. A comparison is given of the relative costs and advantages of construction in various media.

The construction is described, reference being made to difficulties encountered and the means adopted for overcoming them.

INTRODUCTION

In his Paper[1] Mr G. A. Wilson gives a brief description of the transit-shed berth, No. 19 Eastern Dock, London Dock, which he states "illustrates current ideas on planning and has been used experimentally to ascertain the cost and advantages of using prestressed concrete in dock buildings." The object of the present Paper is to give a detailed description of the design and construction of the berth so as to illustrate the planning ideas more fully and to describe some of the difficulties and advantages that were

[1] G. A. Wilson, "Port of London Authority Engineering Works, 1952." Proc. Instn Civ. Engrs, Part II, vol. 2, p. 551 (Oct. 1953).

presented by the use of prestressed concrete in what is a fairly typical dock building.

In doing so the Authors make no claim to expertness in design in this medium and acknowledge their indebtedness to others upon whose knowledge and experience they have leaned heavily.

History

The construction of the Eastern Dock of the London and St Katharine Docks Group was commenced in 1828 by the now defunct London Dock Company. Until 1940 the west quay was occupied by two adjoining and inter-connecting warehouses. The east warehouse, which was two storeys high, was built concurrently with the quay, and the west, which had three storeys, was built in 1842. As will be seen from Fig. 1, Plate 1, the warehouses and, in particular, their foundations differed in construction. The ground floor of the east warehouse was of brick groined arches supported on masonry columns on timber-piled foundations. In the case of the west warehouse brick barrel arches spring from cast-iron beams supported on cast-iron columns. These columns are founded on a lime-concrete raft 7 feet thick. It is intriguing to guess the reason for this avoidance of piles and it may arise from the possibility of pile driving shaking the then existing east warehouse.

Of interest and also of importance to subsequent construction are the vaults which formed the basements of these warehouses. These vaults connect with a vast system underlying many of the warehouses in this dock group which has for well over a century provided excellent storage accommodation for wines, spirits, and other commodities. The re-establishment of the continuity of this system was a requirement of the new construction.

On the night of the 7th/8th September, 1940, the warehouses, and a section of the vaults were destroyed by enemy action ; the extent of the damage is illustrated in *Fig. 9* (facing p. 416).

Requirements

Towards the end of 1950, the Port Authority authorized the reconstruction of the berth to provide transit facilities for the short-sea vessel trade. The transit shed was to be suitable to accommodate the large variety of goods handled in this dock, using mechanical-handling equipment. Ample loading banks and yard area were particularly necessary as the berth is not served by rail. An important consideration was the recovery of the valuable vault space.

Planning

General

To meet these requirements it was decided to construct a single-storey transit shed of approximately 30,000 square feet with a 40-foot quay and

a loading platform running the length of the shed on the landward side. The floor loading was to be 3 cwt per square foot in the shed, and 4 cwt per square foot or a point load of 5 tons on the quay. The shed was to be sufficiently high to accommodate mobile cranes, with wide doors, and clear runs for mechanical-handling equipment.

The scheme finally evolved is shown in Fig. 2, Plate 1 and Figs 3, Plate 2. It was partly determined by the geography of the site and the extent and condition of those parts of the structures remaining after the bombing. The floor over the vaults in the western section was adjudged sufficiently strong to be incorporated in the new construction after repair. Except for a small section to the north, the remaining vaults in the eastern section were to be demolished.

The geography of the site presented many difficulties and no extension was possible. The yard area is smaller than is desirable, and to provide road access to the quay, a ramp (1 in 25) has been provided in the south corner. Road access to the site is at present restricted to one gate. A second access and extended yard area will be provided when the reconstruction of the adjacent site to the north is undertaken.

Transit operations require the minimum obstruction to free passage from quay edge to shed and from shed to loading bank. This need is accentuated when mechanical-handling equipment is used. In a restricted site there is a tendency for ancillary buildings to be placed so as to obstruct clear runs. To avoid this tendency no such buildings (with two minor exceptions) are sited at quay or yard level. Shed offices for the staff of H.M. Customs and Excise and of the Authority are sited on mezzanine floors in the south-east and north-west corners of the shed respectively, the spaces below them being utilized as lock-ups for gear and special goods which are in any case required (Fig. 2, Plate 1). Lavatory accommodation is provided at basement level, access being obtained by a passage under the ramp. The salient feature of this planning is that, although the development is ostensibly the provision of a single-storey transit shed, the site has in fact been developed on four levels, namely, yard level, basement level, quay and shed floor level, and office floor level. From Table 1 it will be seen that, out of a total plan area of 81,600 square feet, the amount not available for the movement or storage of goods is only 3,300 square feet or about 4 per cent. This solution to the avoidance of congestion on a restricted site was partly provided by the original builders of the vaults, and partly by the fact that the modern transit shed, in which height is dictated by the needs of mechanical-handling operations, lends itself to the construction of mezzanine floors as a convenient means of removing shed offices from the operational level.

Shed

The size of the shed was partly determined by the shape of the site, and the existing structures and levels. A width of approximately 150 feet

TABLE 1.—PLAN DIMENSIONS

	Length: ft ins	Width: ft ins	Area: sq. ft
Quay:			
East side . .	324 0	38 3	12,400
South side . .	130 0	38 8	5,000
			17,400
Shed:			
East section . .	257 7½	75 0	19,300
West section .	185 2½	75 0	13,900
			33,200
Loading Bank . .	305 0	11 9	3,600
Yard:			
High level . .	—	—	8,300
Low level . .	—	—	13,800
			22,100
Ramp road. . .	—	—	2,000
Ancillary buildings, stairs, etc. . .	—	—	3,000
Total area . . .	—	—	81,600
Vault area . . .	—	—	52,000

in two spans was adopted. The length of the western section is 257 feet; the eastern section was restricted to 185 feet by an irregularity in the site boundary in the north-west corner. The height to the eaves of the shed had to be adequate for mobile cranes and loaded fork-lift trucks to enter the side as well as the gable end-doors. The height provided is 21 feet 8½ inches and the clear door opening height is 16 feet 5 inches. The width of the loading platform is 11 feet 9 inches.

Portal-Frame Construction

For medium spans the standard roof truss is generally the most economical form, but the bottom ties of such trusses tend to prevent easy movement of mobile cranes working within the shed. This has led to the use of a portal-frame design.

Use of Prestressed Concrete

Transit sheds in the Authority's docks are generally of structural steel and in normal circumstances it seems probable that steelwork would have been used in this case also. However, the work was planned and constructed during a time of acute steel shortage and alternative designs for

the superstructure and suspended floor over the vaults in reinforced and prestressed concrete were considered. Comparative costs of the three forms of construction for the single-storey transit shed, 150 feet wide, in two spans with total floor area of approximately 32,500 square feet, are given in Table 2.

TABLE 2.—COMPARATIVE ESTIMATED CONSTRUCTIONAL COSTS: TRANSIT SHED

	Steel	Reinforced concrete	Prestressed concrete
Superstructure framework . . .	£8,804	£9,443	£12,683
Superstructure framework: per sq. ft of floor area	*5s.* *5d.*	*5s.* 9¾*d.*	*7s.* 9½*d.*
Complete superstructure including framework, doors, aluminium cladding, gutters, rainpipes, glazing, and painting, excluding paving	£17,917	£18,556	£21,796
—do.—per sq ft of floor area . .	11*s.* 0¼*d.*	11*s.* 5*d.*	13*s.* 4¾*d.*
Suspended floor of shed excluding columns: per sq. ft	—	7*s.*	12*s.* 2½*d.*
Weight of steel in shed: tons . .	150	36*	20*

* 8 tons of steel refers to the doors, etc., common to both designs, thus the true comparison is 28 tons for reinforced concrete to 12 tons for prestressed concrete.

From Table 2 it will be seen that, of the designs considered, the cheapest in first cost is the steel-framed shed on a reinforced-concrete floor. The most economical in steel, but most expensive in first cost, is the prestressed concrete floor and framework. The design adopted was the prestressed concrete framework on a reinforced-concrete floor.

Apart from cost there were other objections to the prestressed concrete floor of the type envisaged. This comprised precast concrete beams and hollow floor units, the spaces between being filled with in-situ concrete. There was doubt whether a suitable floor finish would be obtained without recourse to a topping and, if the latter was adopted, whether an adequate key between the topping and the precast work would be obtained. In addition, the shell of the hollow unit was so thin that it would be very vulnerable to dynamic point-loads (such as the corner of a packing case falling from 2 or 3 feet—a contingency not unknown in dock working). Such damage and that arising from abrasion could not be easily repaired. Thus, despite the extra steel involved the conventional reinforced-concrete floor was selected.

For the superstructure the prestressed concrete design was adopted. The main factor in making this choice was the saving in steel and the fact that suitable high-tensile steel cables were available. An important consideration was that a concrete structure would reduce maintenance painting. It will be readily appreciated that a substantial part of a dock authority's annual maintenance expenditure is accounted for by painting, but less often realized is the additional hidden cost arising from the dislocation of normal working that painting causes. A reduction in, or even better, the obviation of, the need for maintenance painting may compensate fully for higher first cost. The longer the life of the building, the greater the compensation. Judging from the buildings in the London and St Katharine Dock Group, many of which are over a century old and still rendering good service, this policy of accepting higher first costs with a reduction of subsequent maintenance is sound.

A further consideration which should not be overlooked was the desire to obtain first-hand experience in what is still, certainly in the field of British maritime engineering, a novel structural medium.

Sheeting

Aluminium alloy sheeting was selected for roof and sides in order to save steel and reduce maintenance costs. The extra cost of this sheeting compared with asbestos-cement roof sheeting and galvanized-steel side sheeting is 1*s*. 7*d*. per square foot of floor area. This is only slightly more than the cost of once painting a shed of similar dimensions. The aluminium sheeting is being left unpainted except for the laps. It will be carefully watched to see if painting is advisable. Even if this is found to be necessary it is anticipated that the periods between painting will be much greater than in the case of galvanized-steel. Asbestos-cement sheeting, though excellent for some purposes, is at a disadvantage in transit sheds of this type owing to its vulnerability to damage, particularly at the eaves, from quay cranes and from mobile cranes working within the shed.

Foundations

An investigation was made into the existing foundations and soil characteristics. As was stated earlier the foundations under the site were of two kinds. A trial hole put down in the west section showed that the foundation consisted of a slab of lime-concrete about 7 feet thick, founded on approximately 6 feet of soft peaty clay and 4 feet of very silty soft clay, overlying the sandy gravel known locally as "Thames Ballast." Four-inch-diameter undisturbed samples of this clay were taken and shear-strength measurements carried out in the laboratory. The safe gross bearing capacity of the peaty clay amounted to 1·1 ton per square foot. Although the apparent safe bearing capacity of the clay was not high, a three-storey warehouse had been in existence on it for about 100 years and it was felt that as the floor loading from the new transit shed would

be less than that previously experienced, there would be no likelihood of further settlement. To confirm the thickness and condition of the remaining section of the concrete slab, a number of 5-inch-diameter holes were drilled through it, using a drifter drill. No serious discrepancies were found and it was assumed that the slab was monolithic. The drawings of the foundations for the eastern half of the shed showed that the vault arches had been founded on piers built upon timber rafts supported by timber piles. As the state of the timber in the piles was not known it was decided that trial holes should be put down alongside the foundation and samples taken. Examination of the timber showed that the top 2 feet had decayed and some of the piles would obviously carry little load. Samples were taken from the clay for laboratory examination as in the western half of the warehouses. It was found that there the clay was softer doubtless as a result of the fact that it had supported less superimposed load.

Following that examination it was decided that the existing foundations of the eastern section would not be safe to support the new structure, and new bearing piles were driven into the sand and gravel stratum.

An investigation was made into the foundations of the centre wall dividing the two warehouses. It was found there that the timber piles upon which the wall was built were showing signs of decay. As a result of this, arrangements were made to carry the centre supports to the shed on cantilever beams supported on new piles.

Some Preliminary Design Considerations

Before dealing with the prestressed concrete design, a general account of the foundations and vaults will be given.

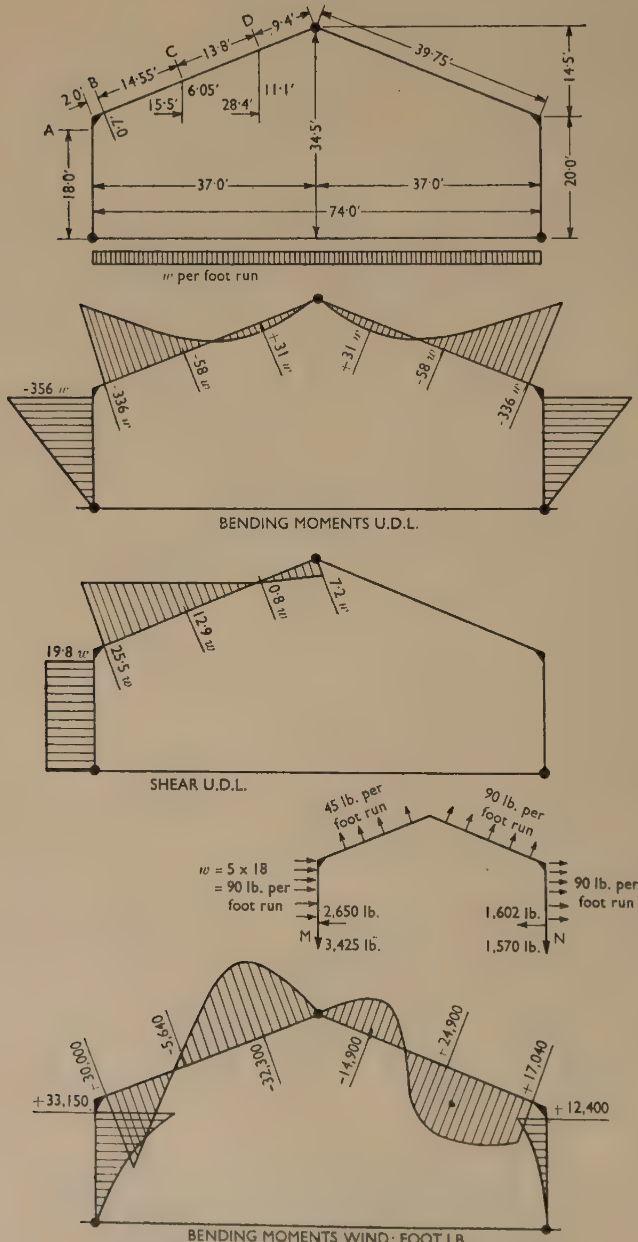
Foundations

The site investigation showed a layer of Thames Valley sand and gravel overlying the London clay. This layer is approximately 20 feet thick and commences 24 feet below basement floor level. From experience it is known that piles driven up to 5 feet in this layer will obtain a satisfactory set; this was confirmed by driving test piles. The loads on the vault columns ranged from 208,000 to 277,000 lb. and groups of 14-inch and 16-inch piles were used, the 14-inch piles being loaded to a maximum of 54 tons, and the 16-inch to 80 tons. The longitudinal spacing of portals, columns, and pile groups was 18 feet $1\frac{1}{4}$ inch; arrived at from the superstructure design requirements and the need to miss existing pile foundations.

Vaults

A section of the old brick groined arches to the north of the site was undamaged. The join between these arches and the new reinforced-concrete beam and slab construction was made by cutting back the arches

Figs 4

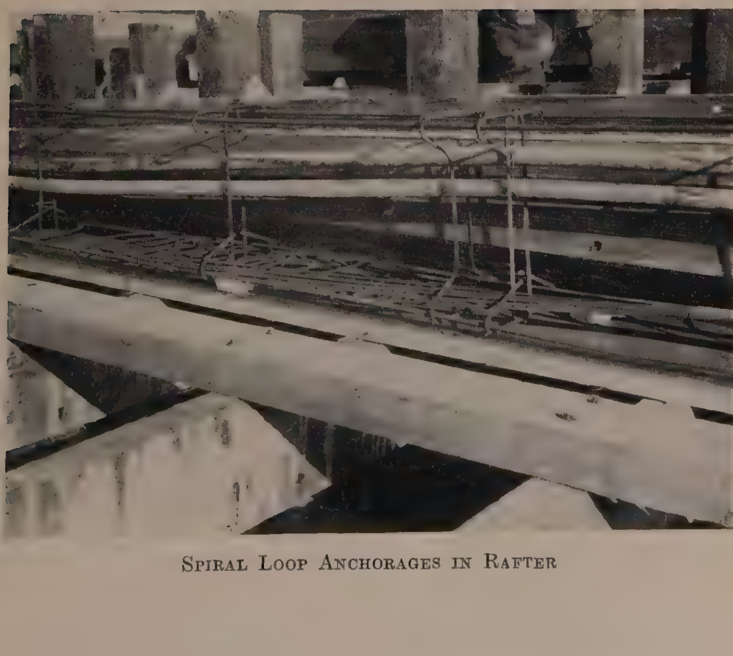


Typical Moment and Shear Diagrams

Fig. 9

Site after Bombing, September 1940

Fig. 10



Spiral Loop Anchorages in Rafter

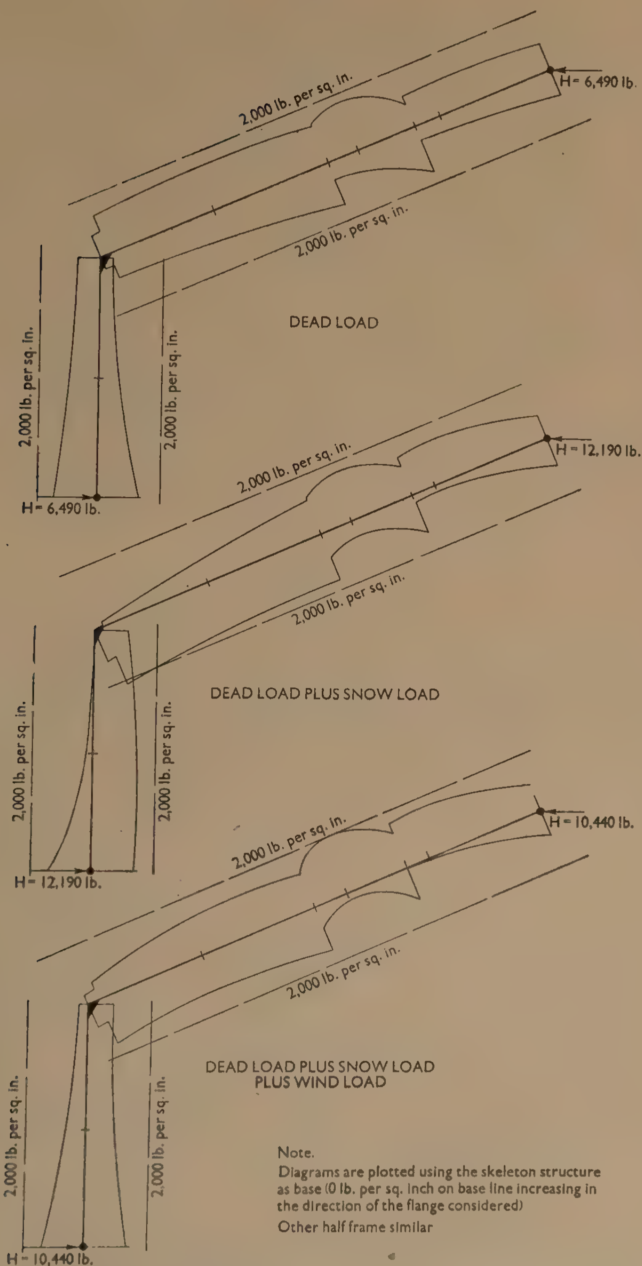
Fig. 11

Arrangement of Anchorages and Reinforcement in Rafter Shoulder

Fig. 12

Rafter Mould before Concreting

Figs 5



Typical Bending-Stress for Half Frames

to the crown and taking the thrust by a facing beam strutted back to the reinforced-concrete frame. The face of the cut-back brickwork is left rough to improve bond and avoid a weak shear-plane.

For about 10 years after the bomb incident, when the vaults had lain in a partly demolished state, the exposed ends had not been strutted and there had been a tendency for the joints in the brickwork to open. Under live-load conditions with proper supports at the springings, these joints would tend to close, but there would be a danger of weakness in shear and the possibility of bricks dropping out. After several trials the most suitable method of filling the joints was found to be by forcing pieces of slate into them.

In the west vaults the roof consists of three ring brick-on-edge barrel arches with sandy gravel filling to a depth of 14 inches over the crown and 48 inches over the springing. This filling was used to make concrete (6 : 1, crushing strength 1,100 lb. per square inch at 7 days) which was then replaced as filling over the arches. The slight additional dead-load was acceptable and the addition of the concrete thickened up the arch rendering it capable of supporting the required loading. Bomb holes in the arches were repaired by shuttering up the soffit and filling to the underside of the floor paving with mass concrete.

Design of Superstructure

Portal Frame

The superstructure framework comprises a series of pairs of adjacent portal frames, each frame being 74 feet 6 inches wide overall. The height to eaves is approximately 20 feet and the rise 14 feet 6 inches. For spans of this order a three-pinned portal is very suitable. It also has many advantages from the constructional point of view and lends itself to precasting. It is of course statically determinate. The frame used is illustrated in Figs 3, Plate 2.

Figs 4 and *5* show some of the bending-moment and bending-stress diagrams used in the design. To a large extent the dimensions are determined by the stresses at the lower end of the rafter B and the top of the column A, and by structural requirements. These critical sections are designed in the normal way. Taking section B as an example, the I-section selected has a depth of 28·5 inches, the flanges are 12 inches wide and the flange and web thicknesses are 4 inches. Four cables of twelve 0·2-inch wires with a resultant eccentricity of $5\frac{1}{4}$ inches and tensioned to 50,000 lb. each, give stresses under prestress alone of 1,980 lb. per square inch and 260 lb. per square inch in the top and bottom flanges, respectively.

Table 3 gives the stresses in top and bottom fibres under various conditions of loading.

The dead load was taken at 10 lb. per square foot and the snow load (live load in Table 3) at 15 lb. per square foot. The wind load, approxi-

TABLE 3.—STRESSES IN CRITICAL RAFTER SECTION UNDER VARIOUS LOADINGS (LB. PER SQUARE INCH)

	(a) Own weight	(b) Prestress	(A) Sum $a + b$	(c) Dead load	(B) Sum $a + b + c$	(d) Live load
Top . . .	−459	+1,980	+1,521	−595	+ 926	−891
Bottom . .	+459	+ 260	+ 819	+595	+1,414	+891

	(C) Sum $a + b + c + d$	(e) Wind load	Sum $e + B$	Sum $e + C$	(f) Wind load	Sum $f + C$
Top . . .	+ 35	+295	+1,221	—	−168	− 133
Bottom . .	+2,305	−295	—	+2,010	+168	+2,473

mately equal to 55 miles per hour at a height of 35 feet was taken as 10 lb. per square foot.

Having selected the section dimensions for position B, the dimensions at the apex which are 12 inches by $8\frac{1}{2}$ inches are largely pre-determined by anchorage requirements. The width of the member remains constant and by adopting a constant taper from section B to the apex all the cross-sectional dimensions are determined. The cable eccentricity and stop-off position are then deduced from the various conditions of bending moment.

Maximum shear occurs at B and amounts to 11,500 lb. The maximum shear intensity is 140 lb. per square inch giving a maximum principal tensile stress under prestress of only 17 lb. per square inch.

In the case of the rafter an I-section was chosen for lightness. Theoretically the thinner the web the more economical is the section. Here a practical minimum of 4 inches was selected and was found to be none too large when the rafters were concreted.

The design of the column is similar but less complex, and because the weight of the member is a minor consideration, a rectangular section was chosen.

In both members the prestress is applied by four cables of twelve high-tensile steel wires 0·2 inch in diameter. The force in each cable is 50,000 lb. after relaxation; this is equivalent to a stress of 59 tons per square inch in the steel, that is, 0·535 × ultimate tensile strength, which is 110 tons per square inch.

In the rafter the cables start at the eaves one above the other. The eccentricity in the lower half of the rafter is positive and at about half

span one cable is stopped off; a second cable is stopped off at about three-quarter span where the eccentricity has become negative. The cables then move out of the vertical plane and terminate side by side on the neutral axis at the apex. The method of stopping-off the cable was to split it into its separate wires and develop each wire into a wide loop. This development takes place over a considerable distance during which the cable was bent down into the bottom flange (*Fig. 10*, facing p. 416). The loop used is approximately a logarithmic spiral curve so that friction provides a uniform resistance along its length; the spirals are maintained in position by bond, and frictional resistance to unwinding. Tests in France on such anchorages in concrete of only 2,000 lb. per square inch crushing strength have shown that the wire invariably fails before the concrete. The cable sheath is stopped off at the tangent point. Grout was injected in the same manner as for standard anchorage arrangements, but air vents were left in the member at the tangent points.

In the column the four cables have a straight path except at the top where they bend over slightly to reach suitable anchorage positions. In both members the principal stresses arising out of shear are quite small and are negligible after the prestress is applied. In the rafter, which was tensioned on the casting bed, nominal mild-steel reinforcement is provided, mainly in the form of links, to help locate the cables. The column was erected before tensioning and mild-steel reinforcement was provided to cater for handling-stresses.

The portal naturally divides itself into two symmetrical halves and for purposes of casting and erection each half was further divided just below the shoulder (Figs 3, Plate 2). The inclination was selected so that the plane was at right angles to the line of the resultant thrust at this section. Thus the forces tending to cause sliding at the joint are zero. Had the plane been horizontal the line of thrust would still have been within the angle of friction. This would have been an improvement in the design as it would have facilitated casting and erection.

The anchorages at the apex and the base are standard, and the mild-steel reinforcement provided to withstand the bursting stresses arising from the concentrated anchorage forces conforms to normal practice. At the shoulder (*Fig. 11*, facing p. 417) there are eight anchorages and the stress distribution is very complex. The provision of mild-steel reinforcement was rendered somewhat difficult by lack of space, and the device was adopted of sheathing two of the vertical cables in mild-steel tubes. It will be noted that the cables in one direction act as reinforcement for the transverse stresses of the compressive forces arising from the cables in the other direction.

The concrete limiting stresses for the purposes of design were + 2,000 and 0 lb. per square inch, but in practice these are slightly exceeded. The worst condition of loading, namely maximum wind load plus maximum snow load, gives a maximum compressive stress in any section of 2,473 lb.

per square inch and a maximum tensile stress of 133 lb. per square inch. These are the stresses after relaxation, but tensile stresses before relaxation were less. The conditions required to produce these stresses, if they occur in practice, will be only momentary and under these circumstances the stresses can be accepted. The stresses were slightly reduced by grouting-up the cables.

In tensioning the cables an allowance of 15 per cent was made for creep in the steel and for creep and shrinkage in the concrete. This amounted to approximately 9 tons per square inch. In addition a measured allowance was made for slip in each cone. The value adopted for Young's modulus was $28{\cdot}5 \times 10^6$ lb. per square inch.

The practical implementation of the theoretical "pin" joints required at apex and bases was the provision of "rocker" joints which are described in the section on "Construction" (Figs 3, Plate 2).

Gable-End Columns

The gable-end columns, whose sole function is to support the sheeting, are of two types, one 31 feet $1\frac{3}{4}$ inch and the other 26 feet $5\frac{3}{4}$ inches long. Both have the same T-section and in both types the flange continues beyond the web and is attached to the portal rafter. The attachment is by mild-steel bolts in oversized holes so as to provide no vertical restraint to the rafter or column but horizontal restraint is obtained. The effective loading on the columns results from wind which may act positively or negatively so that the resulting stresses reverse their signs. In the long column two cables are used, which were not symmetrically placed in relation to the centroid but were unequally tensioned so that a uniform prestress was applied. In the shorter column one cable is provided, positioned at the centroid.

The members described above and also the wind bracing beams were post-tensioned ; the latter and the portal frame columns after erection.

Purlins, Sheeting Rails, and Door Lintels

These members were factory made on the long-line system, using bond-anchored high-tensile wires. The wires are 8-gauge with a working load of $1\frac{1}{2}$ ton. The purlins and rails are 7-inch-by-5-inch-rectangular section and the lintels 10-inch-by-5-inch. The maximum stresses that occur under any state of loading are +2,990 and —220 lb. per square inch and are experienced during handling. Under working conditions they are reduced to +2,900 and —130 lb. per square inch. The minimum concrete crushing strength at transfer of prestress was 5,000 lb. per square inch, and at delivery to site 7,000 lb. per square inch. Slotted holes were cast in the ends to receive bolts cast into the nibs on the portal frames. The purlins are notched to fit over the rafters, the apex purlins being 8 inches by 5 inches. To develop a tie (other than that provided by the sheeting) between the two sides of the roof, the apex purlins were bolted

together through concrete distance-pieces. The holes for the bolts were made oversize so as not to impede the rocker action of the main rafter apex joints. In addition to the bond-anchored wires, holes were cast in the apex purlins to take two-wire cables, and precast anchor blocks are fixed at each gable-end. The apex purlins were stressed together from end to end of the building after erection of the framework had been completed, thus giving a further element of restraint in the longitudinal direction. The maximum deflexion in any member under load is $\frac{3}{4}$ inch and occurs in the purlins.

Shed Sheeting, Perimeter Walls, Doors, etc.

The roof and sides of the shed are sheeted in 18-gauge "Rigidal" aluminium alloy sheeting. The hook and seam bolts are of aluminium. As a safeguard against electrolytic corrosion, all steelwork in close proximity to the sheeting was painted with bituminous aluminium paint and the sheeting laps were similarly treated. No bare overhead electric cables run near the shed.

From experience it is known that the bottom 5 feet of the walls of a transit shed suffer more damage than the remainder. This damage is very severe and the repair and replacement of sheets form a heavy maintenance item. As a departure from normal practice, the bottom 5 feet of the walls of this shed were built in 9-inch brickwork. This thickness is made up of two non-bonded skins tied together with brick reinforcement in every third course. This construction, which is now general practice in the Port Authority for panel walls in certain types of storage and transit buildings, enables two fair faces to be obtained without special bricks. The inner skin is built in light-coloured bricks of the sand-lime type but harder and more durable; the outer in London stocks. By this means the need for painting the internal face of the shed was obviated; indeed there is virtually no painting in the shed. This represents an important first-cost saving, but a much more important maintenance saving.

The doors, the numbers and positions of which are shown in Fig. 2, Plate 1, are of the two-leaf type, top hung, and have a clear height of 16 feet 5 inches. The clear width at the gable is 20 feet and at the sides 17 feet $1\frac{1}{4}$ inch. The frames are made of mild-steel angles and clad with 18-gauge galvanized-steel sheets with 3-inch corrugations. Steel sheets have been used on the doors for the following reasons; of all parts of the shed the doors will receive the most damaging treatment; aluminium alloy sheeting as generally used, although nearly as strong as steel, will deflect much more easily; large sliding doors have a tendency to whip but the greater stiffness and weight of steel sheeting tends to reduce this; in addition the doors are the only part of the structure in which steel framework makes firm contact with the sheeting, and electrolytic action might have been difficult to avoid.

Offices on Mezzanine Floors

Offices are provided on mezzanine floors built in reinforced concrete in the north-west and south-east corners of the shed (Fig. 2, Plate 1 and Figs 3, Plate 2). The former, for the Authority's staff, provides 850 square feet of accommodation and the latter, for H.M. Customs, 1,044 square feet. The area below the floors is separated from the shed by mild-steel framed and weld-mesh partitions and is used for lock-up stores, etc. Access to the offices is only from outside the shed for security reasons, and also to enable them to be used when the shed is locked up. Internal windows are provided so that the shed can be overlooked by office personnel.

Lighting

The electricity supply is A.C. 415–240 volts, 3 phase, 50 cycles. Three-hundred watt distributive fittings are mounted 23 feet above the floor at approximately 36 feet × 20 feet centres, providing 3 foot-candles at floor level. Individual lights are fixed over each door. Flood lighting is provided outside the shed for night working and " police lighting " for use at night when no work is being done. The internal light fittings are suspended from the roof by chains so that they tend to swing away if struck and their height can be adjusted.

Quay

The quay is 38 feet 3 inches wide with a cross-fall of 1 in 60 towards the cope. Crane tracks of 10 feet 1 inch and 13 feet 6 inches are provided for hydraulic and electric quay cranes respectively. The Authority operates a number of hydraulic cranes which still have a number of years of economic life but which will ultimately be replaced by electric cranes.

A subway is provided below the quay to accommodate 6-inch fresh water and hydraulic mains, a 2-inch gas main, and electric and telephone cables (Fig. 6, Plate 2). The subway is formed by partitioning a section of the vaults, and access from the vaults is by a door wide enough to permit a 9-foot length of pipe to be manœuvred through. Steel Victaulic pipes have been used for the hydraulic main. Hydraulic and fresh-water hydrants are contained in pits in the quay deck at 26 foot and 100 foot centres respectively. Contrary to the Authority's previous general practice, crane switches are also located in pits in the deck which are at about 52 foot intervals. The cable is led from the pits into the space between the crane rail and its guard angle by a shaped channel cast into the quay (Figs 7, Plate 2). The crane track is made up of 90 lb.-per-yard-rails (B.S.F.B. 90 R. section) with $5\frac{1}{2}$ inch × 3 inch × 10·02 lb. bulb angles at either side (Figs 7, Plate 2). This is now the Authority's standard type of rail for crane and railway tracks. Telephone connexion boxes for ship-to-quay lines are located in the cope.

Quay Wall Face

The existing quay wall was of brickwork with a granite cope. The face had been badly scored and abraded, and the granite cope was damaged and left projecting. The cope was replaced in reinforced-concrete integral with the quay deck. The quay wall was cut back to $4\frac{1}{2}$ inches behind the new face line. Seventy-five lb. flat-bottom rails were bolted through their flanges to the wall at 2 foot 6 inch centres and connected to one another by $\frac{3}{4}$-inch tie-rods at 2 foot 3 inch centres. These helped to locate the rails accurately and were also used to support steel-mesh reinforcement (610 B.R.C.) fitted between the rails. The face was then shuttered and concreted leaving the rail head projecting $\frac{1}{4}$ inch beyond the concrete. Pencil vibrators were used to obtain dense concrete. This facing has so far stood up well to the very heavy wear that is experienced, mainly from the swim ends of steel barges.

CONSTRUCTION

Piling

The assumption, based on previous experience, that piles driven about 4 feet into the ballast layer would take up a satisfactory set was confirmed by driving two test piles before general casting commenced. The required set was calculated on the Hiley formula. In all, ninety-two 14 inch by 14 inch and thirty-one 16 inch by 16 inch 30-foot-long piles were site-cast and driven in 11 weeks. A standard 50-foot sykes frame was used, powered by a 4/8-ton double-drum diesel-winch operating a 3-ton drop hammer. To reduce to a minimum the time lost in moving the frame between rows of piles, it was mounted on a low cross-carriage of 40-foot span, enabling piles in four rows to be driven before moving the carriage to the next group. By this means it was possible to drive as many as eight piles per day in a congested basement site. In Table 4 are given the safe bearing capacities as calculated by various dynamic formulae.

Concrete

The concrete mixes specified for this contract were in accordance with mixes employed by the Port of London Authority. They are based on the D.S.I.R. Road Research Station Technical Paper No. 5 and Road Note No. 4. Generally the mixes used were as given in Table 5.

The water/cement ratio was reduced in the light of site tests, particularly when vibration was used in placing.

Mixes Cg, Dg, and Eg were made with granite coarse-aggregate and were used in places where severe abrasion could be expected, such as in the shed floor, quay, and coping. As-raised ballast was used for mix C and graded river gravel and sand for all other mixes.

Mix E was used in the reinforced-concrete retaining walls, columns, piles, and beams under the shed floor.

TABLE 4.—PILE BEARING CAPACITIES

Formula	Test pile No. 1 (14 inch × 14 inch): tons	Test pile No. 2 (16 inch × 16 inch): tons
Engineering News .	118	122
Hiley	129	150
Wilcoxen . . .	131	152
Dutch	167½	187

TABLE 5.

Grade	Aggregate/cement ratio	Water/cement ratio	Specified minimum crushing strength 28 days : lb. per sq. in.	Actual average crushing strength 28 days : lb. per sq. in.
C . .	7·0	0·60	2,500	4,200
Cg . .	6·0	0·60	2,500	—
D . .	4·8	0·50	3,000	4,500
Dg . .	4·5	0·50	3,000	5,300
E . .	4·1	0·45	3,750	6,000
Eg . .	3·9	0·45	3,750	5,760
F . .	3·6	0·40	4,500	6,980

A number of samples of aggregate were obtained and sieve tests were carried out. It was found, to get consistent results, that it was important to have all the supplies from one pit. This was insisted upon, and a very consistent grading of coarse and fine aggregate was obtained conforming to B.S.S. 882.

The concrete for the piles was mixed in a ⅓-yard closed-drum mixer and high crushing strengths were obtained by using a water/cement ratio of 0·40 and vibrating the concrete with immersion vibrators of the poker-type. The opportunity was taken to test a number of high-speed vibrators, which were then on the market, to obtain the best results with the very dry mixes likely to be used at a later date when carrying out the prestressed work. The speed of vibration was measured in air and, when the vibrating head was immersed in concrete, using a stroboscope. A popular make

giving a speed of 11,000 revolutions per minute was adopted as being the most efficient and most robust in use.

Little need be said about the concreting of the yard, basement, etc. The main concreting operation, that of casting the quay and shed floor of the eastern section, was carried out using a "Benford BR.25" continuous mixer and 6-inch-"Pumpcrete" concrete-pump.

Site Casting of Frame Units

The site-made precast units were cast on the completed floor of the western half of the shed while the construction of the eastern half was in progress. There were insufficient units to warrant the use of steel and, in general, the moulds were made of $1\frac{3}{4}$-inch wrot and thicknessed boards screwed or bolted to 6-inch-by-2-inch bearers. These proved to be sufficiently robust to withstand vibration without distortion.

Concreting of Frame Units

To obtain high early compressive strengths a dry concrete-mix was used, the water/cement ratio being about 0·32. This was unusually low for site work, but with careful attention to grading, mixing, and placing, no difficulties were experienced. The aggregate/cement ratio was 3·6. An ordinary 14/10 closed-drum mixer with volumetric batching in gauge boxes was used. The aggregate moisture-content accounted, on the average, for up to a half of the required water; the additional water was added by hand from a graduated container. The aggregate moisture-content was regularly assessed by heating and weighing, and also by using a steel-beam moisture-meter designed by the Building Research Station, which was found to be speedy and reasonably accurate. To guard against segregation, the whole batch from the mixer was discharged on to a metal plate from which it was loaded into wheel-barrows and transported to the moulds.

The minimum crushing strength to be achieved before tensioning was permitted was 6,000 lb. per square inch. Cubes were regularly made from samples taken from the moulds and tested for 3-day and 7-day strengths. To ensure that the cubes were made under conditions similar to the main work, they were vibrated on a small table for the same period as the main work. The tests for the concrete used in the precast work gave an average 7-day strength of 5,680 lb. per square inch with a standard deviation of 875 lb. per square inch and a coefficient of variation of 15·4 per cent. This illustrates the high degree of consistency that can be achieved by volume-batching, provided that the work is carefully and expertly supervised.

Rafter

The joint in the portal-frame between the rafter and the column was positioned below the shoulder. Thus the shoulder, containing eight

anchorages, was cast homogeneous with the rafter (*Fig. 12*, facing p. 417). To accommodate this, the mould soffit was raised 3 feet. The taper of the rafter was taken up in the soffit so that the upper surface remained level during casting. The concrete was vigorously vibrated using twenty-two 230-volt shutter-vibrators spaced 4 feet apart on each side of each mould. The vibrators were fixed with quick-release pins enabling them to be quickly transferred from mould to mould. In addition, where possible, immersion-vibrators with an operating speed of 11,000 r.p.m. were used ; these were particularly valuable at the ends of the units. To prevent the concrete arching across the narrow web, the lower flange was filled first and then the concrete worked up. A dense, hard concrete was obtained, with no honeycombing or air pockets.

The four Freyssinet cables are anchored one above the other in the vertical face of the shoulder. Two of the cables are anchored side by side in the vertical face at the apex, and the other two are bond-anchored in the bottom flange. The path taken by the cables is somewhat complex and, as accurate positioning was important, careful attention had to be paid to the methods used. It is understandable that designers of pre-stressed concrete are loath to provide more mild-steel reinforcement than is dictated by design considerations, but the site engineer welcomes a light mild-steel cage as a convenient means of locating the cable duct formers and on this occasion, happily, the designers were persuaded to provide it.

The cable ducts are formed by means of "Ductube." This proprietary tube-former is now widely used and a description of its characteristics is, therefore, unnecessary. Short lengths of mild-steel bars were welded to the reinforcement to form support brackets at about 2-foot intervals through which the tubes were threaded. This method, in addition to accurately locating the tubes, prevented their tendency to rise in the concrete under vibration. Another method was used successfully where there was no mild-steel reinforcement. The tubes were wired to steel rods inserted temporarily into the moulds and removed only after vibration was completed. It was not found possible to wire the tubes to the moulds sufficiently securely. To ensure adequate rigidity in the tubes it was found essential to inflate them to the maker's specified pressure. On inflation the tubes tend to reduce in length, resulting in a strong pull on the female cones through which they are threaded. Any displacement of the cones brings the risk that grout may enter the duct. Figs 8, Plate 2, illustrate a method of fixing which was devised on the site whereby a mild-steel plate is holed so as to grip the inflated tube firmly, and fixed to transmit the pull directly to the end of the mould. It also illustrates the method of fixing the cones to the mould ; this fixing must be very rigid.

The bond-anchored cables are encased in plastic sheathing since no light metal sheaths were obtainable, but the plastic material was not

wholly satisfactory. On one or two occasions it became damaged during concreting operations, allowing cement grout to penetrate into the cable. The junction between the sheath and the female cone was important for a similar reason. The most satisfactory method adopted was to use a female cone with a metal spigot protruding from the inside, over which the sheath was threaded and secured with insulating tape.

The spiral loops on the ends of the bond-anchored cables are described in the section on "Design" (*Fig. 10*, facing p. 416). No difficulty was found in forming these loops with ordinary bar-bending equipment, but care had to be taken that the angle formed between the cable and the plane of the loops was accurately formed to avoid distortion of the cable position. The bond-anchored cables in their sheaths were located similarly to the Ductubes.

The shoulder of the rafter is illustrated in *Fig. 11*, facing p. 417. Eight anchorages are contained in a comparatively small compass together with a considerable amount of mild-steel reinforcement required because of the high local stresses. The construction of this part of the member is manifestly costly and is an example of prestressed concrete at a disadvantage compared with steelwork. On the other hand, the joint is tested during tensioning when the worst stresses are experienced, which is not the case with steelwork. The shoulder reinforcement was greatly simplified by the replacement of two Ductubes by mild-steel tubes which were left in the concrete member to act as reinforcement.

Other Units

The casting of the other units which comprised portal-frame columns, gable-end columns, and eaves-beams presented no special features. The portal-frame columns and the eaves-beams were tensioned after erection and thus contained sufficient mild-steel reinforcement to withstand handling stresses.

Tensioning

Standard Freyssinet equipment was used. In all cases the prestress was checked by measuring the extension, the pressure-gauge reading being used as a guide only. The rafter cables were tensioned and anchored on the casting beds. In the case of the portal columns, the cables were anchored at the lower end and left protruding at the upper; after erection the protruding ends were fed through the ducts left in the shoulder of the rafter units. They were subsequently tensioned and anchored in the top edge of the rafter, thus forming a monolithic rafter-column unit. However, before erecting the columns it was necessary to cast on to their lower ends a rocker section, which completely encased the lower anchorages. It was advisable to take up the slip in these anchorages before casting on this section so that the slip could be measured. This entailed applying preliminary tensioning to the column-cables while they were on the

ground. The top end of the column-unit has no anchorages and is not reinforced to withstand high local stresses, thus a special steel frame was made to receive the jack and distribute the pressure over a large area of the end of the columns. The end of the column is not at right angles to the cable path ; the frame is made on a corresponding taper so that the jacks pulled in the line of the cables.

On the rare occasions when a cable did not give the desired elongation at the correct pressure-gauge reading, the best method of freeing it was found to be by removing the wire spiral-core. This might be recommended as a useful procedure before attempting to extend any cable. In all cases before threading a cable through the duct, one wire was marked for its full length to enable any twist in the cable to be taken out. Where the wire spiral core has been removed, special care must be taken to ensure that the wires are not twisted before being placed in the jack.

All cable ducts were grouted after tensioning ; the grout was inserted through the male cone and pumping continued until it flowed freely out of the other end. In the cases where the other end was not free, vent holes were provided in the casting.

Pin Joints

The apexes of the rafters and the bases of the portal columns were designed as pin joints. The practical realization of this design conception had to allow for a certain measure of location without an unacceptable degree of restraint. The joints adopted are illustrated in Figs 3, Plate 2.

In the case of the columns, a rocker joint was formed. Before erection, the base of the column was extended beyond the anchorage, the lower surface of the extension being rounded. This face rests on a block with a curved upper-face which was cast into the foundation pocket. A mild-steel pin protruding from the foot of the column enters a hole in the block which is slightly larger in diameter than the pin. The foot of the portal can thus rock on the foundation block ; the diameter of the pin is such that little restraint is caused to this rocking motion, whilst considerable restraint (by the pin in shear) is presented to any movement at right angles to the plane of the portal.

At the apex, sections were cast on to the rafters so that the meeting faces have line contact only and a slight rocking action is permitted. Light reinforcement inter-penetrating the ends gives no appreciable restraint to this motion while restricting sideways movement. The ends of the cables were spread open to give continuity between the main body of the rafter and the cast-on joint section. The method of making this joint was to cast the end section on one rafter of a pair before erection, the end section of the other rafter being cast on after the pair of rafters had been erected and positioned and the frame tensioned. This ensured perfect alignment.

Erection of Superstructure

The sequence in erecting the portal frame superstructure was as follows :—

(1) Positioning of the portal-column-unit on the preset rocking pads, making sure that the dowel bar was housed correctly in its socket.
(2) Holding the column unit in position while the rafter was being assembled.
(3) Lowering the rafter unit into position and threading the Freyssinet cables through the ducts in the shoulder.
(4) Supporting the rafter units during jointing and tensioning.
(5) Making the shoulder joint.
(6) Tensioning the column through the shoulder of the rafter.
(7) Making the apex joint.
(8) Fixing the purlins, sheeting rails, and lintels.
(9) Removing the false work ready for the next portal frame.

There were three main problems to be decided before the erection of the framework could go ahead.

(*a*) It was necessary to support the first frame in such a way that it would remain rigid until a sufficient number of units had been erected and the wind-bracing fixed. This was achieved by erecting a tubular-scaffolding framework which held the columns firmly, and also supported the rafter units and gable columns.

(*b*) To hold the columns and rafters in position during assembly and stressing, three tubular scaffolding towers mounted on wheels were used. The two end-towers supported the column units and were equipped with a working platform upon which men could stand for making the shoulder joint. These towers were also equipped with a bearer to take the weight of the rafter unit while it was being assembled. The centre tower supported the apex of the two rafter-units and provided a working platform to enable the apex joint to be made. As soon as the apex joint was set and the purlins fixed in position, the towers were travelled forward to their next position.

(*c*) The third problem was the choice of plant for handling and positioning the units. It was originally intended that the erection should be carried out using a travelling derrick which would erect the eastern section of the shed first and then be moved to carry out the erection of the western section. The use of the derrick, however, would have prevented the erection of the two spans of the sheds proceeding simultaneously. Mobile cranes enabled the erection to be speeded up as they could travel from one section to another very quickly. A 5-ton "Lorraine" crane, equipped with a 45-foot jib, was used for handling and erecting the rafter units, purlins, and ridge

beams. In addition, an "Allen Oxford" 5-ton mobile excavator dressed as a crane was used in the casting yard for handling the units, and during the erection period for handling the column units and the sheeting rails.

Using the equipment described above, the whole of the erection was carried out within a period of 6 weeks and 2 days between the 28th January and the 13th March, 1953. The best week's output was five frames completed, that is, one per day, and the average over the job approximately four frames per week. *Fig. 13* illustrates a stage in construction.

Making of Shoulder Joints

The threading of the Freyssinet cables through the rafter unit did not present any great difficulty as the 5-ton cranes were able to lower units with adequate precision. Initially, the $\frac{1}{2}$-inch joint between the column and rafter was made by "buttering" the meeting face of the column unit with a stiff cement-mortar, allowing the rafter unit to sit down on to it. After a few units had been stressed small cracks appeared in the top of some of the column units. The positions of the cracks indicated that the compression was not being distributed over the face but was being localized, probably owing to the unequal setting of the rafter on the prepared bed. The method of "buttering" the joint was abandoned and all further joints were made in the orthodox manner of caulking the joint with "earth-dry" cement-mortar. No further cracks appeared. After the two-column- and rafter-units had been assembled and tensioned, any final adjustment at the apex was taken up and the joint made using a rapid hardening cement. In this way it was possible to remove the towers 24 hours after the joint had set.

Conclusions

The use of prestressed concrete in the superstructure, although determined by other factors, was looked upon as experimental by the Authority and the Contractors, between whom the closest co-operation was maintained throughout the design and construction. It is appropriate, therefore, to consider what conclusions can be drawn from the experiment from each point of view. The Authority have been provided with a structure which, though strictly utilitarian, has clear, and efficient lines (*Figs 15* and *16*). If it fulfills expectations, it will be comparatively inexpensive in maintenance costs. On the other hand, the first cost of the structure has been greater than would have been the case with other materials.

From the contractor's standpoint, construction in this medium involves the most careful supervision throughout all stages. This requirement not only derives from its comparative novelty and the consequent need to train operatives in new techniques, but is inherent in a construction medium

which requires high-strength materials and consistency and in which this strength and consistency are generally subjected to the most stringent tests they will undergo before loading occurs.

There can be little doubt about the value of prestressed concrete standardized factory-produced articles, particularly where the wires are straight and bond-anchored and long-line manufacture is possible. For less specialized uses its value in relation to other media must depend upon a balance between those of its characteristics which are favourable, for example, its use of high-quality materials to their greatest advantage with consequent saving of materials, its low depth-to-length ratio for long spans and its obviation of tension cracks in the concrete, and those which are unfavourable, namely, the high labour costs involved, the need for most careful supervision and accuracy in construction, and a certain inflexibility in design at least as so far developed.

Fig. 14 illustrates two aspects of prestressed concrete. It shows a failure in a gable-end column during tensioning, as a result of the accidental omission of mild-steel mesh reinforcement. It may serve to illustrate the necessity of the closest supervision and the difficulties inherent in the anchorages, which represent the "Achilles heel" of this medium. On the other hand, the fact that the unit failed before erection is a point greatly in favour of prestressed concrete.

As far as the structure which has been described in the Paper is concerned, had steel been available, a portal frame in this medium would have been superior in economy and ease of erection, although inferior in maintenance requirements. The complex construction of the shoulder of the portal would have been reduced to a few inches of site welding. This complexity occurs because of the number of anchorages involved and it seems probable that similar involved joints will occur in other types of prestressed concrete frames. No doubt, however, improved design and construction techniques and greater familiarity with the medium will, in time, remove the difficulties and the making of such joints will become commonplace.

Where the cables depart from straight or simply curved paths, their placing calls for "finicky" and expensive operations and indeed the technique of prestressing suffers from this disadvantage in comparison with bar-bending and fixing in ordinary reinforced-concrete. It must be admitted, however, that his attitude may be engendered by the relatively greater familiarity which most workmen and engineers, including the Authors, have with the latter. In addition, much greater simplicity in construction techniques may be expected in the near future.

Acknowledgements

The Authors are indebted to Mr G. A. Wilson, M.Eng., M.I.C.E., M.I.Mech.E., Chief Engineer of the Port of London Authority, for

Fig. 13

MOBILE TOWERS USED IN ERECTION

Fig. 14

GABLE COLUMN, FAILURE DURING TENSIONING

Fig 15

EXTERIOR VIEW FROM THE SOUTH-WEST

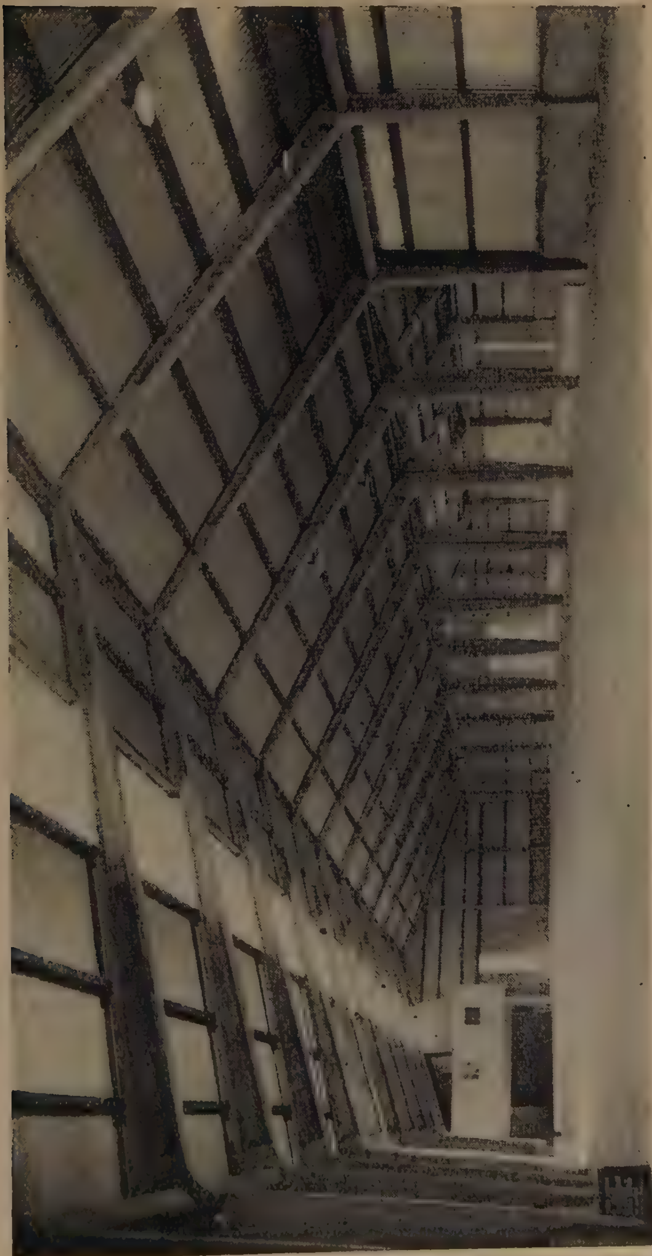


Fig. 16

INTERIOR VIEW OF SHED

permission to publish this Paper and to the Directors of Messrs John Mowlem & Co. Ltd, for making available certain records and information. They wish to express their appreciation to their colleagues who have assisted in providing information, drawings, and comments.

The Paper is accompanied by fifteen photographs and eight sheets of diagrams, from some of which the half-tone page plates, folding Plates 1 and 2, and the Figures in the text have been prepared.

Discussion

Professor A. L. L. Baker, complimenting the Authors upon the Paper, said that it was gratifying to see the Port of London being reconstructed in that way. After making trips to Le Havre and other Continental ports and returning feeling a little frustrated, he was pleased to see that things had happened, close at hand, in London.

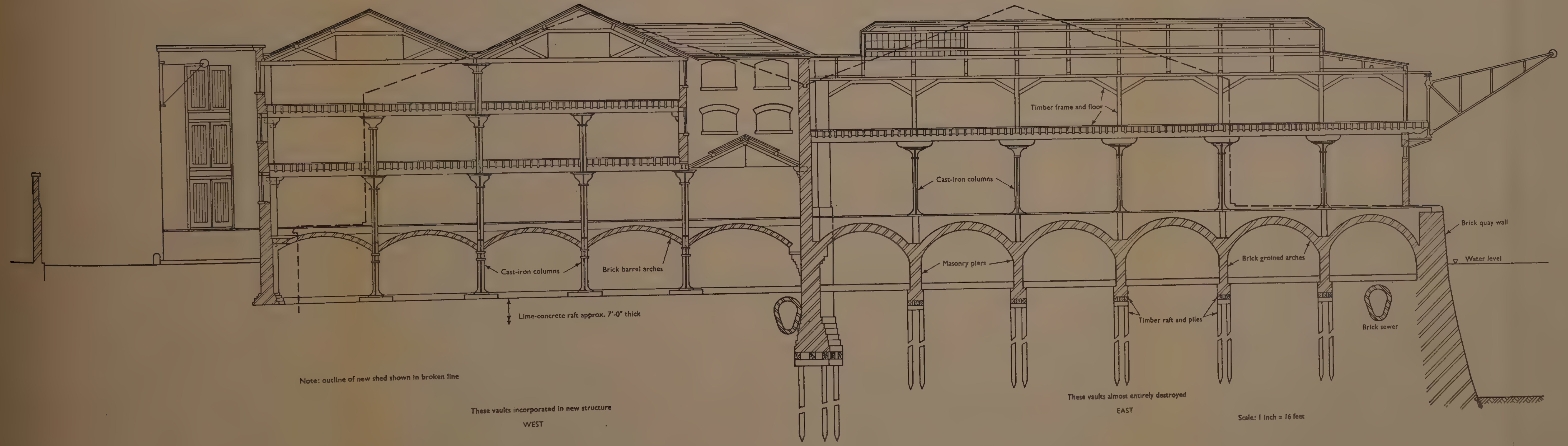
He felt that that particular job showed many significant points ; the most significant being the comparison of costs between the three different schemes. The Authors mentioned that steel—and steel for a function in which it usually held the field, the long-span shed—cost very nearly as much as reinforced concrete and about £3,000 less than prestressed concrete, without taking maintenance into account. He had inquired from a contractor what would be the cost of painting structural steelwork of that kind and had been told that, at the present-time, the cost was about 4*s.* per square yard of roof. That would make the cost of painting once about £550, so that after painting the cost was practically the same as for the reinforced-concrete design.

He had then considered whether there were possible ways of reducing even the cost of the reinforced-concrete design or the prestressed-concrete design. First, the very high strength of the concrete had struck him as a notable feature. A strength of 5,680 lb. had been reached in 7 days, which would mean that in 28 days there would be a strength of about 9,000 lb. per square inch. The Authors had very thoughtfully provided the value of the standard deviation of the cubes, which was only 875 lb. per square inch, and only 5 per cent of the cubes tested, roughly, would come outside the range of double standard deviation, which was 1,750 lb. per square inch ; that was to say, if at 28 days the mean strength was 9,000 lb., then the minimum strength would be 7,250 lb., apart from 5 per cent of the cubes. It therefore seemed to him that the crushing strength in the structure itself could be assumed to be 7,250 lb. per square inch, so that, using a load factor of 2, which was normally applied to the reinforcement, a design working stress of 3,625 lb. per square inch could be worked to.

He did not know what strength the designer had used in arriving at the reinforced-concrete design ; he doubted whether so high a stress as that

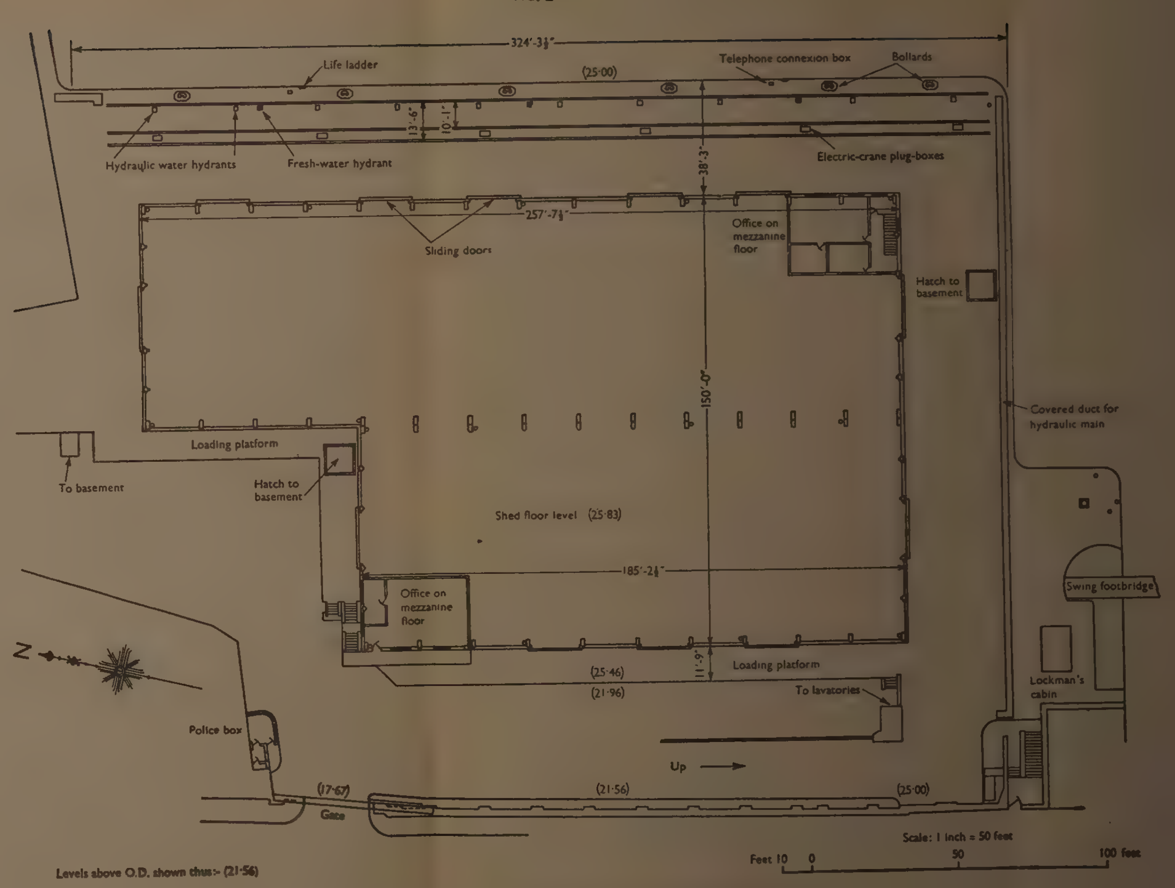
DESIGN AND CONSTRUCTION OF A PRESTRESSED CONCRETE FRAMED TRANSIT SHED FOR THE PORT OF LONDON AUTHORITY

FIG. 1



No. 19 BERTH. CROSS-SECTION THROUGH OLD WAREHOUSES, LONDON DOCKS

FIG. 2

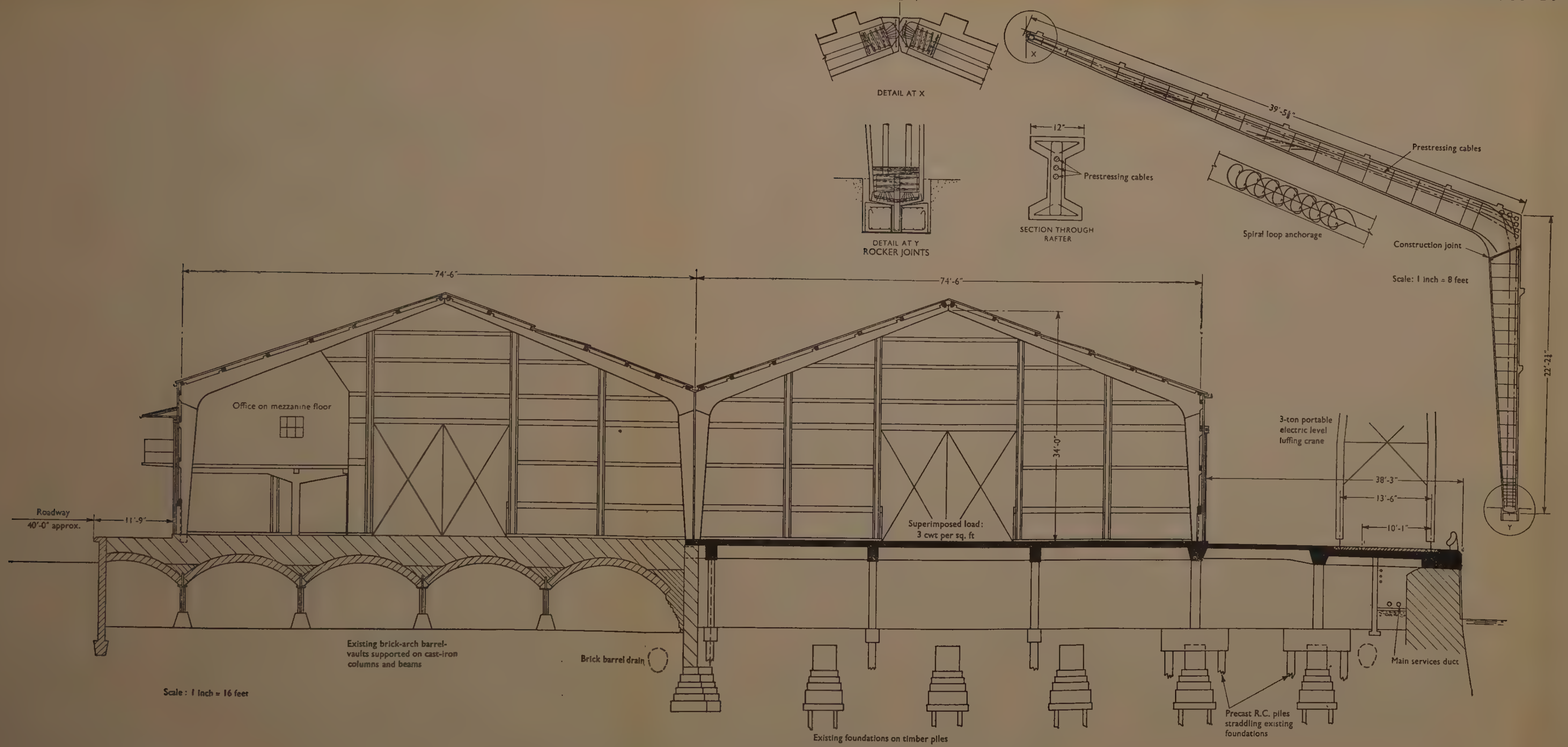


No. 19 BERTH. LONDON DOCKS, AS RECONSTRUCTED

N. N. B. ORDMAN AND I. S. S. GREEVES

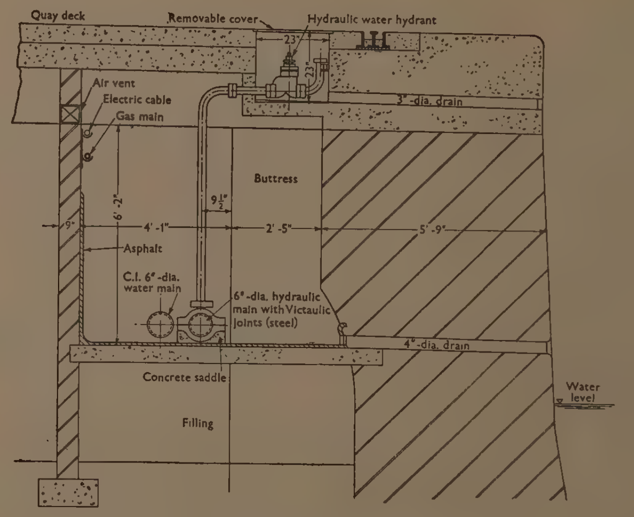
DESIGN AND CONSTRUCTION OF A PRESTRESSED CONCRETE FRAMED TRANSIT SHED FOR THE PORT OF LONDON AUTHORITY

FIGS 3



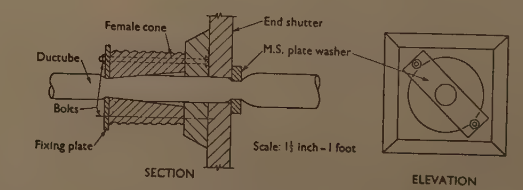
No. 19 BERTH CROSS-SECTION AND DETAILS OF PRESTRESSED CONCRETE PORTAL FRAME

FIG. 6



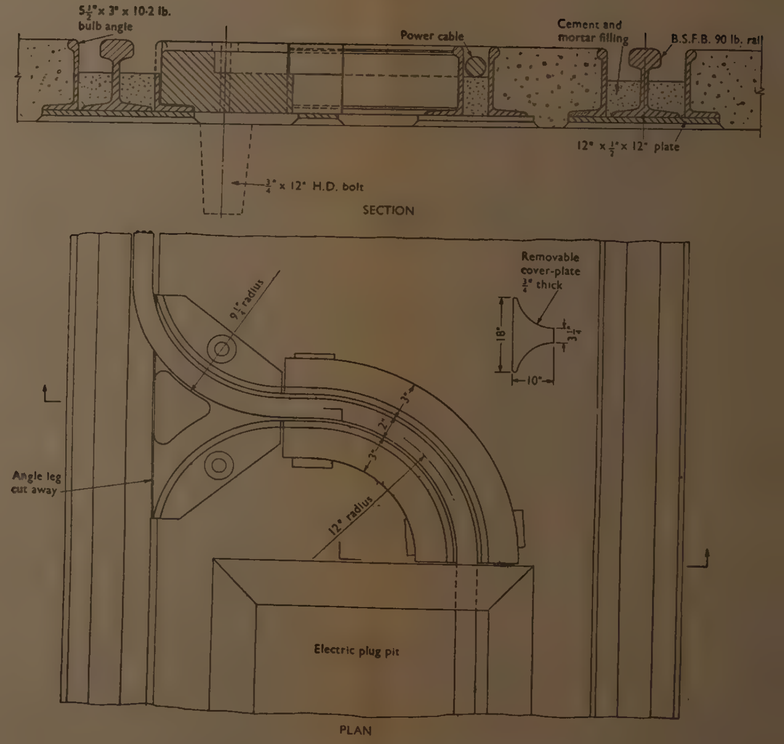
SECTION THROUGH SERVICES SUBWAY

FIGS 8



DUCTUBE FIXING AT CONE

FIGS 7



CABLE GUIDE AND CRANE TRACK DETAILS

N. N. B. ORDMAN AND I. S. S. GREEVES

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WILLIAM CLOWES & S

would have been used, and if it were, he thought that even a slightly cheaper job could be produced. In any case, that particular achievement in concrete strength did seem to point the way to future design for that type of structure.

He asked the Authors whether, if they achieved a standard deviation in the cube strength of 875 lb. per square inch, they would consider that in the structure itself the minimum strength, say at 28 days, could be relied upon to be 9,000 lb. per square inch, less double the standard deviation, that was, less 1,750 lb. per square inch. That assumption would mean that 5 per cent of cubes that came outside the scatter would be either rejected by the inspector, so that those slightly weaker bits of concrete would certainly not occur in the structure, or else it could be assumed that the 5 per cent beyond the range of $\pm$ 1,750 lb. per square inch had occurred because of the process of testing, that was, that the machine had slightly eccentrically loaded the cubes and that, in the structure itself, the concrete in the compression zone would be subject to no such eccentricity but would have a perfect bearing against the adjacent concrete, and therefore an assumption of 9,000 lb. per square inch plus or minus twice the standard deviation would be justified. He was not certain of the right view to take about that problem. It was very much a problem for civil engineers to solve at the present time, and he thought that the engineers who had carried out a job of that type should be in a position to give a very valuable opinion on it.

He asked the Authors why they had used rocker bearings. Those seemed to come into most designs of that type, and he felt that they were not necessary. A spliced joint could easily be inserted in the peak of the arch and at the base of the columns. If the outline of members was such as shown in the sections, then the bending moments—the restraint moment at the base of the column and the bending moment at the crown of the arch—because of the narrow depth, would be so small that the narrow section there would be quite capable of taking them, and a slightly better distribution of bending moments would result and a slightly cheaper type of construction. He also thought, though he knew it would be very difficult, that it would be an advantage if that type of precast member could be made with the long-line system of prestressing in order to cut out that rather difficult post-stressing operation on the rafter after it was in position. He thought it was a little difficult to make a joint at the knee and to prestress the rafter at the same time.

At Imperial College they had tried to produce a method of jointing precast prestressed bonded members by the use of a stretched high-tensile steel wire across the joint and with a special tool to do the stretching, so that one would just have to make the joint locally and not have to prestress the member at the same time.

Mr G. A. Wilson said that he wished to amplify the reasons given in the Paper for using prestressed concrete. From the facts given it could

be deduced that the transit shed had cost the Port of London Authority about £6,000 more than the cheapest form of construction which could have been devised. The reasons given to justify that increase were to gain information on actual costs and experience in that type of design and construction. The shortage of steel had presented the opportunity, and the possible saving in maintenance expenditure might balance the extra outlay.

But that was not the whole story. All progressive commercial and industrial concerns were conscious of the great mass of research work which was continually inventing new materials and making new methods possible; in many quarters it was realized that the value of those discoveries was not obtained until they were applied to production. The application, like the research, might be costly, but industry thrived on such investments. The return was both material and psychological.

In that case, the Port of London Authority would get, on the one hand, maintenance savings and would increase production from the easier working conditions; on the other hand, there was the stimulus given to all concerned by the practical application of any new product. That stimulus came first from the demonstration that new ideas were welcomed and introduced. Meeting and overcoming the difficulties encountered with a new medium engendered mental alertness. All who were connected with the work, either in design or construction as engineers or foremen, experienced satisfaction, and those who used the finished product could take a new pride in their work.

It did not seem to be desirable for large industrial or commercial concerns to undertake basic research which, if required, could best be undertaken at research centres. The province of industry was to see that the results obtained were turned to practical purposes, and it was in that spirit that the construction of the transit shed in prestressed concrete had been undertaken.

Mr A. J. Harris said that the Authors were to be congratulated not only on a very well-written and complete Paper, but on a very frank one. What was interesting about most engineering works was not only what went right, but what occasionally went wrong, and the Authors had pointed out everything that had gone wrong.

Having said that, he would now set to with a will to minimize the difficulties which the Authors had described, starting with the shoulder joints. The Authors maintained that that was an extremely complicated joint—with eight anchorages, much mild steel, and many cables. So it seemed until one looked at *Fig. 11* (facing p. 417) and compared it with a typical corner detail in a reinforced-concrete portal, where it was then difficult to fill the spaces between the bars with a larger aggregate than ¼ inch.

In connexion with the jointing and assembly of the rafter to the column, Professor Baker had mentioned the possibilities of using stressing over

extremely short lengths. He was not greatly in favour of that; the advantage of the type of assembly employed on that structure was that no additional concrete or steel was required, and only one supplementary operation, which was the filling in of the joint with a well-packed dry mortar. The "buttered" joint was never to be recommended.

A comparison had been made with welding—he thought in somewhat loose and light terms. The phrase used had been "merely a few inches of welding." Would that welding were always so simple.

The other difficulty to which the Authors had referred was that of the cable position. There was, he found, a great tendency in designing cables for prestressed concrete beams to stick to a parabolic trace. A parabola could be drawn without any great difficulty, and fitted the bending-moment diagram, but on the site it meant that at every few feet, an intercept had to be set out, and the cable fixed to correspond, instead of using a number of cables, each on a different parabola. The arrangement was being increasingly adopted in which the cables were carried straight through the bottom of the beam, and one cable was bent up through a small angle at regular intervals. That system was statically equivalent to the parabolic trace; it had the advantage that only straight lines were used.

The next point which he thought worthy of mention, in view of the cost of the job, was that it had been a first job. At the time the shed had been designed, there had been no other three-pin arches of that dimension anywhere, and he thought the number was still small. The cost of a new application or a new technique was best judged, perhaps not on its first application, but certainly on its second. In the first application many of the difficulties might have been over-insured against by all concerned, and in that particular case there were a number of design points where he thought that too much insurance had been made and which, in a subsequent design, would be considerably simplified. For instance, there was the question of the hinges—the rockers. The reason for putting in those rockers had been very understandable and human; it was desired to be quite certain about exactly what was happening. If three hinges were provided, the thrust line was known with some exactitude and the bending-moment values followed.

Moreover, when that structure was designed some years ago, very little experimental work on continuous redundant prestressed concrete structures had been done. Since then there had been the experiments of Professor Baker and Guyon and others, which indicated that the redundancies did in fact increase very considerably the safety of structure.

Without doubt, one of the first things that would be done in a repetition of a design of that sort would be to cut out that rather tricky detailing at the hinges. Erection with temporary hinges at those points might be made but then, by quite simple means, those hinges could be made solid and would add very considerably to the redundancy of the structure and consequently to its factor of safety.

Mr H. Allen said that, as an operating man, he was grateful for the privilege of being able to say a few words on how the Port of London Authority used its quays, sheds, and warehouses after the engineers had built them.

There was, in the Authority, a happy marriage between the engineering side and the operating side, and after so many of the sheds and warehouses had been destroyed during the war they had had to get together and discuss matters. The views on operating had changed very much since before the war because of the development of mechanization. They had always had a fairly set idea about the width of quays, and they had always been interested in wide quays up to 50 feet. London was a barge Port, a great deal of work was done on the quays, and that position had not changed.

When it came to the sheds, however, the operating method had completely changed during the war. Before the war, the operating method had been to have a shed with many doorways, narrow and not particularly high. All that had been wanted was that the trucker should get to his pile in the shed by the shortest possible route, because labour was expensive ; deviations from the route meant extra trucking costing more money.

In the shed itself they had not been particularly interested in height, especially in transit sheds, since, if piling by manual effort were to be high, the gang had to be increased, and the gang had also to be increased for delivery. For ordinary transit work they had been accustomed to pile at normal shoulder height. But the development during the war of the mobile crane and more particularly the fork-lift-truck had altered all dock operators' views on what they wanted an engineer to construct for them.

The Port of London Authority was concerned now with specifying not a number of narrow doorways, but fewer and wider doorways ; in London they had agreed on a doorway about 20-feet square, based upon the passage of a mobile crane into the shed or a fork-lift truck with a 12-foot lift.

There was also the question of labour. Many people had said that the docks had not heard of mechanization, but to say that was to deny the tremendous steps which had been taken. The Port of London Authority had found that where they had introduced a completely mechanized shed, in which labour was reduced to a minimum and every machine did its full job, they had been presented with some problems, because if the gang, through mechanization, was reduced, there could be no change half-way through—the reduced gang had to stay.

For the shed, they wanted a height of about 20 feet and the minimum of obstructions inside. There should be about a 13-foot gangway to take mobile cranes and fork-lift trucks. Therefore, instead of having many narrow gangways, a smaller number of wider gangways were needed, so that the shed floor area used was much the same, but instead of piling 4 feet or 5 feet high, the piling was up to 16 feet or 20 feet.

There was, then, the further problem of loading banks. The P.L.A.

had always been very happy to have loading banks for delivery. The shed described had been designed to some extent for handling green fruit, which was delivered to various descriptions; but with an export shed, which was completely mechanized, they were not quite so sure whether they were pleased with the loading bank. In completely mechanized sheds, a mobile crane or a fork-lift truck could be taken right out alongside the vehicle or, if necessary, the vehicle could be brought into the shed. If there was a loading bank, there was a restriction in that respect. However, if there was no loading bank and if there was a cargo which did not lend itself to complete mechanized handling, the loading bank was badly missed. Therefore, in some places there was no loading bank, and in other places the loading bank had been widened to about 15 feet or 20 feet.

Mr Cecil Peel said that he wished to make some observations of a more general character, rather than refer to particular items in the actual construction.

No doubt in the circumstances prevalent at the time, the use of that type of construction with prestressed concrete and portal design had been justified. There were, however, on general grounds, a number of points concerning which criticism could be made which would not be altogether of an adverse character.

Roof structures, generally speaking, should obviously be as light as possible, because the live-load they had to carry was of itself quite small. For that purpose, if aluminium were cheaper, it would be an ideal material, and he thought it was very desirable that the price of aluminium for that purpose should be reduced.

Generally speaking, the ordinary open braced steel structure had no equal whatever for speed, simplicity, and economy; but in the use of reinforced concrete, a totally different type of problem appeared. Basically, a concrete structure was heavy and in a sense, therefore, concrete was an unnatural material for that purpose. Weight increased costs, especially if it occurred in a beam, and it also affected foundation costs. Therefore, such a structure, if it was in reinforced concrete, should be as light as possible; that could only be achieved by prestressed work or by a shell.

Unfortunately, in a pitched roof there was induced a lateral thrust, unlike a portal structure with horizontal beam members. That, in turn, meant that either a portal framework with heavy bending moments at the knee had to be erected, or else a tie had to be used. There was a third method of overcoming the difficulty; an entirely pin-jointed portal frame, statistically determinate and rigid, could be used, by making a space frame in which there was transverse bracing in the two planes of the roof. It would be interesting to know in that particular case, in view of the design that had been adopted, what economy, if any, could have been effected either by the introduction of a tie or by a space-frame design.

On p. 412 the Authors referred to the bottom ties of steel roof trusses as tending to prevent easy movement of mobile cranes working within

the shed. Perhaps they did within the centre space, but he was not sure that was not a good thing. If in the centre of the shed and the centre of the spanning, a crane went in and lifted its jib which slewed round, there was quite a danger that it might crash through the glazing, or sheeting, or even damage one of the purlins on the rafters. If, on the other hand, a definite ceiling was imposed by the presence of ties, then everybody working in the shed with cranes knew that they should not be luffed above a certain level; the cranes could be so marked, and all would be well. Perhaps the Authors would indicate whether they had considered what would happen to one of the members if quite a large chunk of the concrete work was spalled out by collision, in view of the fact that it was under such high stress and that an eccentric thrust occurred at once in the member.

The Authors had referred to the question of maintenance painting of steel roofs. That affected the comparison of costs and, in the Authors' view, justified the increased cost. The painting of the steel framework of an ordinary transit shed was not a heavy item when spread over the life of the structure, because that steelwork was painted only infrequently. On the other hand, the painting of the sheeting of the conventional type of transit shed which had been in use for many years in the Port, was an exceedingly heavy item. That corrugated steel sheeting had to be coated outside every few years, and sooner or later it corroded at the laps and had to be renewed. Therefore, the adoption of aluminium sheeting was an innovation; its use in that particular shed was not the first, but it was one of the first occasions where it had been used; and it should prove a very valuable innovation, despite the increased cost.

Mr Allen had referred to the introduction, since the end of the 1939–1945 War of mobile plant, namely, cranes and fork-lift trucks. As Mr Allen had said, that had imposed an entirely new set of conditions upon the design of new Port structures above ground and had affected the reconstruction and reconditioning of older structures. First, it had meant an abnormal increase in height compared with the height of such sheds before the war. That was not such a serious matter with a new shed, but it had proved a very expensive matter in dealing with existing sheds, many of which had to be raised. Whilst, in certain cases, such an increase of height could prove of value in giving extra space for stacking, in certain cases he thought it was true to say that transit sheds which were not entirely filled with goods and export cargoes, for instance, tended to be spread about to a moderate height, in spite of the extra height available, with the result that all the extra height merely produced empty space for the purpose of luffing up the jibs of the mobile cranes. He would like to make an earnest plea to designers of mobile cranes to try to reconsider the basic design of the structure of such machines. It should be possible to arrange a crane which would alter its radius without altering its height, with some form of pantograph motion, or something of that sort. If that

could be done, it would help in cheapening the cost of such works as that described in the Paper.

The Chairman said he thought Mr Harris had generalized on one point in a manner which might be rather dangerous. The suggestion that an increase in redundancy could always increase the strength of the structure was, he was afraid, somewhat heretical. He would not dogmatize about it, but he would very much like to see any proof which had been deduced for that statement, because he had a vivid recollection of a test nearly 40 years ago on two similar aeroplanes in one of which there was a redundant cable which was missing from the other. The load factor on the aeroplane without the redundancy was 7, and on that with the redundancy 5·5. It was probably true in most structures that as a result of redundancy some increased strength did accrue or, at any rate, strength was not reduced; but he thought it was not difficult to imagine cases where that was untrue and where redundancy could reduce the strength.

He had been struck by the remark on p. 414 that "A further consideration which should not be overlooked was the desire to obtain first-hand experience in what is still, certainly in the field of British maritime engineering, a novel structural medium." That was an important statement, and he had been delighted to hear it emphasized so strongly by the Chief Engineer of the Port of London Authority. There was little use in research establishments and individual research workers carrying on their work if nobody was courageous enough to test whether the results could be applied to produce better and more efficient engineering structures. It was, therefore, really refreshing to find a remark of that sort in the Paper. It was generally only large undertakings like the Port of London Authority who could afford to experiment; but he was sure that Mr Wilson was right and that it was part of their function to see what could be done with new methods.

There were two points about which he wished to ask questions. First, the sheeting was referred to as aluminium alloy and then, a few lines later, as aluminium. He would be interested to know whether there was reason to expect, or any evidence in service, that the aluminium alloys were not affected so much as other materials, such as steel, by corrosion. Early aluminium alloys, he believed, had corroded rather badly, and it had been necessary to protect them very thoroughly. He would like to know whether there was any evidence about the present, because the object in using that sheeting was to save painting.

The second point concerned wind loading. On pp. 418 and 419 it was stated "The wind load, approximately equal to 55 miles per hour at a height of 35 feet, was taken as 10 lb. per square foot." That was more appropriate, he thought, to 60 miles per hour than 55 miles per hour, but that was immaterial. What he was not clear about was how the effective wind load had been distributed on the structure, and he thought that with structures of that type it would be worth while to carry out a wind-tunnel

test to get a closer idea of the distribution. It was not a very expensive matter to carry out such a wind test, and much more information about wind pressure was needed; perhaps the Port of London Authority would next time add to the debt owing to them by trying to give further information on the distribution of loading on stanchions in areas such as the London docks.

Mr A. Goldstein said that he wished to join with other speakers in congratulating the Authors on an "honest job honestly done."

The first point he wished to raise was on the question of comparative costs. He agreed that it was very courageous of the Authors to give those figures, but he would like them elaborated a little. First, had any other systems of prestressing been tried in the competitive cost comparison? If other schemes had been tried the figures, if available, would be very useful information.

It had not been explicitly stated but it was implicit in the Paper, and in the discussion, that the actual job had not been one of competitive tendering. He was perhaps taking an implication that had not been meant, but if it had not been competitive tendering, what reliance did the Authors place on the estimates? If, on the other hand, it had been competitive tendering, had there been any competition in the design?

When comparing costs, if the Authors had prepared the steel design and details and the contractors had prepared the prestressed-concrete scheme, and the reinforced-concrete scheme, then somebody had had to pay for that design work, and he strongly thought that the cost of the design was included in the figure given in the Paper, since the contractors, in pricing, would allow for it. For comparison, the figures should, of course, include the cost of the design of the steelwork, and perhaps the Authors could give those adjustments.

The next point referred to the shoulders of the portals. On p. 420 the Authors stated "At the shoulder (*Fig. 11*, facing p. 417) there are eight anchorages and the stress distribution is very complex." He thought that was a masterly understatement, and he wondered whether the Authors had considered the problem in any detail and whether they would enlarge upon that matter. Perhaps they had made some tests. Quite apart from the complex local anchorage stresses at that point, it was an extremely difficult problem even when considering load stresses and the "counter stresses" from prestress. There was, first, the question of the lap effects of the cables; secondly, there was the difficulty of correctly assessing the effective cross-section, bearing in mind that the anchorages were set within the section and that there were spaces left for mortar filling, etc.; thirdly, and most important, there was the stress distribution from the localization and "bunching" of the anchorages. In prestressed portals, with an unknown stress diffusion at a standard section, it would be most important to ascertain the effects with some precision.

Possibly one of the simplest ways was to test one of them; he felt that

some very interesting information would be obtained by testing to destruction, not a whole portal, but 'idealized' shoulder joint only. It was a problem about which he did not think there was a rigorous answer as yet.

Finally, on the question of end reinforcement, it would be interesting if the Authors could say whether any cracks had occurred where mild-steel end reinforcement had been placed. The unfortunate photograph to which reference had already been made was a case where no mild-steel had been used. Prestressed-concrete designers went to great detail in regard to local reinforcement for the end-stresses. If the Authors could confirm that there had been no cracks where reinforcement had been used, it would be a useful practical verification of present-day practice.

Mr J. A. Derrington felt that the most interesting part of the Paper was Table 2 on p. 413, which dealt with costs. The objects of the experiment, they had been assured, were to discover whether a new form of construction justified itself when compared with more conventional methods. He felt that the Authors' conclusions on that point were rather incomplete.

It was to be noted that, if the steelwork design was considered as a basis, the prestressed-concrete design was about 40 per cent dearer (he was considering the structure only), and the reinforced-concrete design only 7 per cent dearer. He felt that, with those two facts in mind, it would be interesting to know more about the reinforced-concrete design, and, in particular, whether it was based upon the same working stresses as the prestressed one, about 2,500 lb. per square inch on the concrete. He had had the temerity to put one or two figures to a reinforced-concrete section and it seemed that, if stresses of that order were tolerated—and they seemed to be justified in relation to the cube strength—the section would not be increased in size and the detail at the eaves would not be as formidable as was at first apparent, especially if a high-tensile steel reinforcement was used.

The second point concerned the knee joint, which Major Harris had assured them was very simple. He felt that even Major Harris would hesitate to put any figures to the stresses that occurred at that point; but having accepted the fact that it worked, the question was why it had been done in the air. He felt that, since prestressed concrete was still going through some teething troubles, the object should be to make the joints in the simplest manner possible and in an easy position. He asked the Authors why the design had been based upon joining the rafter to the column after erection, and why the legs had not been cast in one piece on the ground, where the whole prestressing could have been done quite effectively and simply. He felt that if the frame had been cast in two pieces only, the erection would have been very much quicker and a great saving in the cost of temporary supports would have been made.

Mr Francis Walley, referring to the question of cost, asked whether the estimates had been for comparable designs or whether they had been for

the cheapest method of design with any of the materials. He also noted that the figures related to the estimated costs, and asked whether it would be possible to hear what the final cost of the scheme, as erected, had been. Would it be possible to have also the cost per cubic foot of the actual beams, subdivided into post-tensioned and pre-tensioned units? He had usually found that that gave quite a good check on design work.

He felt that costs given in that way in the Paper, although they were extremely useful, had to be treated with a certain amount of caution. He had in mind at the moment, not a comparable design, but a job with which he was associated where, for a three-bay portal workshop, the estimated costs were in the region of 100 per cent for steel, 120 per cent for concrete and 102 per cent for prestressed concrete. In that particular case there would be the advantage that all three workshops would be erected, so that the actual costs would be known. It was, however, on a different scale from the job described in the Paper.

There were two small points that arose directly from the Paper. He was very surprised, but pleased as well, that the strengths and consistency had been obtained with volume batching and not weigh batching. It was a little unusual to see that volume batching had been used on the job, although a coefficient of variation of 15 per cent, as quoted, was perhaps a little higher than he would have expected for that class of concrete.

Finally, he wished to ask when the failure during tensioning had occurred. Had it occurred immediately, 24 hours later, or at what time interval?

Mr J. A. Williams said that he had recently been concerned with the design of transit sheds for two major port schemes overseas; he thought it might be interesting to compare the way problems had been solved abroad with the way the Port of London Authority had tackled them.

The sheds were respectively 85-foot clear span and 100-foot clear span. For various reasons, steel had been selected and there had been no question of a comparison with normal reinforced or prestressed concrete.

The first point of comparison was in actual design of the steelwork. Referring to Table 2 in the Paper, the steel design had come out at 150 tons, which he had worked out at 10 lb. of steel per square foot of floor area, for a span of 75 feet. In the sheds with which he had been connected, the 85-foot span had worked out at 9 lb. of steel per square foot which, bearing in mind the inevitable differences in detail, he thought was a good check. The steel for the 100-foot span had come out a little higher at $11\frac{1}{4}$ lb.

In the job to which he was referring, for the whole of the cladding to full height, they had decided in favour of blockwork and had adopted the more usual asbestos-cement sheeting for the roof. The eaves were a danger, and they had carried the blockwork up past the eaves to have hidden eaves and gutters, thus presenting a flush-face to the quay, which it was hoped would afford the necessary protection.

In regard to the development of frame shapes and spacings, the frame which they had first tried in the 100-foot span sheds had given rather high bending moments, so they had departed from the normal rigid portal and used a broken ridge ; it might not have looked particularly beautiful, but it reduced maximum bending moment by approximately 10 per cent and steel required in the main frames by about the same amount. Because of certain constants in purlins, and so on, it had reduced the total steel required by 7 per cent and had been considered worth doing. The spacing of frames in both cases had come out at approximately a quarter of the spans ; in other words, 20 feet for an 85-foot span and 25 feet for a 100-foot span. They had worked out quite a few figures before deciding on that. They had tried spacings for the 100-foot spans from 18 feet to 32 feet ; but were ultimately convinced that a quarter-span, or 25 feet, was favourable.

With regard to transit sheds in general, they had also been faced with the problem of a double-storey shed where the client was interested in keeping the bottom storey completely clear of columns. After a good deal of consideration, they had decided to make the whole of the top storey in the form of Vierendeel girders, with external joints rigid and the others, points of simple suspension. He did not suggest that that was the cheapest way of building a two-storey shed, but where a clear bottom storey was required he suggested that as one solution.

He concluded with two questions. He gathered from the Paper that, in the Port of London, glass was used for external lighting. Was that the normal practice in the Port of London, and was it satisfactory ? In the job to which he had referred, they had decided to use Perspex.

He noticed that all the piles driven for the foundations were vertical. He would have thought that the portal reactions were inclined, and expected to see some raking piles, but perhaps there was a straightforward explanation of that.

The Authors, in reply, agreed wholeheartedly with the Chairman's comment on the need for large undertakings to take the lead in applying the results of technological research, and counted themselves fortunate that the engineering department of the Port of London Authority, under the guidance of Mr G. A. Wilson, had provided them with the opportunity of carrying out that very interesting work. There was no doubt, as Mr Wilson had said, that meeting and overcoming novel difficulties was stimulating.

Mr Allen, the Assistant to the General Manager of the Port of London Authority, had referred to the close co-operation that existed between the operating and engineering departments and his account of the changing character of operating requirements was very interesting. The engineer relied upon a clear statement of the functional requirements of the structures he was to provide, and it was obvious that Mr Allen had a clear realization of the trends of the very complex organization under his control.

The costs given in the Paper for the prestressed-concrete structures were

actual costs, those for steel and reinforced concrete were estimates. Taking those estimates as being reasonably accurate, the reinforced-concrete structure was, as Professor Baker had pointed out, almost as cheap as steel, and, in fact, the reinforced-concrete design had not been ruled out on grounds of cost. In that connexion it would be interesting if Mr Walley could arrange for the publication of the comparative costs of the three similar buildings, which were to be erected each in one of the three media. In accordance with the Authority's normal practice the competitive designs were not called for, and for reasons which had been given by Mr Wilson (see reference 1, p. 409) competitive tenders had not been invited.

Professor Baker had commented on the high strength of the concrete, but it should be made clear that although good-quality concrete had been maintained throughout the work, the figures which he had quoted referred to the concrete used in the prestressed-concrete members. The design stress was 2,000 lb. per square inch, and the minimum cube crushing strength before tensioning was 6,000 lb. per square inch. The aim had been to obtain that requirement as early as possible so that the members could be tensioned and removed from the beds without delay. The Authors had had no doubt that the strength could be obtained, but had been concerned to obtain good consistency. The standard deviation had been used as a measure of their success and to confirm the assumptions regarding degree of control which should be made in all concrete-mix design.

Mr Derrington had suggested that if a reinforced-concrete structure had been designed on the basis of using concrete of the strengths actually used for the prestressed-concrete work the sections would have been comparable. That might be so, but it would appear that reinforcement difficulties might well occur, and the factor which always arose in considerations of high design stresses in reinforced concrete, namely tension cracks, would undoubtedly have to be considered.

Mr Harris and Mr Goldstein as well as Professor Baker and Mr Derrington had commented on the shoulder joint. Despite Mr Harris's remarks the Authors considered that its complexity was one of the weaknesses of that type of design. However, Mr Derrington's suggestion that the joint between column and rafter should have been made on the ground appeared to overlook the difficulties of raising the awkwardly shaped member that would have resulted. The contractors had considered that possibility, but had rejected it as uneconomical in view of the very rigid and heavy strong-back that would have been required to prevent undue handling stresses. Mr Goldstein was correct in stating that a rigorously precise design was not possible for the shoulder section. That comment might be applied to many structural elements in common use. The method of design was that suggested by Mr Guyon and had been guided by experience. Although no tests to destruction had been carried out, the member had been adequately tested during tensioning and no failures had been en-

countered. Where mild-steel reinforcement had been introduced in anchorage zones, no cracks had occurred. The failure that had been eliminated was attributable to the accidental omission of reinforcement and had occurred immediately on tensioning.

Mr Peel had made some very pertinent remarks about the general question of roof designs, with which the Authors agreed. They did, however, consider that a principal tension tie could be objectionable where freedom of movement was required for cranes and that some expense in obviating it was merited under those conditions.

They were in wholehearted agreement with his comments on the design and operation of mobile cranes. His suggestion regarding an entirely pin-jointed frame was ingenious, and no doubt was well worth consideration for a steel or timber structure, but from a superficial examination it did not appear to lend itself to reinforced- or prestressed-concrete construction. However, profitable consideration could no doubt be given to the introduction of statical indeterminacy to reduce or re-distribute the bending moments by removing one or more of the pin-joints.

The Chairman had suggested the use of a wind tunnel to assess more accurately the distribution of wind stresses. In the structure under discussion, which was fairly low, the calculation of wind stresses had been based on the Code of Practice, but it was apparent that for other types of structure and particularly bridges, the wind-tunnel technique was a welcome development.

It was felt that the heavy corrosion of aluminium alloys in the early stages of their use, to which the Chairman had referred, might have been due to a lack of knowledge of the causes contributing to this corrosion. It was important to ensure that electrolytic action could not be induced by nearby power cables, or by contact with uncoated steel or other metals. Great care had been taken to rule out those possibilities in the structure and good results were anticipated.

In reply to Mr Williams, the use of reinforced glass roof-lights was partly dictated by the requirements of the Fire Offices Committee and H.M. Customs and Excise. The use of battered piles was unnecessary, for there were massive structures at either side to take lateral thrust.

Correspondence

Mr D. H. New referred to the description on p. 426 of the procedure adopted to guard against segregation. That was a problem which occurred too frequently with many types of mixers, and was particularly acute when high-class concrete was required. Would the Authors consider their procedure to have been necessary if the mixing had been done by a horizontal-drum paddle mixer?

The reference on p. 427 to the use of plastic sheathing could hardly be interpreted as a recommendation. Many engineers insisted on the use of

a reasonably stiff metal sheath to the complete exclusion of plastic sheathing. Did the Authors consider that there were any conditions in which plastic sheathing would have advantages ?

On p. 432 the Authors had drawn attention to the unfortunate results produced by the " buttering " of joints—a method sometimes required by specification. It should by now be generally recognized that it was a practical impossibility to form a perfect joint of that description without using the method of dry packing finally adopted.

The Authors, in reply, referred to Mr New's statement that the " buttering " of joints was sometimes specified. The Authors had also noticed that, and from their experience they agreed that it was not a satisfactory alternative to dry mortar packing. There was no objection to plastic sheathing as such, but the plastic sheathing used on the job had not proved satisfactory. The material of which the sheath was made had to be strong enough to withstand the treatment it might be expected to receive and not more inflexible than the cable it covered, and in the Authors' experience light metal sheathing met those requirements to a greater degree than the plastic sheathing used. Where those requirements were met the only other criteria were price and delivery.

As stated in the Paper, no segregation of the concrete had in fact been observed and the precaution of tipping the batch on to a plate had been part of the system of controls. However, the Authors were of the opinion that for dry rich mixes a horizontal-drum paddle mixer had advantages, although the primary guard against segregation should be the careful design of the mix.

Correspondence on the foregoing Paper is now closed and no further contributions will be accepted.—Sec. I.C.E.

WORKS CONSTRUCTION DIVISION MEETING

23 March, 1954

Mr A. C. Hartley, Member, Chairman of the Divisional Board, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Works Construction Paper No. 25

"Design and Construction of Rama VI, Surat, and Bandara Bridges in Thailand"

by

Oleg Alexander Kerensky, B.Sc.(Eng.), M.I.C.E., and Kenneth Edwin Hyatt, B.Sc.(Eng.), M.I.C.E.

SYNOPSIS

The Paper explains the importance of Rama VI, Surat, and Bandara Bridges to the Thai Railway system.

The old bridges demolished during the 1939–45 war are compared with the new ones, which are fully described.

Rama VI and Surat Bridges are combined rail and road bridges and the Bandara Bridge is for rail traffic and pedestrians only. All railways are of metre gauge.

Rama VI Bridge has five openings with a simple span at each end, anchor arms occupying the intermediate spans extended as cantilevers supporting a suspended span in the centre opening. The total length is 445 metres (1,460 feet). Surat Bridge consists of three simple spans, one of about 80 metres and two of 60 metres (197 feet). Bandara Bridge has a central suspended span supported by cantilevers each extending over the two shore openings as anchor arms. The total length of this bridge is approximately 262 metres (860 feet).

The designs of the bridges are explained and the considerations governing the particular types adopted are noted.

A brief description of the demolition of the remains of the old superstructures is given and also the combined demolition and reconstruction of the foundations.

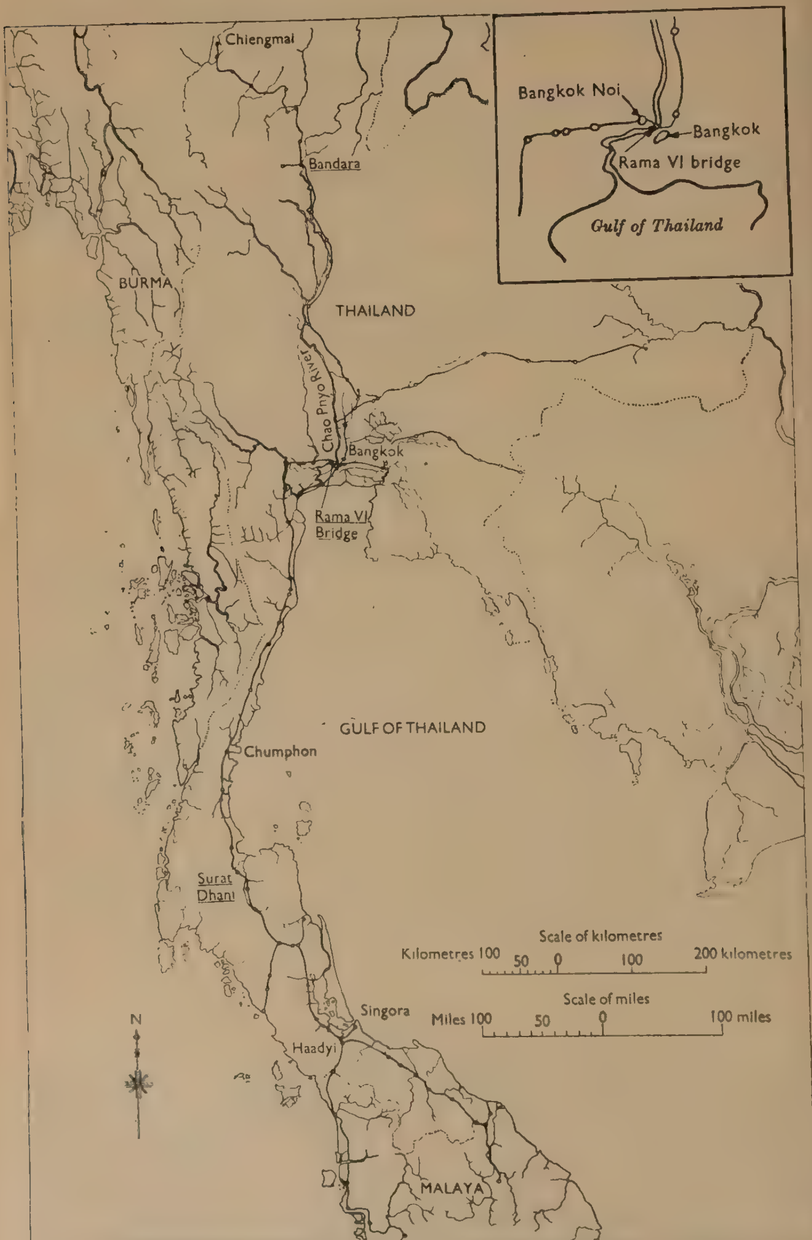
The erection of the steel superstructures is described.

Introduction

The Rama VI, Surat, and Bandara Bridges are vital crossings in the Thai Railway system, as will be noted from the map of the country (*Fig. 1*). Rama VI Bridge connects the capital city of Bangkok with the Southern line to Malaya ; Surat Bridge is also on this line, about 400 miles to the south of Bangkok. This is the most important line in the country and carries international expresses. The Bandara Bridge, 200 miles to the north of Bangkok, carries the line to the old northern capital of Chiengmai.

The original Bandara Bridge was built in 1907, that at Surat in 1914,

Fig. 1



MAP OF SIAM SHOWING LOCATION OF THE THREE BRIDGES

and the Rama VI Bridge in 1927. All three bridges were demolished by Allied air bombing towards the end of the war and wagon ferries were built at Rama and Surat by the British when Siam was occupied. A temporary trestle-bridge diversion was built at Bandara.

The wagon ferries and their piers were constructed of military pontoon and Bailey bridge units and could carry a locomotive or a few freight cars. Their capacity was very limited and high floods put them out of action. Passengers were ferried in small boats at Surat—a somewhat exciting experience during flood periods. The alternative station of Bangkok Noi was used as the southern terminal at the capital.

The trestle bridge at Bandara had to be rebuilt after severe floods.

The banks of all three rivers are low lying, particularly at Rama and Surat, and the difficulty and expense of building long new approach banks to suggested new sites resulted in the railway administration deciding to rebuild on the old centre-lines in spite of the difficulties which were expected to attend that method.

Flooding normally occurs at Rama VI Bridge from May to October; at Surat, from August to December; and at Bandara from April to September. The usual variations in water level are 3·80 metres, 4·10 metres, and 10·30 metres respectively.

In December 1948, the Thailand State Railways invited competitive international tenders for the design, fabrication, and erection of new superstructures and for the repairs and strengthening of the existing foundations for Rama VI, Surat, and Bandara Bridges. Tenders, with complete sets of drawings and calculations were to be in by 15 May, 1949.

Design

Tender Design Considerations

The consulting engineers' approach to the design of the bridges was, of necessity, somewhat unusual. The task was to produce, in less than 4 months, three highly competitive designs, at the same time avoiding the many undesirable details of the cheaper forms of construction.

Although outline drawings of the demolished bridges were available, it was clear from the start that different types of trusses would have to be evolved to suit modern methods of fabrication and erection, and parallel-chord trusses were adopted throughout, since they give simpler connexions to top laterals and are cheaper to fabricate than the slightly lighter trusses with curved chords. It was also highly desirable to lighten the superstructure as much as possible so that greater live loads could safely be carried by the old foundations, and it was decided to make extensive use of high-tensile steel (B.S. 548), which resulted in great economies in weight and considerable saving in cost, particularly in the high freight charges. (In 1949 the cost of shipping one ton of steel to Bangkok ranged from £6 10*s*. 0*d*. to £9 10*s*. 0*d*., depending on the size of pieces.) The basic price of plain mild steel at that time was about £21 per ton.

The steel situation in England at that time was fairly desperate and the decision to adopt high-tensile steel was taken with some misgivings, for it was felt that it would be even more difficult to obtain than ordinary mild steel. Throughout the design, the supply situation was borne in mind and variations of sections were kept to the minimum by making all three bridges as similar as possible, adopting nearly the same panel lengths for Rama VI and Surat Bridges, and using similar types of members throughout, so that large tonnages of each section would be required. As a result, in the case of Rama VI and Surat Bridges, all roadway stringers were 18-inch-by-6-inch-by-55-lb. joists ; all railway stringers consisted of 44-inch-by-$\frac{3}{8}$-inch web, with four 6-inch-by-$3\frac{1}{2}$-inch angles of variable thicknesses, and all cross-girders consisted of 54-inch-by-$\frac{3}{8}$-inch webs, with four 6-inch-by-$3\frac{1}{2}$-inch-by-$\frac{1}{2}$-inch angles and 13-inch-wide flange plates of variable thicknesses. The same size angles were also used in cross-girders for Bandara Bridge.

Since one of the contractors was, in fact, also a steel producer, the supply difficulties were overcome and the fabricators were ultimately able to obtain the necessary high-tensile steel. However, the special precautions in the design were not wasted, for they resulted in savings in handling and fabrication costs.

The Thailand specification did not provide for the use of high-tensile steel, but fortunately permission was granted by the Purchasing Commission to prepare designs in accordance with the best current British practice, whilst using the loadings and clearances given by the State Railways' Specification.

Specification

Rama VI and Surat Bridges provide almost identical facilities, namely, a single metre-gauge railway line, placed near one truss, a 6-metre-wide roadway alongside it, and 1·5-metre-wide cantilevered footways (two on Rama VI Bridge and one on Surat Bridge). Unfortunately, with this arrangement, the total loads carried by each main truss are substantially different, and although similar types of members were used in both trusses, the thicknesses of component elements had to be varied considerably.

Bandara Bridge carries a single metre-gauge railway track and one cantilever footway. The footway loading is not significant and the trusses for this bridge were made identical. The specified loadings were as follows :—

(1) *Railway live load and its effects*

Unit loading equivalent to 14·75 units of B.S. metre-gauge loading (1937).

Impact : $\dfrac{50W}{L+45}$ where L denotes loaded length in metres and W the load per track.

Lurching : $\frac{300W}{2S}$ where S denotes spacing in millimetres between centres of stringers or longitudinal girders and W the load per track.

Longitudinal force : 20 per cent of all wheel loads or 25 per cent of loco driving wheels.

Lateral sway : 3·5 metric tons single force at rail level.

(2) *Highway live load and its effects*

(*a*) Train of two 12-ton trucks on each 3-m. wide lane ;

or (*b*) one 18-metric-ton roller with U.D. loading of 300 kg. per square metre ;

or (*c*) Uniformly distributed load = $(500 - 2L)$ kg. per sq. m. (not less than 350 kg. per sq. m.),

where L denotes span in metres.

Impact on (*a*) and (*c*) only $= \frac{25}{L + 60}$

where L denotes loaded length in metres.

Longitudinal force : 10 per cent of live-load on structure.

Live load on footways : 300 kg. per sq. m. (in the case of Rama VI Bridge, 450 kg. per sq. m.).

Wind : 150 kg. per sq. m. on structure and on train 3·5 m. high and/or on highway load 4·5 m. high.

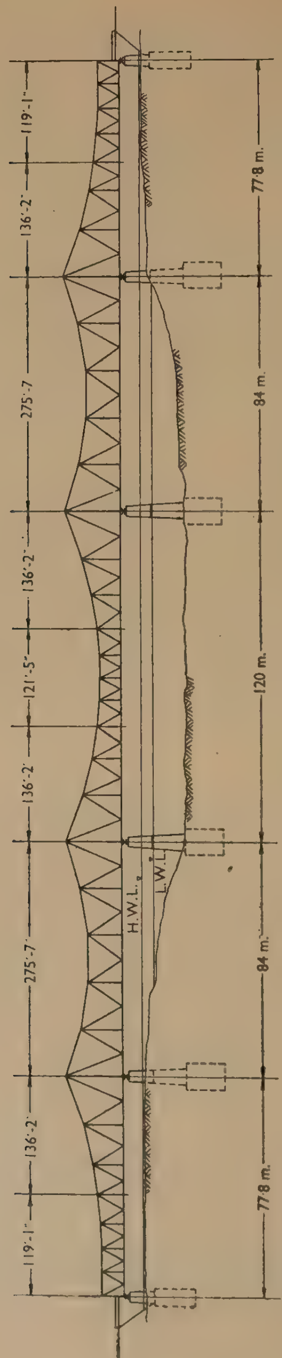
Temperature range : $\pm$ 30° C. ($\pm$ 54° F.)

(3) Provision had to be made for jacking each span clear of its bearings.

With a few minor modifications the steelwork was designed in accordance with the Code of Practice for Simply Supported Steel Bridges (prepared by the Joint Committee of the Institution of Structural Engineers and of the Institution of Civil Engineers) which at that time was available only in draft form. It may therefore be said that these are the first bridges designed in accordance with the Code, which forms the basis of the revised B.S. 153, parts of which have been circulated recently for comment.

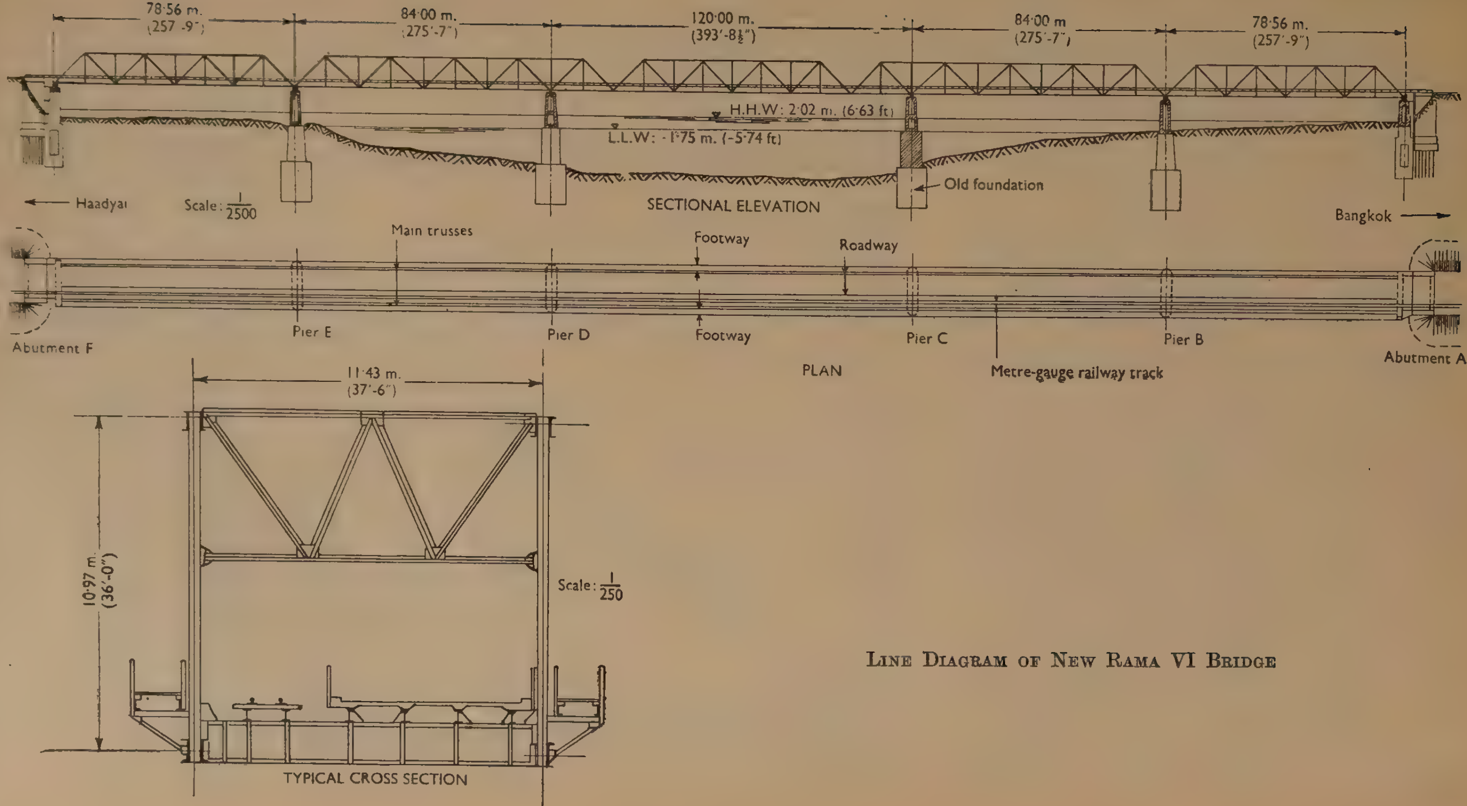
Steel roadway stringers were designed to act in combination with the reinforced-concrete roadway slab, angle shear-connectors being provided throughout their length to ensure inter-action. No allowance, however, was made for distribution by the reinforced-concrete slab of the heavy roller loads to adjacent unloaded stringers, because the various experiments carried out by the Building Research Station and the Illinois University had at that time not been completed and the available information was not considered to be sufficiently reliable. In any case the distribution would have very little effect on the size of the steel beams. The three-span continuous roadway slab was designed using the Pigeaud distribution method with maximum stresses in concrete of 1,000 lb. per square inch ; ½-inch wearing

Fig. 3



LINE DIAGRAM OF MAIN TRUSSES. OLD RAMA VI BRIDGE

Figs 4



Line Diagram of New Rama VI Bridge

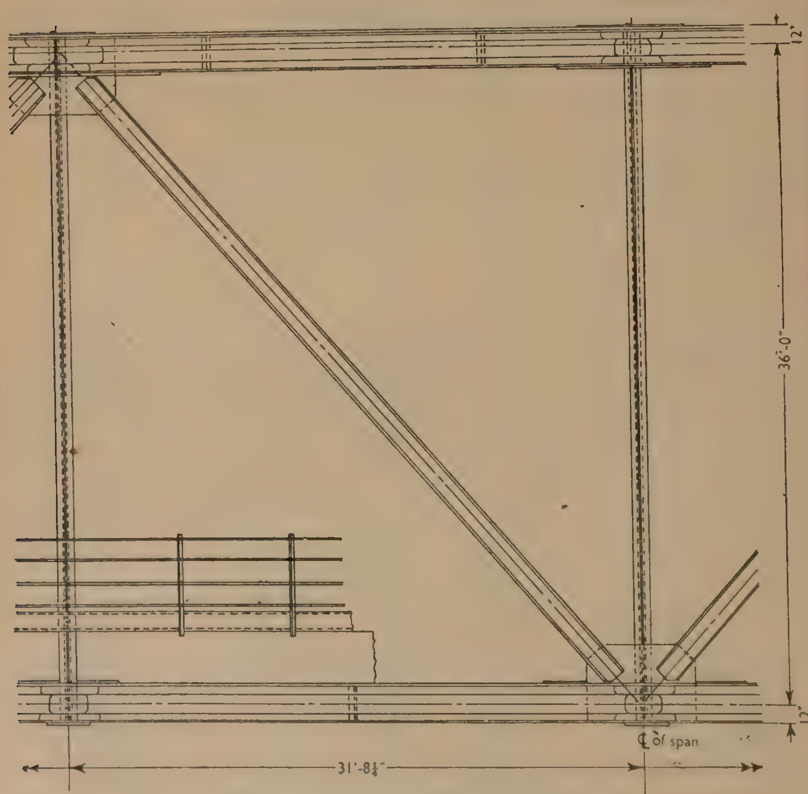
surface was provided by increasing the cover over the top reinforcing bars, and all expansion joints at cross-girders were reinforced with steel angles.

Rama VI Bridge

The old bridge (*Fig. 2*, facing p. 472), built by *Les Etablissements Daydé* of Paris, was of cantilever type with a total length between abutments of 443·6 metres (approximately 1,455 feet), divided into five spans of 77·8 metres, 84 metres, 120 metres, 84 metres, and 77·8 metres. The arrangement of main girders, which are 10 metres apart, is as shown on *Fig. 3*. It was designed to carry a railway loading equivalent to about 10 units of the B.S. loading and a 5-metre-wide roadway which, however, was never built.

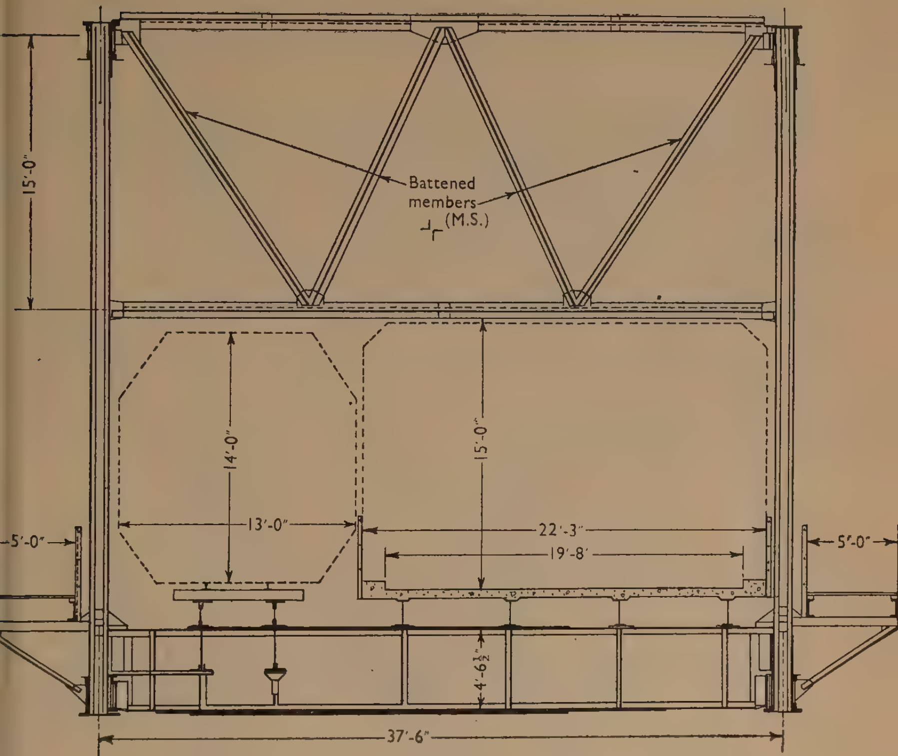
The new bridge has, of course, the same spans, but the arrangement of main girders is quite different (see *Figs 4*). The bridge now has three almost identical simple spans of 253 feet 6 inches (one of them suspended in the centre opening) and two unsymmetrical anchor spans of 343 feet 11 inches

Fig. 5 (a)



ELEVATION OF TYPICAL PANEL OF NEW RAMA VI BRIDGE

Fig. 5 (b)



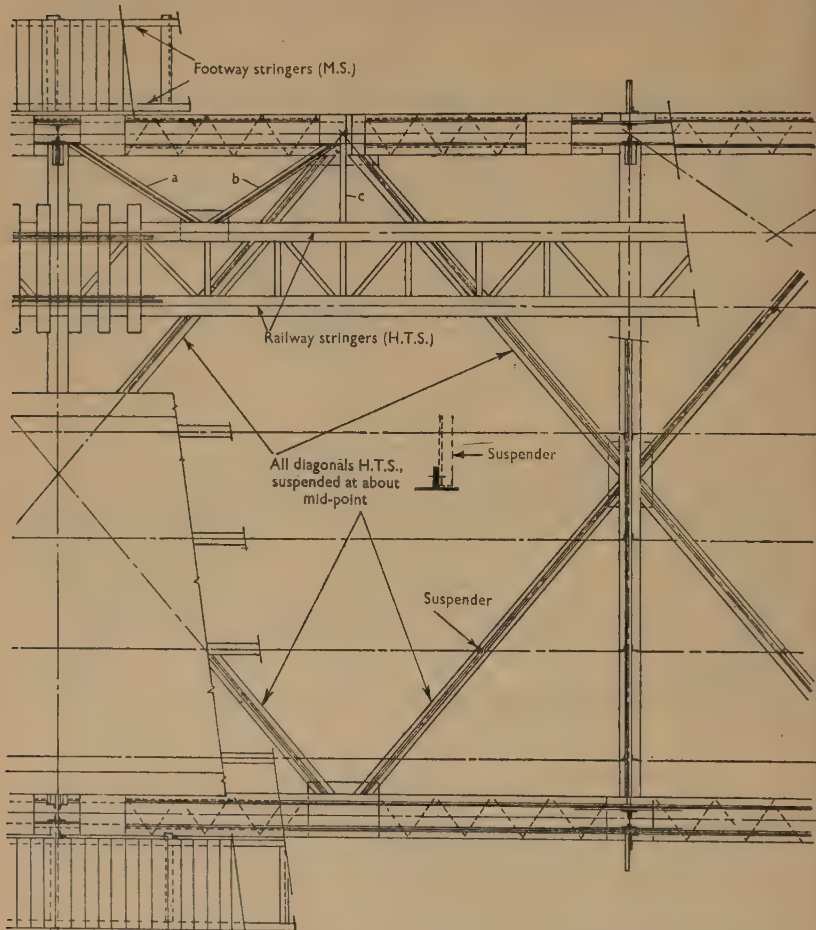
CROSS-SECTION OF TYPICAL PANEL OF NEW RAMA VI BRIDGE

overall length. The excessively long cantilever arms of 136 feet 2 inches in the old design have been shortened to 70 feet 1 inch, thus reducing the loads on the piers and also making the great depth of trusses (64 feet) at the piers unnecessary. The long shore spans were specially provided to make the cantilever erection scheme practicable. High-tensile steel rivets were used throughout, even in mild-steel elements, to avoid possible errors on site.

Care has been taken in the design to reduce maintenance troubles so far as possible.

To minimize corrosion, solid webs are used in all members, suspension links are made out of solid slabs, all inaccessible surfaces are either lined with gunmetal or metallized or protected by concrete filling. It is common practice in Thailand for the timber sleepers to rest direct on stringer flanges, and keen competitive conditions, unfortunately, did not permit the introduction of bearing packs. However, they can always be added by the railway authorities at a later date.

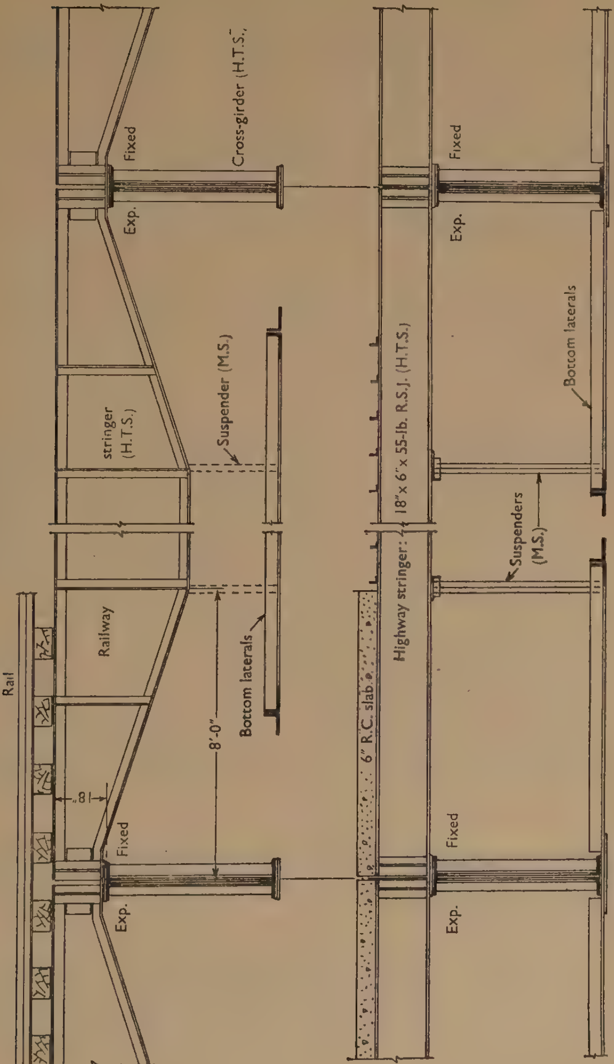
Fig. 5 (c)



Plan of Typical Panel of New Rama VI Bridge

To avoid loosening of rivets connecting stringers to cross-girders and cross-girders to main trusses as a result of strains set up by the interaction between deck and main trusses, expansion joints are provided at each cross-girder (approximately every 32 feet). Also all stringers are placed on top of cross-girders so that they are reasonably free to articulate under live load. (See *Figs 5 (d)*.)

A diamond lateral system of bracing was adopted between both top and bottom chords. In this system the lateral diagonals do not participate in stresses in chords of vertical trusses.



Figs 5 (d)

STRINGERS. NEW RAMA VI BRIDGE

In addition to lateral forces the laterals are capable of resisting a cross shear of $2\frac{1}{2}$ per cent of the load in the compression chord. Top lateral diagonals consist of battened star angles, which, while possessing good rigidity, are fully accessible for painting. The battens are designed for resisting a cross shear of $2\frac{1}{2}$ per cent of the load in the member, assumed as acting in the plane of least radius of gyration, that is, on skew axis. Bottom laterals consist of angles placed in contact, back to back, and suspended by

flexible hangers from the stringers above. To comply with the requirements of the Specification, sway frames are provided at each panel point, all members being also of starred angle section. Fewer sway frames could be used without any detriment.

The system of bracing to resist lateral and horizontal forces on rail stringers is rather unusual (see *Figs 5* (*b*) and (*c*)). It consists of :—

1. Horizontal bracing angles between stringers connected direct to the top flanges of stringers (without gussets).
2. Three vertical cross frames, one at each end and one at mid panel point.
3. Members a, b, and c, which connect the bottom chord of the outer stringer to the bottom chord of the main truss and to the main lateral system.

Lateral forces from trains are resisted in the normal way by the horizontal bracing in the plane of the top flanges of the stringers and transmitted to the cross-girders and thence to the main lateral system by the end cross-frames, but the longitudinal forces, particularly those on the inner stringers, follow a somewhat complex path, straining all the bracing elements in the panel before reaching the bottom chords of the main trusses, which carry the load to the main fixed bearings.

Another unusual feature is the location and design of the bearings supporting the suspended span (see Figs 6 and 7, Plate 1). The suspension pin and the end cross-girder are located eccentrically to the panel point, one on each side of it. The small moments resulting therefrom are resisted in bending by the main truss-members. This arrangement avoids the usual difficulty of combining a cross-girder with a suspension pin and gives a clean and simple connexion, permitting the use of a fairly long solid suspension link at the expansion end.

Abstracts from the calculation sheets showing stresses and analyses of sections for the main trusses are given in Tables 1 and 2, pp. 15 and 16, and in Table 3, Plate 1.

The total weight of steelwork in the old bridge was 2,630 tons and in the new bridge is only 2,060 tons, although the bridge now carries approximately 50 per cent heavier live load. The greater part of this saving in weight is attributable to the 65-per-cent increase of allowable stresses, and the rest to the re-arrangement of trusses and more economical detailing. The steelwork weights on the superstructure are shown in Table 4.

Surat Bridge

The old bridge (*Fig. 8*, facing p. 472) consisted of three simple spans of 80 metres, 60 metres, and 60 metres, carrying only a single metre-gauge railway and one cantilevered footway.

The new bridge carries in addition, a 6-metre-wide roadway. The same arrangement of spans was adhered to but the old piers had to be widened

TABLE 1.—MAXIMUM-STRESS COMBINATION NO. 1
FOR 253′–6″ SPAN OF RAMA VI BRIDGE (RAILWAY TRUSS)

Member		Dead load: tons	Loaded length: feet	Live load: tons			Impact: tons		Lurching	Total: tons
				Railway	Roadway	Footway	Railway	Roadway		
Bottom chord	$L_0 L_2$	+ 102	254	+ 156	+ 20	+ 22	+ 64	+ 4	± 6	+ 374
	$L_2 L_4$	+ 216	254	+ 312	+ 44	+ 47	+ 127	+ 8	± 10	+ 764
Top chord	$U_1 U_3$	– 174	254	– 260	– 35	– 38	– 106	– 6	± 9	– 628
	$U_3 U_5$	– 231	254	– 330	– 46	– 51	– 133	– 8	± 11	– 810
Diagonals	$L_0 U_1$	– 154	254	– 235	– 30	– 33	– 100	– 6	± 8	– 566
	$U_1 L_2$	+ 109	218	+ 178	+ 23	+ 24	+ 85	+ 5	+ 6	+ 430
		+ 109	32	– 8	– 2	– 3	– 7	– 1	0	+ 88
	$L_2 U_3$	– 66	68	+ 27	+ 4	+ 4	+ 20	+ 1	+ 1	– 9
		– 66	182	– 131	– 17	– 17	– 68	– 4	– 4	– 307
	$U_3 L_4$	+ 22	146	+ 88	+ 12	+ 11	+ 50	+ 3	+ 2	+ 188
		+ 22	110	– 54	– 7	– 7	– 36	– 2	– 2	– 86
Hangers .		+ 27	63	+ 63	+ 10	+ 7	+ 50	+ 4	± 2	+ 163
Posts . .		– 6	—	—	—	—	—	—	—	– 6
Reaction .		132	254	213	26	29	87	5	± 7	499

considerably. The longest span, 262 feet 6 inches, is fortunately almost identical to the simple span of Rama VI Bridge. The two smaller spans, each of 196 feet 10½ inches, were made very similar to it, using the same panel length of 32 feet 9¾ inches throughout and parallel-chord main trusses of the same depth, reducing the sizes of members as necessary (see *Figs 9*). The old panel lengths were approximately 26 feet and 22 feet for the larger and smaller spans respectively, and the main trusses had curved top chords. All the details and arrangement of laterals, sway frames, and stringer bracing were made identical to those of Rama VI Bridge.

Typical details of expansion and fixed bearings are shown in *Figs 10*. These are made from mild-steel slabs with "knuckle half-pins" or "knuckle blocks" welded to them. Rollers are also machined out of mild-steel slabs, with locating bars welded at each end. These bearings are light and easy to fabricate. They were designed for 40 tons per square foot pressure on concrete, but with modern high-quality concrete the pressure could be safely doubled and the bottom slabs would then be only

TABLE 2.—MAXIMUM-STRESS COMBINATION NO. 2 FOR 253′–6″ SPAN OF RAMA VI BRIDGE (RAILWAY TRUSS)

Member		Combination 1 stress: tons	Wind stresses: tons		Sway: tons	Longitudinal force: tons	Total combination 2 stress: tons	Design: stress tons
			Lateral	Overturning				
Bottom chord	$L_0 L_2$	+ 374	± 48	± 7	± 4	± 80	+ 513	+ 513
	$L_2 L_4$	+ 764	± 65	± 16	± 6	± 63	+ 914	+ 764
Top chord	$U_1 U_3$	− 628	± 9	∓ 12	—	—	− 649	− 628
	$U_3 U_5$	− 810	± 11	∓ 18	—	—	− 828	− 810
Diagonals	$L_0 U_1$	− 566	± 8	± 10	—	—	− 584	− 584
	$U_1 L_2$	+ 430	—	± 7	—	—	+ 437	+ 430
	$L_2 U_3$	− 307	—	± 5	—	—	− 312	− 307
	$U_3 L_4$	+ 188	—	± 4	—	—	+ 192	+ 231
		− 86	—	± 2	—	—	− 88	− 129
Hangers . .		+ 163	—	± 2	—	—	+ 165	+ 163
Posts . . .		− 6	—	—	—	—	− 6	− 6
Reaction L_0	Vertic.	499	—	± 14	—	—	513	499
	Later.	—	± 23	—	± 2	—	25	25
	Longl.	—	—	—	—	± 80	80	80

half the size in plan and about two-thirds in thickness. Although these bearings are satisfactory it is better to attach the knuckle pins and blocks to the main slabs by dowel pins or studs, instead of by welding. This makes the fabrication easier and permits the use of non-weldable high-tensile steel, resulting in further reduction in sizes of bearings.

The steelwork weights in the superstructure are shown in Table 5.

Bandara Bridge

The demolished bridge (see *Figs 11*, facing p. 472 and *12*, p. 465) was built by the Cleveland Bridge & Engineering Company Limited, and consisted of three spans of 80·6 metres, 101·2 metres, and 80 metres. It was of cantilever type with a central suspended span of 131 feet 3 inches supported by two cantilever arms of 100 feet 4½ inches each. The main trusses placed 16 feet 4⅝ inches apart were of the unusual "Japanese Warren" type with deck panels of 16 feet 4⅞ inches and main-truss panels of 32 feet 9¾ inches, except immediately over the piers where, for some

TABLE 4

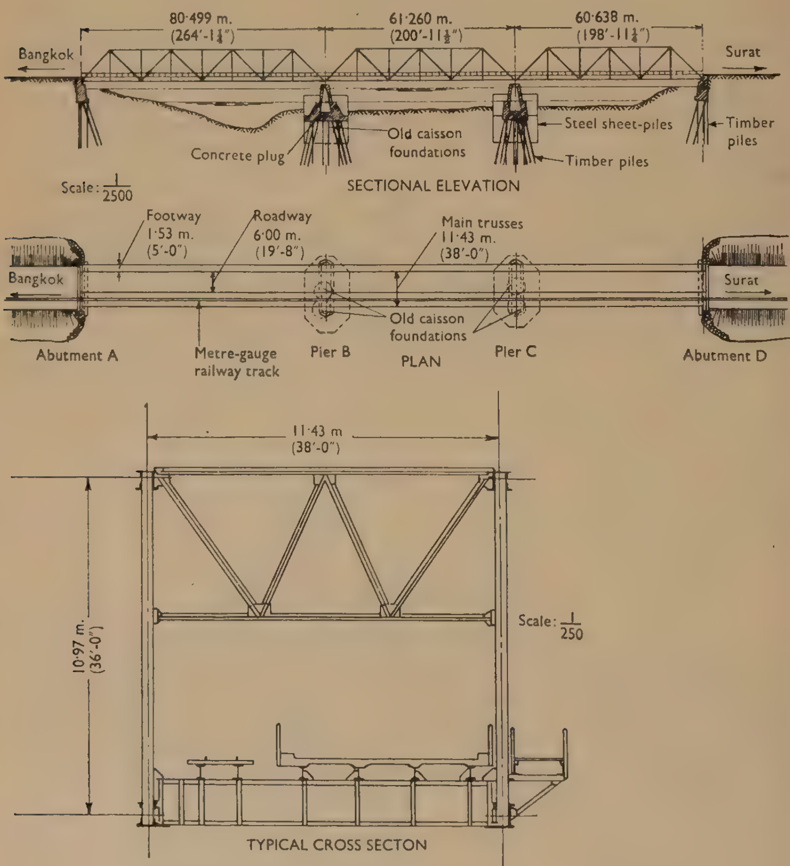
Item	Quantity : tons					
	In three 253′–6″ spans		In two 343′–11″ anchor spans		Total in bridge	
	H.T.S.	M.S.	H.T.S.	M.S.	H.T.S.	M.S.
Railway stringers and bracing	77·4	32·4	73·8	27·1	151·2	59·5
Roadway stringers (joists, etc.)	78·2	8·0	69·8	6·0	148·0	14·0
Cross-girders	78·7	16·5	70·2	17·1	148·9	33·6
Top laterals, sway bracing, and portals	—	65·6	—	52·9	—	118·5
Bottom laterals	24·0	10·3	22·9	9·9	46·9	20·2
Footway brackets, hand rails, etc., and stringers	—	90·7	—	80·1	—	170·8
Main truss, railway side	263·0	34·8	302·0	33·2	565·0	68·0
Main truss, roadway side	181·0	51·6	205·0	40·4	386·0	92·0
Total : superstructure	702·3	309·9	743·7	266·7	1446·0	576·6
Fixed bearings and grillages	—	7·4	—	13·0	—	20·4
Expansion bearings	—	8·2	—	7·4	—	15·6
Total steel	702·3	325·5	743·7	287·1	1446·0	612·6

TABLE 5

Item	Quantity : tons					
	In 262′–6″ span		In two 197′ spans		Total in bridge	
	H.T.S.	M.S.	H.T.S.	M.S.	H.T.S.	M.S.
Railway stringers and bracing	24·6	11·9	37·5	16·0	62·1	27·9
Roadway stringers	26·7	4·8	41·0	8·6	67·7	13·4
Cross-girders	28·1	4·3	44·2	6·1	72·3	10·4
Top laterals, sway bracing, and portals	—	21·3	1·7	29·7	1·7	51·0
Bottom laterals	8·4	2·5	12·6	2·6	21·0	5·1
Footway brackets, stringers, hand rails, etc.	—	14·7	—	22·2	—	36·9
Main truss, railway side	86·2	11·5	97·9	16·0	184·1	27·5
Main truss, roadway side	62·8	17·6	74·8	21·6	137·6	39·2
Total : superstructure	236·8	88·6	309·7	122·8	546·5	211·4
Fixed bearings and grillages, Expansion bearings	—	6·2	—	12·4	—	18·6
Total steel	236·8	94·8	309·7	135·2	546·5	230·0

reason, longer panels were introduced. The depths of these trusses varied from 23 feet for the suspended spans to 52 feet 6 inches at the piers. The bridge carried a single metre-gauge railway and one cantilevered footway but it was probably built originally for 4 feet 8½ inches gauge. The railway loading used at the time was equivalent to about 7 units of the British Standard Loading.

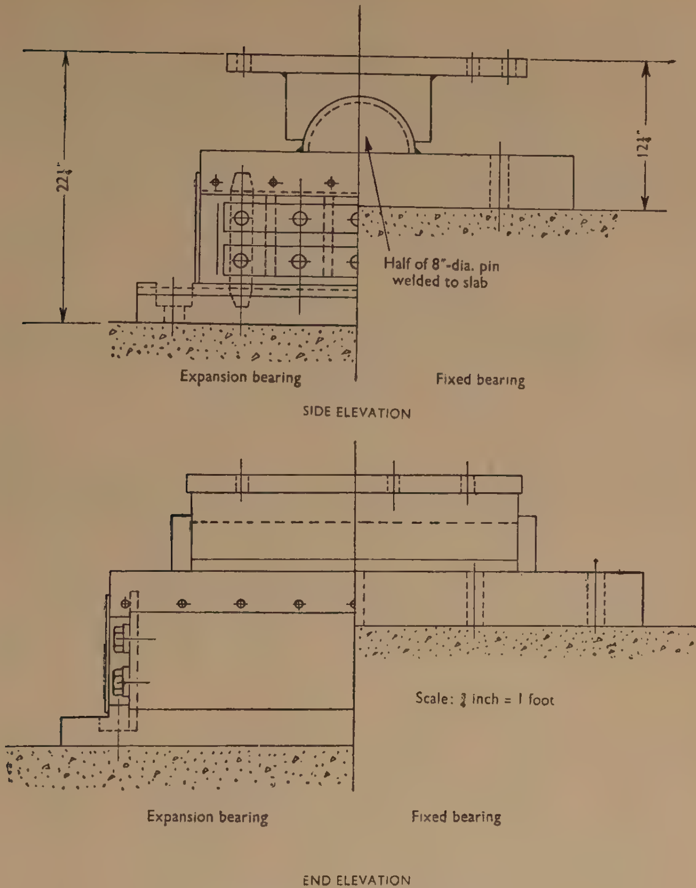
Figs 9



Line Diagram of New Surat Bridge

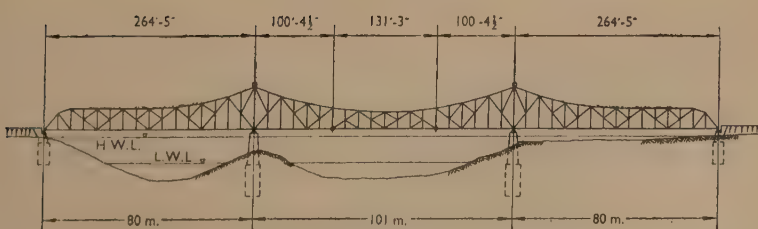
The new bridge (see *Figs 13*) had to be built on the same piers and was required to carry about 100 per cent heavier loading. To reduce the loads on the two river piers, the cantilever arms were shortened from 100 feet 4½ inches to 66 feet 1¼ inches with a suspended span of 199 feet 9¾ inches, and an unusual arrangement of bearings was adopted. The whole bridge

Figs 10



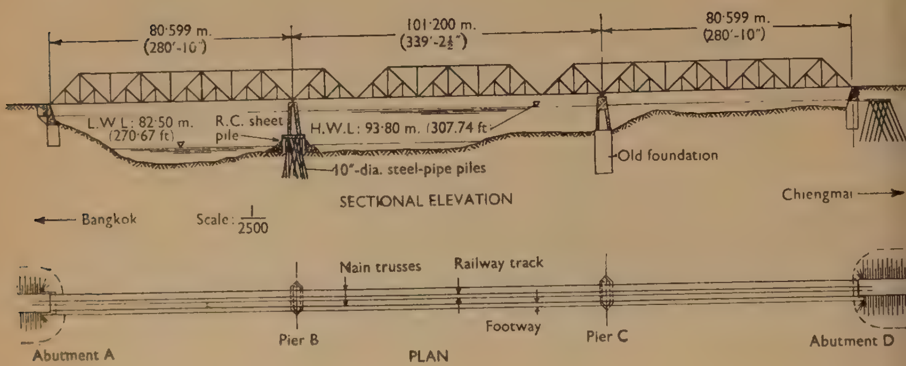
BEARINGS. SURAT BRIDGE

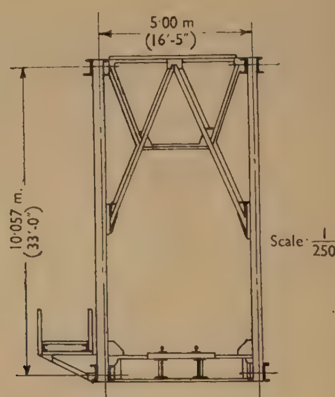
Fig. 12



LINE DIAGRAM OF MAIN TRUSSES. OLD BANDARA BRIDGE

Figs 13





LINE DIAGRAM OF NEW BANDARA BRIDGE

was tied to one of the newly reconstructed abutments, with expansion roller bearings on both river piers, and on the other abutment. With this arrangement the river piers are not subject to longitudinal forces from the superstructure whilst the maximum vertical bearing reactions were increased by only about 25 per cent. This re-arrangement of bearings resulted in large expansion movement at the free abutment ($\pm 4\frac{1}{2}$ inches) but greatly simplified suspension details for the central span. A simple pin could be used at each end, which is almost ideal.

For such a narrow long-span bridge the economic panel length for the deck is short compared with that for the main trusses. As a compromise, and rather reluctantly, a sub-panelled truss was adopted with all main panel lengths of 33 feet $0\frac{5}{8}$ inch and deck panels of 16 feet $6\frac{5}{16}$ inch, which are nearly the same panel lengths as those in the original bridge.

Large secondary stresses may develop in this form of truss and special precautions were taken to reduce these. Members were made as slender as possible in the vertical plane and the sub-verticals and sub-diagonals were made of such lengths as to ensure that when the bridge is fully loaded the secondary stresses in the bottom chord are at the minimum.

The " diamond " top lateral system was adopted to avoid participation in top-chord stresses and also to increase the lateral rigidity of chord members. When subject to maximum lateral forces considerable secondary stresses may be set up in top-chord members, but when combined with all other stresses these would not exceed the permissible increase in working stress.

The bottom lateral system consists of double intersection diagonals with braking girders in each panel, formed by adding cross-members between stringers, at points of intersection with the laterals. The lateral diagonals and cross-members are connected to bottom flanges of stringers by attachments capable of transferring longitudinal forces. This lateral system participates in live-load chord stresses and the sizes of members and connexions were made capable of resisting wind and live-load stresses without increasing the normal working stress by more than 25 per cent.

The total weight of steelwork in the new bridge is 762 tons, which compares well with the weight of 615 tons in the old one which was designed to carry half the new live load at three-fifths of the new working stresses.

Details of steelwork weights are given in Table 6.

TABLE 6

Item	Quantity : tons					
	In one 200′ suspension span		In two 330′ anchor spans		Total in bridge	
	H.T.S.	M.S.	H.T.S.	M.S.	H.T.S.	M.S.
Plate-girder railway stringers . . .	13·3	—	44·4	—	57·7	—
Plate-girder cross girders	7·8	2·8	29·4	11·1	37·2	13·9
Top laterals, portals, and sway frames . .	—	7·8	—	27·1	—	34·9
Bottom laterals and stringer bracing . . .	2·9	3·3	9·5	8·0	12·4	11·3
Handrail standards, footway brackets, and stringers . . .	—	7·6	—	25·7	—	33·3
Main trusses . . .	68·0	27·0	345·0	100·0	413·0	127·0
Total superstructure .	92·0	48·5	428·3	171·9	520·3	220·4
Fixed bearings . .	—	—	—	1·7	—	1·7
Expansion bearings . .	—	—	—	19·2	—	19·2
Total . . .	92·0	48·5	428·3	192·8	520·3	241·3

Erection

From the outset it was decided that all three bridges would be erected by cantilevering from the shore spans, which could be built on stagings (see Figs 15, Plate 2), and all three bridges were designed for this method. For the last stage of erection of the cantilevered span, that is to say, when making the actual lifts of the members in the end panel with the crane at its farthest-out position, maximum wind pressure of 20 lb. per square foot, instead of the normal 30 lb. per square foot, was taken on the whole of the superstructure and only 10 lb. per square foot on the crane itself with the jib right out. This reduction of wind for the last critical operation just prior to reaching the far support is justified on the grounds that no one would attempt to work in winds higher than 50 miles per hour and the structure would be quite safe with the crane tied up and the last panel not erected, in a gale of 90 miles per hour.

In the case of Rama VI Bridge, sections of certain members had to be increased on account of cantilever erection stresses, and these were tabulated separately so that if cantilever erection eventually was not adopted, reversion could be made to normal sections. The extra weight of erection material for Rama VI Bridge amounted to :

permanent :	10 tons
temporary :	26 tons

For Surat and Bandara Bridges no additional permanent material was required and the erection ties made for Rama VI Bridge were to be adapted and re-used for these two bridges (see Fig. 14, Plate 1, and Figs 15, Plate 2).

Demolition of Existing Structures

Figs 2, *8*, and *11* (facing p. 472) are reproduced from photographs taken after the bombing and show the state of affairs before reconstruction was commenced. In general the steel superstructures were completely removed before any other work was started and no particular difficulties were encountered, although some of the work proved very laborious.

The following brief particulars indicate the methods employed at each site.

Rama VI Bridge (River Chao Phya)

Although nearly all piers and abutments of Rama VI Bridge had been badly damaged, only the centre suspended span had actually fallen (*Fig. 2*). Both side and anchor spans remained perched precariously on their supports. However, the standing steelwork had been badly knocked about and some main members were completely severed. It was, therefore, decided to construct piled falsework under all standing spans as being a more certain and economical scheme than any attempt to make good the old structure so that it could be safely demolished by cantilevering.

Electric derrick cranes running on tracks alongside each side span and mounted on pontoons as floating cranes were used to handle the pieces, and all demolition was carried out by burning. Part of the centre span still hanging on the Bangkok anchor span was recovered, and preparations were made for raising the main part of this span which was partially embedded in the mud on the bed of the river in about 40 feet of water. When the position was examined by divers a number of large unexploded bombs were found, one of which was recovered, lying very near the steelwork. It was considered that the use of explosives for breaking up the span would involve serious risk of damage to the existing piers, and this steel-work (amounting to approximately 40 tons) was abandoned. The amount of steel recovered was approximately 2,600 tons.

The demolition of the piers and abutments of all bridges was carried out concurrently with their reconstruction and (except where noted) did not present any very difficult problems.

Surat Bridge (River Tapee)

The tops of both river piers of Surat Bridge had been broken down so that the centre span lay in the river (*Fig. 8*, facing p. 472) and the river ends of the two side spans had dropped, although the shore ends still rested on the abutments.

Although the two shore spans were not much damaged structurally, their displacement and the presence of the centre span in the water largely blocked the river and held up great quantities of timber from the rafts which are floated down from the forests. As a consequence, the profile of the river bed was entirely changed, and there was a difference in water level of nearly 1 metre between the upstream and downstream sides.

Temporary piled trestles were constructed to support the side spans, the steelwork of which was then dismantled by cutting out the rivets.

The centre span, lying on its side, was almost entirely covered in silt and this had to be removed by means of compressed air jets handled by divers. Underwater cutting torches and explosives were used to break up the span. It proved impossible economically to recover all the material so that one truss and part of the deck of this centre span were left in the river.

Floating plant was used to load the steel (amounting to 510 tons) on to pontoons for removal.

Bandara Bridge (River Nan)

The north-side span and cantilever of Bandara Bridge were still standing but the south pier had collapsed, letting down the whole of the south end of the bridge and leaving the suspended span hanging into the water from the north cantilever. Stagings were built to support the north end of the bridge, including the hanging suspended span, and the steelwork was dismantled by cutting out the rivets. The south-end steelwork was

burnt off above water and jacked up progressively from the temporary piles exposing more steel for burning. A small amount was broken up by explosives under the water. Altogether 593 tons of steelwork was recovered.

RECONSTRUCTION OF FOUNDATIONS

Rama VI (*see* Figs 16, Plate 2, and *Fig. 17*)

The original piers for Rama VI Bridge were founded on pairs of pneumatic caissons. The masonry-faced middle sections were built as two shafts united at the top by portals. These portions of piers C, D and E were badly damaged, the curious condition of pier E being shown in *Fig. 17*. Cavities in the pier legs originally housing the air shafts to the caisson enabled internal inspections to be carried out.

The upper portions of all the piers, and abutment A, were demolished by means of pneumatic tools and explosives and were rebuilt in concrete. The top landing of abutment F was also reconstructed to take the new bearings. A feature of both abutments was, and still is, that the portion carrying the loads from the ends of the shore spans is separate from the portion retaining the embankment filling. Short steel spans connect the two.

The middle bodies or twin shafts of piers B and E were found to be in order and oval piers were constructed in reinforced concrete upon them. These were solid, apart from two voids, from about level — 1·75 (metre) upwards. More or less of the original concrete was incorporated in the parts under the bearings according to its condition. Timber cofferdams were employed to enable work to be carried out in the dry in rising water levels.

The middle body shafts of pier D were slightly damaged and heavy reinforcing cages were concreted into the old air-shaft chambers running up into the upper portion and extending downward into the tops of the caissons.

The middle sections of pier C were so badly damaged that special measures had to be taken which form the only really interesting part of the work.

Each leg was encircled in a reinforced-concrete cylinder, 4 inches thick and 7·40 metres (24 feet 3 inches) outside diameter, which rested on the top of the old caisson at level — 13·90 (metres). Each cylinder was cast over its location in lifts of 1·30 metre ($4\frac{1}{4}$ feet) and the bottom section was hung from three plate-link chains supported by sand jacks, by which the cylinder was progressively lowered as new sections were added (see Figs 16, Plate 2).

When each cylinder reached its final level a concrete plug was cast under water on top of the old caisson. The water was then pumped out and the interior was filled with concrete to low-water level, although the cylinder extended above high water.

The damaged pier legs were embedded in this new concrete which was vibrated to assist the filling of the large cracks. The upper portion of the pier was built as two separate legs to high-water level and the top portion of each cylinder, which had been acting as a temporary cofferdam, was then cut down to low-water level. Above high-water level the pier was built to the full oval shape and the two flat sides were closed with precast concrete slabs between high and low water for the sake of appearance.

The upper part of pier D is similar in construction.

Surat Bridge

Owing to the increase in the width of Surat Bridge, all foundations had to be extended during reconstruction. The old abutment D was intact and was extended laterally by building a new concrete section on the downstream side. Timber piles, 16 metres long, were driven to support the base slab to pick up a stratum of sand at level 82·00 (metres) and the new section of abutment was bonded to the old by means of chases and mild-steel reinforcing bars. Abutment A had been so badly shaken by bombing that it had to be entirely demolished and rebuilt. The rubble was cleared and excavation carried out at low water when fifty old piles, 12 metres long were withdrawn. It is interesting to note that the timber was in perfect condition after 36 years. This abutment was also supported on 16-metre-long timber piles and the upper portion was constructed in concrete.

During the life of the old bridge, large rubble stones had been dumped round both river piers to mitigate the effect of scour. The addition of broken masonry from the piers together with waterlogged trees, all in a matrix of hard silt, made the work in the river difficult and tedious.

The debris was removed by teams of divers working all possible hours. The Thai divers employ a helmet only which rests on the shoulders and is weighted with pig iron to keep it down. A simple air-pump, operated by one man, supplies air through a small rubber tube. If the diver gets into trouble he merely jettisons his helmet and swims to the surface. With this simple apparatus, locally made, a large number of men could be employed, but clearing operations took a long time. Both river piers were surrounded by a steel sheet-pile cofferdam of Dorman Long KIII section, 16 metres ($52\frac{1}{2}$ feet) long, the area enclosed including that needed for the extensions. The tops of the dams were at level 98·20 (metres) which was only likely to be topped for 10 to 15 days during the highest flood.

Pier C caused very little trouble and the old masonry was demolished down to the tops of the supporting caissons in the dry. Excavation round the caissons and over the extension to the pier was carried out to level 89·50 (metres), after which timber piles, both vertical and raking at 1 in 4, were driven round the caissons and in the area of the extension. The rising concrete was then cast and the sheet piles cut off at low-water level.

In the case of pier B, the sheet piles met many obstructions of stone and timber and some parted from their clutches so that it was not possible to

dewater the dam. The old caissons had been found to be out of position and out of plumb, and the dam was enlarged beyond the intended size to allow more bearing piles to be used. At level 89·50 (metres) an underwater concrete plug, 1·5 metre thick, was cast over the whole area of the dam after driving ninety-four timber piles. The dam was then pumped out, a modified base constructed and the upper pier-shaft concreted. Work on this pier occupied 12 months.

During the execution of the work at Surat, three unexploded 1,000-lb. bombs were removed from the river, one of which had had its detonator removed by the driving of one of the sheet piles.

The original caissons, octagonal in shape, were built below level 91·00 (metres) with shells of 3-inch hardwood laid in horizontal strakes fastened to vertical 2-inch-by-2-inch steel angles tied in with circumferential steel rings. About level 91·00 (metres) the timber gave way to brick reinforced by steel ties, ending in plates embedded in the mortar joints.

Bandara

The badly damaged front wall of abutment A was demolished down to the top of the existing foundation which consisted of two solid concrete blocks 4·00 by 3·75 metres (about 13 feet by $12\frac{1}{3}$ feet) in section. The new wall was built on these and carried a rearward cantilever slab to make use of the imposed weight of earth filling to balance the overturning moment. The wing walls were also cantilevered to retain the embankment filling.

Abutment D which had to withstand all the longitudinal forces on the bridge was anchored to a cluster of reinforced-concrete piles of 0·30 metre (10 inches) square section by $14\frac{1}{2}$ metres (46 feet) long, driven on opposite batters behind the existing structure, as shown on *Figs 13* (p. 466). The top of the old wall was cut down and rebuilt in concrete with short cantilevered wing retaining walls and the anchors from the piles were embedded in this portion.

The old foundation to pier C was in good condition so that a new concrete top was all that was required. Pier B, however, was very badly damaged. The two old cylindrical caissons, 5·80 metres (19 feet) diameter, were thought to be in good condition. Fifty-six steel tubes, 10 inches diameter, fitted with cast-iron shoes were driven as bearing piles round the old caissons to a good sand stratum, pumped out, and filled with concrete. Since no steel sheet-piles could be made available for this bridge, reinforced-concrete sheet-piles, 5·15 metres (17 feet) long were then driven round the perimeter of the new outline to the pier, to retain sand filling. Short concrete columns were built from the old caisson tops to support the base slab but were not reckoned in the final calculation as carrying load. A reinforced-concrete base-slab, 1 metre thick, was cast at level 84·50 (metres) on which the pier shaft was built in reinforced concrete. The outside curtain of this pier was protected by dumping rubble all round it.

Fig. 2

RAMA VI BRIDGE BEFORE RECONSTRUCTION

Fig. 8

SURAT BRIDGE BEFORE RECONSTRUCTION

Fig. 11

BANDARA BRIDGE (LOOKING TOWARDS CHIENG MAI END) BEFORE RECONSTRUCTION

Fig. 17

RAMA VI BRIDGE. PIER E IN ITS DAMAGED CONDITION

Fig. 18

RAMA VI BRIDGE. LAST PHASE OF ERECTION—LINKING SUSPENDED SPAN TO ANCHOR SPAN

Fig. 19

COMPLETED RAMA VI BRIDGE

Fig. 20

COMPLETED SURAT BRIDGE

Fig. 21

New Bandara Bridge in Course of Erection

(The end of the suspended span is carried on the heavy trestle and the first panel of the Bangkok Anchor span is attached by its temporary ties)

Fig. 22

Completed Bandara Bridge

Erection of Superstructures (Figs 15, Plate 2)

Generally similar schemes were adopted for all three bridges, namely, the erection of the first spans on false-work followed by cantilever erection of the remainder. Temporary ties were designed which fitted all bridges by varying the outstanding lengths of the gussets to which the ties were attached (see Fig. 14, Plate 1).

The ties were of similar construction to the permanent chords and had a field splice offset from the mid-point. They were supported during erection by temporary posts which were joined by sway bracing made from permanent members.

The ties were connected to the gussets by $8\frac{1}{4}$-inch-diameter pins. The size of pin and section of the ties were dictated by the conditions at the Rama VI site where the loading was heaviest.

Tables of deflexions, jack loads, and quantities of bolts and drifts required were supplied to the site with the erection memorandum for all bridges.

Rama VI Bridge (*Figs 4*, p. 455, and *18*)

Although it had been intended to travel the erection cranes on the deck of the new structure in a similar manner to the other two bridges, it was decided to use a floating crane after the experience gained during demolition.

The stockyard was set up at the Bangkok end of the bridge and furnished with sidings connecting to the State Railway system. A small jetty was built into the river on the same side, on to which there was a rail connexion. A 5-ton locomotive crane on standard-gauge tracks served both yard and jetty and a 5-ton hand derrick was also erected at the jetty head. Two Diesel-driven metre-gauge shunting locomotives were provided, one being employed on each side of the river. A siding was laid on the Haad Yai bank. The two sets of sidings were immediately adjacent to the train ferry so that material could be transferred from one bank to the other on rail trucks when necessary.

The falsework in the two side-spans which had been used for the demolition of the old bridge had been so arranged that it would serve for the new side-spans. Some repairs had to be carried out on the timbering owing to the depredations of *teredo* and land beetles in the interval between the two operations.

For the erection of the abutment steelwork and the side spans, tracks for the $6\frac{1}{2}$-ton electric derricks were laid on each river bank parallel to the bridge centre-line. The derricks, mounted on bogies incorporating short timber trestles, travelled on the tracks and were fed from the sidings on either side. Subsequently one of the cranes was mounted on an assembly of " port-construction " pontoon units for use as a floating crane and the other was substituted for the hand derrick on the jetty to handle the

heavier lifts in the anchor spans. The members were fed to the floating crane on rafts towed by a steam launch.

Erection of the steelwork was on the same general lines of the other two bridges (*q.v.*), the only special features being occasioned by the fact that each anchor span was flanked by a simple span and that the central slung span was erected last. Erection of the bridge was started from both ends.

For each side-span the moving bearings at the abutments were set in their mean positions and the fixed pier-bearings were set in their correct positions. The spans were erected on their camber blocks and longitudinal adjustments to suit the fixed bearings were made by jacking endways. The falsework was flexible enough to conform to this movement. The spans were then jacked up at each end and lowered on to their bearings, the maximum load on a corner being 87 tons.

Each intermediate span, or anchor arm, was cantilevered out from its respective side span by means of the temporary ties and bottom-chord packings, the first members being temporarily supported on auxiliary piled trestles. The shore end of the east side-span was tied down by long bolts previously cast into the new abutment and the west span was anchored by kentledge on the deck, since this abutment was not rebuilt. The droop of each anchor span over piers C and D was calculated to be 32·6 inches including an allowance for clearances in the holes at the connexions and checked as a little less. The outer ends of each intermediate span were jacked up when the cross-girder over each pier had been erected. The pins in the temporary ties and lower-chord packings were then removed and the span was lowered on to its bearings. The maximum jacking load was 106 tons under the railway truss. This operation was carried out at a temperature as near as possible to the mean. Throughout, the joints between members were sufficiently filled with service bolts and drift pins to sustain the dead load of each span. The erection of the wind bracing and sway bracing was carried forward as far as possible as each bay was completed.

During cantilever erection of all bridges, the bottom-chord member of a bay was placed first and the other members were built upon it. Where the web member was inclined backwards towards the erection crane, this was inserted without difficulty and held up the outward end of the bottom-chord member. In alternate bays, however, the diagonal inclined outwards and this, with the vertical at the outer end, caused deflexion of the bottom chord so that the gap was too wide for connecting the top chord member. The outer end of the top chord member, therefore, was connected to the gussets at the joint between diagonal and post, and a simple pulling jack was used to draw the inner end of the top member into its place. This method avoided the need for temporary supports or for erecting from the top downwards by using divided slings.

The use of a floating crane allowed great flexibility in the order of erection which was of particular value since erection could then take place im-

mediately upon delivery of steelwork from the United Kingdom. The order of erection was, in fact, generally maintained and fair progress was possible. The east, or Bangkok, end of the bridge had a lead of 2 to 3 weeks over the west end, and erection of the suspended span was started from the east. The temporary ties from the shore end of the anchor span were transferred to its outer end and the shore end was ballasted with 17 tons of kentledge to take care of the uplift induced by the suspended span. Owing to the weight of the end rakers of this span, it was necessary to erect and guy them without their top gussets which were added afterwards.

The suspended span was cantilevered out and, as the last member in each truss was secured, a jacking frame was attached between the end of the lower-chord member and that of the end member of the west cantilever which then lay above it (see Figs 6, Plate 1 and *Fig 18*). The screwed rods of the jacking frame were tightened sufficiently to raise the end of the suspended span very slightly, all rods being loaded with the same tension so far as could be ascertained by inspection. Jacks were inserted in the top of each frame and were operated to lift the end of the suspended span, the nuts being tightened down on the rods as lifting proceeded. As soon as tension was relieved in the temporary ties to the eastern cantilever, the connecting pins at one end of each tie were removed, and, when the permanent pins in the connecting links with the west cantilever were inserted, the weight was taken on them and all temporary gear was removed. The deck work was then completed.

The net erection time for this bridge was 5 months. *Fig 19* shows the completed bridge.

Surat Bridge (*Figs 9*)

Originally it had been intended to provide electric derricks both for use in the stockyard on tracks and on the bridge as an erection crane. However, owing to the change in programme and the fact that the Surat steelwork was first unloaded and sorted at the Rama VI site near Bangkok, a 5-ton hand derrick replaced the electric yard-crane. The yard was established at the Bangkok end of the bridge and, as the area was liable to flooding, the sidings laid by the Royal State Railways were carried on filling. These sidings gave a certain amount of trouble and delayed erection from time to time.

The erection derrick was arranged to travel at the level of the bridge deck on three bogies. The king post and one leg trailing were accommodated on the same track with a specially made short leg transverse to the bridge centre-line on a second track. The crane was stooled up from the bogies sufficiently high to allow material to be brought forward beneath it.

The permanent rail stringers were used to carry the king-post track but they were temporarily displaced 21 inches towards the bridge centre-line to give sufficient clearance for slewing. The roadway stringers were

adapted to carry the other crane track and material track. The material wagons were run directly on the stringer flanges. The outer roadway stringer was fixed in its permanent position but the others had to be re-positioned after the main erection was completed.

Temporary ties connected the mid-points of the roadway stringers and these ties were extended to connect with the braced railway stringers. Sufficient kentledge (including the weight of the electric generator) was provided just to balance the crane for travelling and the crane bogies were anchored to the stringers when lifting. Extra provision had also to be made to secure the stringers to the cross-girders.

Erection was commenced with the 80-metre span at the Bangkok end of the bridge, and this span was supported at the correct camber on timber falsework carried on piles. Owing to the troubles experienced in rebuilding pier B, it was not possible to commence erection as hoped in the summer of 1951. The falsework, therefore, had to remain in the river during the flood and extra fender piles were driven to protect it. Fortunately the falsework suffered no damage from floating logs and debris. The fixed bearing at the shore end was grouted in at the start of operations. Sufficient drifts and bolts were provided to enable the span to be swung as soon as erection reached pier B, although riveting was carried forward as closely as possible behind the erection. Packings on the falsework were released by jacking under the end cross-girders, the maximum load at any corner being 110 tons. The river, or moving, end of the 80-metre span was landed on its bearings, which were then grouted in, and the adjacent fixed-end bearings for the first 60-metre span were set up on greased packings. Temporary timber supports for the first panel of the 60-metre span were cantilevered from the pier, and this panel with the ties at the top and packings at the bottom to the 80-metre span was erected.

Erection of the 60-metre span then continued until the far end was over its bearings on pier C, the clearance allowed being about 2 inches. The end of the span was jacked to release the ties and packings and then lowered on to the expansion bearings. This operation was scheduled to take place as nearly at mean temperature as was possible when the fixed bearings for the 60-metre span were in their correct positions. These bearings were grouted in as soon as the two spans had been disconnected. The expansion bearings at the other end of the span were also grouted in before the work on the second 60-metre span was started.

The second 60-metre span was erected in the same manner as the first, but the Bangkok end of the first span was first anchored down by connecting it to the last cross-girder of the 80-metre span. This was achieved by securing two groups of five of the bridge roadway stringers to the last two cross-girders of the 80-metre span, so that a cantilever portion over-lapped the first cross-girder of the 60-metre span. Since each stringer was only long enough to span one bay, three in each group were displaced longitudinally to reach the cross-girder of the 60-metre span. These were inter-

laced with the other two, which were securely clipped to the last two cross-girders of the 80-metre span.

Fig. 20 (between pp. 472 and 473) shows the completed bridge.

Bandara Bridge (*Figs 13*, p. 466)

Although the general arrangements for erection were similar to those for the Surat Bridge, the different design (cantilevers and suspended central span) and the narrow cross-section called for some special treatment.

Since the material for this bridge was delivered direct from the docks to the site, an electric derrick was used in the yard, which was established at the Utaradit end, and a modified hand derrick was employed on the bridge. This crane was carried on bogies running centrally on the permanent rails with one leg trailing. Since there was no room for another leg, a special telescopic arm was employed to take its place. This could be attached to the top chords of the span by means of a quick-release clamp. The arm had to be made adjustable for length and inclination owing to the varying heights of top-chord members and the width of the clamp could be altered to suit the different widths of the flanges.

When travelling, the king post of the crane was steadied laterally by means of a member bolted across the top of the back leg of the crane and embracing both top chords with suitable clearances. The trailing leg of the crane was tied down to the bridge deck when lifting and the track was prevented from moving laterally by timber cleats spiked to the underside of the sleepers.

To take the crane reactions on the top chord of the truss, all wind and sway bracing was erected close behind the erection crane before the main lifts were made. A simple pole attached to the trailing leg was used for the purpose.

Material was brought forward on the deck of the bridge on a 2-foot-gauge track made up from footway stringers. It was just possible to pass material between the mast of the derrick and one truss.

The first anchor span from the Utaradit side was erected on piled trestling which supported each panel point to L4 and thereafter alternate panel points. When the steelwork reached pier C, temporary packings were placed on the pier to enable members L8–L9 to be erected and a number of counter-sunk rivets to be driven at joint L8 before further erection. This done and the panel 8–9 erected, the span was jacked at both ends to remove the camber blocks, the permanent bearings on pier C were fixed and the span lowered on to its bearings at both ends. The maximum jacking force was 90 tons.

Thereafter, the erection proceeded to the end of the cantilever. The assembly and riveting of joint L0 had to be carried out to a careful plan owing to the presence of connexions and bearings of the suspended span.

The suspended span was then erected as a cantilever using the temporary ties to connect it to the anchor span. Care was taken to complete

the temporary top lateral bracing system to the temporary ties (usi permanent material for the purpose) before starting erection of the su pended span, and a counterweight box was provided at the shore end the first span to balance the uplift induced by the erection of the suspende span.

During the erection in the dry season of the cantilever arm and th majority of the suspended span, it was found possible to support the ou standing ends of the lower-chord members successively (while the othe members of a panel were placed) by means of a light trestle running o rail tracks on the river bed which saved time and trouble in rigging ten porary supports to these members. When about half the suspended spa had been erected, wire-rope storm guys were attached between the lowe chord members and anchorages on the Utaradit bank. Owing to th narrow width of the bridge, the action of wind tended to cause considerabl lateral displacements at the outward ends of the steelwork during erectio

A heavy trestle bent was provided in the river to receive the outer en of the suspended span and jacks were inserted under the diaphragms in th lower-chord members near joint L6. This end of the span was then jacke up about 24 inches to permit removal of the temporary ties, and the lowered to a predetermined level.

The temporary ties were then transferred to the outward end of th span and erection of the Bangkok cantilever proceeded, the length of tie and height of trestle being calculated to bring the steelwork well above th top of pier B. Erection was carried on to one panel beyond pier B (pan point 7) when the bottom flanges of joint L8 were riveted and the bearing fixed on the pier. Jacking was again carried out from the trestle and th steelwork lowered until it just made contact with the pier B bearings. Th full reaction was still carried on the trestle at this stage and the jackin load on each truss at this point was 107 tons.

Erection of the Bangkok anchor span then proceeded, storm guys bein attached as before, and the load was automatically and progressive transferred from the trestle to the bearings on pier B. When the erecti of the span was completed the trestle packings were released, the shore en of the Bangkok span was jacked up to enable the temporary ties to be di connected, and then lowered on to its bearings. The jacking distance this case was 31 inches and the load per truss 55 tons.

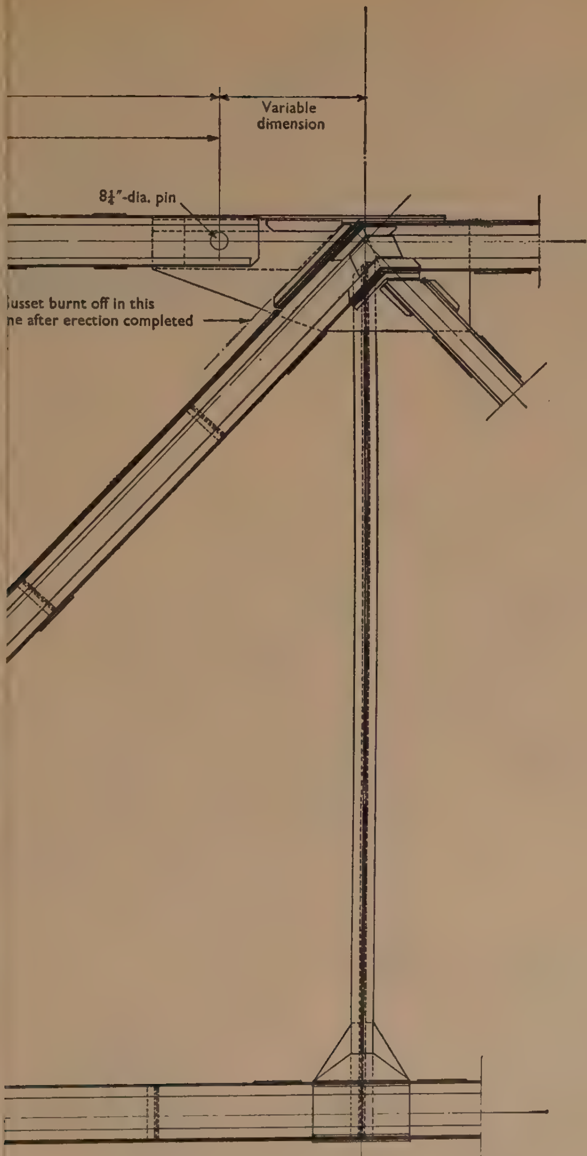
The stringers were re-positioned and the remainder of the footwa material erected by hand.

The use of a hand crane for the erection of this bridge was no di advantage for not much lifting was necessary. The best speed for placi steelwork was four panels a week.

Fig. 21 (facing p. 473) shows the Bandara Bridge in course erection and *Fig. 22* the completion of the bridge.

D

PLATE I
RAMA VI, SURAT, AND BANDARA BRIDGES



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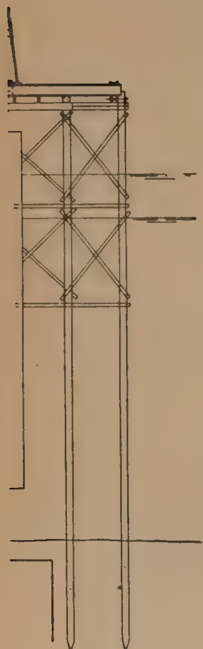

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Costs

The original programme for the rebuilding of these bridges extended over a period of 53 months as it had been intended to carry out work successively on the three sites. However, by providing some extra items of plant and increasing the staff with the agreement of the State Railways, operations on the three bridges were made to overlap in spite of the comparatively large distances between them. This resulted in a saving of 12 months in the total time of construction.

The approximate values of work carried out on each site were as follows :

Rama VI.	Demolition	£70,000
	Foundations	£75,000
	Provision and erection of new superstructure	£343,000
Surat.	Demolition	£16,000
	Foundations	£124,000
	Provision and erection of new superstructure	£140,000
Bandara.	Demolition	£24,000
	Foundations	£41,000
	Provision and erection of new superstructure	£149,000

Acknowledgements

The whole of the work was under the control of the Chief Civil Engineer to the Royal State Railways of Thailand, Luang Videt Yontrakich, and acknowledgements are also due to Messrs Sandberg, whose long association with the Royal State Railways was of great assistance in the early stages. The fabrication of the steelwork was carried out by The Cleveland Bridge & Engineering Co. Ltd, who were the main contractors. Messrs Dorman, Long & Co. Ltd, were responsible for the co-ordination of the work in Thailand, the demolition of the Rama VI Bridge and all the erection. Messrs Christiani & Nielsen (Thai) Ltd, designed and carried out the foundations and the balance of the demolition. The designs for the steel superstructures were in the hands of Messrs Freeman Fox & Partners who acted as consultants to the main contractors.

The Authors' thanks are due to these firms for their help in the preparation of the Paper and for permission to consult their records.

The Paper is accompanied by ten photographs and fifteen sheets of drawings, from some of which the half-tone page plates, folding Plates 1 and 2, and the Figures in the text have been prepared.

Discussion

The Authors introduced the Paper with the aid of a series of lantern slides. Mr Kerensky paid tribute to the late Sir Ralph Freeman, under whose guidance the bridges had been designed.

He emphasized the economies obtained by the use of high-tensile steel and pointed out that in the case of the Rama VI, Surat, and Bandara bridges the number of trains per day was so small that no question of fatigue should arise in any foreseeable future.

Mr H. Shirley Smith said that he had had an early interest in the bridges described because he had visited Thailand in 1949 to obtain particulars for his Company, who wished to tender.

They had wisely asked Messrs Freeman, Fox & Partners, to prepare the designs for them, and they had been produced, as Mr Kerensky had described, in a very short time. On those designs, Mr Shirley Smith's firm had tendered against international competition and secured the contract. Subsequently, it had been arranged that Messrs Dorman, Long & Company should erect the bridges, and Messrs Christiani & Nielsen, who had a contracting organization in Thailand, should reconstruct the foundations and piers.

Fig. 23 showed one of the anchor spans of the new Rama VI bridge erected in the works at Darlington. Owing to the absence of sub-panelling, prestressing was unnecessary and the steelwork was fabricated to the camber drawing. In use, the members were subjected to dead- and live-load secondary bending stresses, but they were small and no increases in section on their account were required. The upper lateral system was light and there were semi-portals at every post. They had been required by the former British Standard 153, but were not called for in the new specification, now in preparation, provided stiff top laterals were provided, which made an independent system.

Fig. 24 showed the suspended span of the new Bandara bridge. The I-members had solid webs—not the old type of flat-bar lacing, which collected water. To assist maintenance, care had been taken to avoid any kind of pockets. The bridge was sub-panelled and prestressing had been adopted in the short sub-verticals and diagonals to obviate heavy secondary bending stresses, which would otherwise have occurred in the bottom chord.

All the expansion had been provided for at one end of the new Bandara bridge, and Mr Shirley Smith's company had designed what they thought was a very ingenious expansion joint in the rails, which allowed for 9 inches of expansion at four different places close together and not all at the same point. Would the Authors say whether that detail had finally been adopted, or what had ultimately been done on the site.

The use of high-tensile steel enabled the chords in all the bridges to be kept parallel, reduced the weight, and saved cost and freight.

Fig. 23

One of the Anchor Spans of Rama VI Bridge

Fig. 24

The Suspended Span of Bandara Bridge

Fig. 8, facing p. 472, showed the Surat bridge in the state in which Mr Shirley Smith had first seen it. It had had three simple spans of 60, 60, and 80 metres respectively. The two middle piers had completely disappeared and one end abutment was shattered. The site of the piers would obviously be obstructed, not only by the wreckage of masonry and steelwork, but also by the pitching of heavy stones, which had been deposited round them annually during the life of the bridge as a safeguard against scour. Moreover, since the new bridge was wider than the old, the piers would in any case have to be increased in width. He had therefore suggested that the right thing to do was to locate the new bridge 200 feet or so downstream. The embankments were not high and coolie labour cost very little. It would have meant shifting the embankments and relaying the track, but that could all have been done while other work was proceeding, and the new piers could then have been built without difficulty. In the event, Mr Shirley Smith's suggestion had not been accepted and 12 months had been spent on one pier trying to drive sheet-piling through the obstructions to make a watertight cofferdam.

The outstanding points on which he considered that those responsible should be congratulated were :—

(1) The amount of standardization, not only of spans and panel lengths but of sections used in the members, which he thought was one of the hall-marks of good design.
(2) The decision to use high-tensile steel in face of the supply risks which had existed 4 or 5 years ago.
(3) The combination of the reinforced-concrete roadway slab with the steel stringers, and the provision of expansion joints in the stringers at each of the cross-girders, which had been necessitated by the greater expansion of high-stensile steel compared with mild steel owing to its higher working stress.
(4) The original design of the slab bearings with half-pins welded on top of them.

Finally, Messrs Dorman, Long should be congratulated on cutting 12 months out of the overall construction time, which was not an easy thing to do in a country such as Thailand.

Mr J. F. Pain made a few comments on the Paper from the point of view of one who had been fairly closely associated with the erection of the new superstructures.

One of the points which had impressed him very much was the extraordinary versatility of the commonest of their ordinary erecting tools, the Scotch derrick crane, and the very great debt which the bridge contractor owed to the crane makers. In the work described, in addition to its conventional use on shore in fixed positions, it served as a travelling crane in the stockyard mounted on bogies on rail tracks, as a floating crane on pontoons, and as an erection traveller on the bridge decks. The last

use introduced various complications, because the bridges were not wide enough to accommodate a crane with ordinary legs—especially the Bandara single-track railway bridge—and special short legs and other improvisations had to be provided. It was, perhaps, the first time that a Scotch derrick crane had been used on a single-track through railway bridge in that way. On the whole, the adaptations worked very well and the cranes proved perfectly satisfactory; there was no doubt that there was a vast advantage in being able to use a standard erection tool of that sort in so many different ways.

One possibly controversial feature of the design of the bridges was the use of high-tensile rivets for the jobs erected under the conditions that existed in Thailand. There could be no doubt that the use of high-tensile steel for the main members achieved very great economy; on that point Mr Pain could confirm everything which Mr Kerensky had said. There had been a great saving in weight and transport costs, and erection was easier. Mr Pain thought, however, that the use of high-tensile rivets in the site connexions of the members under such conditions was a little more controversial.

Under conditions where skilled local riveters were unobtainable or scarce, there was more risk of unsatisfactory work by use of high-tensile rivets than by mild rivets. In the case in question, they had experienced a good deal more difficulty with the high-tensile rivets than could have been expected with mild-steel rivets. He felt that, unless a very material economy could be achieved, it was very doubtful—under conditions where it was difficult to get skilled riveters—whether the use of high-tensile rivets was justified. He realized, of course, that it would involve some sacrifice of economy in material to use mild-steel rivets in conjunction with high-tensile members, particularly in the tension members, but that could be reduced by careful design methods. He would very much like the Authors to give their views on that point.

Mr Gilbert Roberts observed that the point on which he wished to comment had been emphasized by Mr Pain, namely, the use of high-tensile steel rivets. As Mr Pain had said, to achieve maximum economy in the use of high-tensile steel, it was necessary to have proportionately greater strength in the rivets, because otherwise the connexions became excessively large. About 1933 or 1934, Mr Roberts had had a good deal to do with some investigation work into the composition and driving properties of high-tensile steel rivets specified for use with high-tensile steel by the Dutch Government. It had been shown quite clearly by tests on various qualities of steel that, provided the steel was suitable, of the right quality with a low carbon content and a reasonable proportion of manganese, and even of chromium, there was virtually no difference in the driving properties between rivets made from that steel and from ordinary mild steel.

It was well known that in driving a rivet, and in the self-stressing of

the rivet during cooling, the tensile and shear strength of the material was increased by about 5 tons per square inch ; that was, a mild-steel rivet of 24-tons-per-square-inch tensile strength before driving, increased its tensile strength to more than 30 tons per square inch in the driven rivet. The British Standard specification for high-tensile steel rivets, therefore, laid down the limits of 30–35 tons per square inch. That was all very well provided the steel was to the lower limit, but if it was to the upper limit of the tolerance claimed by the steelmakers, there might be difficulty from excessive hardening and cracking in the driven head, and possibly from any slight flaws in the surface of the rivet bar, which would open into cracks at the sides of the head during driving.

Mr Roberts suggested, therefore, that a much closer specification was necessary for high-tensile steel rivets. If the proper steel was used for the rivets, there should be no trouble whatever in driving them, and they would show undoubted economy even in mild-steel structures of reasonably substantial material by reducing the size of the connexions.

Mr F. M. Fuller said that with regard to the use of high-tensile steel, in the three bridges in question, the ratios of high-tensile steel to mild steel varied as 71 to 29, 72 to 28, and 70 to 30. As a matter of interest, in the case of Wandsworth bridge, a deck cantilever, of heavier construction because of the heavier road loading and different types of member, there was a ratio of 68 to 32. He wondered whether a 70 to 30 ratio would apply to any type of bridge if it were designed of high-tensile steel.

Mr Kerensky had emphasized the use of shear connexions on the top of the stringers. They undoubtedly saved weight, but it would be interesting to know how much, and what had been the increased rate per ton for fabricating the stringers with those cleats on the top, and what the Authors estimated was the actual money saving from the use of the cleats.

Mr Fuller did not like the $\frac{1}{2}$-inch wearing surface of concrete, and wondered how it was to be renewed. He thought that the authorities would have been well advised not to use $\frac{1}{2}$-inch of concrete but a thin layer of asphalt, which could be renewed without affecting the structure of the bridge.

The Authors stated that all the members had solid webs, but the diagonals shown in the photographs seemed to have lattice members for their webs.

On p. 458 the Authors referred to avoidance, by providing expansion joints at each cross-girder, of loosening of rivets connecting stringers to cross-girders, and cross-girders to main trusses, as a result of strains set up by the interaction between the deck and main trusses. Their subsequent comments appeared to suggest that there were other reasons for those details, especially for the shape of the stringers and expansion joints at every cross-girder. If it had been merely to avoid loosening rivets, a few turned bolts might have done the job without the use of such elaborate structural methods.

On p. 461 the Authors referred to a bearing stress on concrete of 40 tons per square foot. He did not know what quality of concrete was obtained in Thailand, but that figure seemed to him to be a little on the high side, and he would be interested to know whether any cube tests had been made of the concrete.

From the details of cost given on p. 479, the disproportionate cost of the foundations for the Surat bridge was obvious, for those foundations had cost almost as much as the superstructure. He wondered whether the Authors had considered the possibility of eliminating one pier. That would have given them one opening of 400 feet, which corresponded roughly to a span of one of the other bridges ; it could have been covered with a span supported on one abutment at one end and on a cantilevered arm projecting from the other span at the other. That might have given some economy.

The Authors had now had some time to reconsider the problem, and it would be very useful if they would say whether, if asked to do the job again, they would make any substantial alterations in the design or in the method of construction.

Mr T. J. Upstone, congratulating the Authors, said it was not often that a Paper presented to the Institution covered three different types of bridge.

The new superstructures of the Rama and Bandara bridges had been designed for live loads 50 per cent, and 100 per cent, respectively, greater than the old bridges had been asked to carry, yet they were supported on the original foundations. It would be interesting to know the magnitude of the old foundation pressures, if available, and to compare them with the new ones. Good use had been made of cantilevered and suspended spans in two of the bridges, Rama and Bandara, but in the Surat bridge, simple spans had been retained. He wondered whether some economy could not have been effected at the Surat bridge by using a 60-metre suspended span combined with a 20-metre cantilever arm to cover the 80-metre opening. The centre 60-metre span would then become an anchor span, and the other 60-metre opening would be retained as a simple span. There would, accordingly, be two 60-metre spans identical except for modifications in the end attachments ; the 80-metre anchor and cantilever steelwork should be lighter than an 80-metre simple span.

In the three bridges in question there were six lateral systems, and it was interesting to note that, of those six systems, five were made of the diamond type. He had often found that type of lateral system economical. It cut down the unsupported lengths of the members, enabling smaller sections to be used. The diagonal members were supported at their intersections, and the stresses from their own weight were negligible. The diamond system by itself, however, was not statically stiff and, unless the lateral bending strength of the chords at their junctions with the diagonals

was utilized, it was necessary to insert an additional member to make the system stiff.

He had some observations to make about the erection of the bridges, with which he was closely connected, which he thought would be of interest.

In the various erection schemes, good use had been made of permanent material for temporary purposes. In a country where all steelwork required had to be imported, that was an important point. The platforms of temporary kentledge boxes had been made of highway stringers, as were also the temporary vertical anchorages of the tails of the temporary anchor spans where they were connected to adjacent spans. The temporary erection ties were supported at their middle points by posts, and those posts had been braced with a permanent sway frame by providing special gussets to attach them. In the Bandara bridge, the temporary ties themselves had to be braced with a lateral system, and permanent members of the bridge had been used for that purpose.

All three schemes of erection were cantilever schemes, and not only the strength of the structure but also its deflexions in various stages had to be investigated. When calculating the deflexion of a structure in which some of the joints contained only drifts and service bolts, it was necessary to allow for the effect of the slip which took place at the bolted joints. It was Mr Upstone's practice to assume a $\frac{1}{64}$-inch slip at each bolted joint. Thus, in the 32·6 inches given as the deflexion of the anchor spans of Rama bridge when cantilevered out to piers C and D, 30 inches was elastic deflexion and 2·6 inches deflexion from joint slip. The average actual deflexion for that condition for four trusses had been 29·1 inches, which was rather less than the calculated figure. In the case of the cantilevering of the Rama suspended span, the length of the temporary erection ties had been adjusted so that the outermost panel point of the span would be about $44\frac{1}{2}$ inches below the panel point of the cantilever to which it was to be connected. That had been done so that the process of jacking-up the suspended span with the special jacking frame, which had been referred to in the Paper, also released the tension in the temporary ties; thus, one upwards jacking operation served two purposes: it got the spans into line and released the temporary ties at the same time. No jacking down was needed.

In the case of Surat bridge, the calculated deflexions of the first and second 60-metre spans, when cantilevered out, had been 10·6, and 11·8 inches respectively, with a joint slip included of $1\frac{1}{2}$ inch. The actual deflexions had been less, 8·8 inches, and 10·9 inches respectively. On all three bridges the actual deflexions had been less than the calculated figures. That might have resulted from the stiffening effect of the gussets and cover material at the joints, which had not been taken into account in calculating the deflexions. Joint slip definitely took place. They knew that it had occurred at the drifted joints because, on jacking-up the spans, there had been audible evidence from bangs and cracks that some of the

members were slipping to the opposite sides of the joints, pushing the bolts in the opposite direction.

Mr W. T. F. Austin remarked that it was 5 years since those bridges had been designed, and he wondered what advances had been made in engineering knowledge during that time which would have been of use when the bridges had been designed. First, a considerable amount of work had been done on composite action between steel stringers and reinforced-concrete slabs. Much of that had been done at the University of Illinois Experimental Station and was published in their Bulletins. They were perfectly straightforward to design. One divided the area of concrete from the slab by M, the modular ratio, generally taken as 10 for live load, and considered it part of the steel section. The width of the slab acting was taken as the same as in a reinforced-concrete T-beam section. The resultant saving in stringer steel weight had been found, generally speaking, during those studies, to be about 8 per cent with rolled-steel beam stringers. That was not very great, but the top flange near the neutral axis was not highly stressed. Where loads and spans became bigger and it paid to use a built-up stringer with almost no top flange, say, a 4-inch-by-$\frac{3}{8}$-inch plate ; the saving might reach 30 per cent.

When a concrete slab was poured on top of unpainted steel stringers, at first there was almost complete interaction between the two but, after the bridge had been used for traffic for some time, to all intents and purposes it was certain that that initial bond was destroyed. It was, nevertheless, very desirable that it should be maintained so far as possible ; therefore, shear connectors should be of such a type as to tie the slab down to the top of the stringers. Mr Kerensky had described how that had been done by drilling holes through the stringers for reinforcing bars. In the United States it was common to use bulb instead of plain angles for the shear connectors, and the bulb angles tended to hold the slab down. In certain positions of live load, a slab might be caused to lift away from the top of an adjacent stringer.

The University of Illinois tests had shown that the pressure between the shear connector and concrete slab took place mostly in the lower part of the shear connector. It had been common in some offices in which he had worked in America to design the shear connectors on the assumption that the pressure was about 3,000 lb. per square inch for ordinary concrete over a height equal to the thickness of the leg which was fastened to the stringer, plus $1\frac{1}{2}$ times the thickness of the upstanding leg. That high stress was in order because there was a complicated condition of triaxial compression in that area ; the concrete was restrained and therefore could not fail.

Another point on which there had been a good deal of experimental work done and published during the past 5 years was the distribution of point-load between bridge-deck stringers. At the time that the bridges described in the Paper had been designed, the approach had been by means

of hard mathematical work, but curves and Tables had now been produced which made it a matter of hours only. Those Tables had been drawn up also at the University of Illinois, and were published in the Bulletins of their Engineering Experimental Station; those Bulletins were available in the Library of the Institution. A Paper by Thomas and Short [1] had been read before the Institution and a Paper [2] gave curves which were the result of work by Guyon and Massenet in France. He did not think that those curves would have helped in the particular case in question, because when there was composite action between stringers and deck, the beams were stiff and, being stiff, they did not deflect much and therefore allow the slab to spread the load to adjacent stringers.

He would like to ask the Authors whether they thought that the time had now arrived when it was safe to use high-tensile steel bolts tensioned to a high figure, about 85 per cent of the yield point, instead of rivets in connexions on their bridges. Work had been done on that in experimental establishments in America, and notably the University of Washington and the North Western Technological Institute. High-tensile bolts were used with a $\frac{1}{16}$-inch clearance on the diameter, and the idea was that the stress was transferred from one plate to another by friction at the faying surfaces. There was better resistance to fatigue than with the normal riveted joint. The difficulty was to decide when there was the required tension in the bolt. The last proposal which he had heard was that it should be done by using a hard washer, both underneath the head and on the nut, tightening up with a torque spanner. He did not think that the experiments had reached the stage where it was safe to use the method, but it had advantages. The erection bolts were tightened up, and there was no bolt slip; moreover, they were the final bolts; there was no need to take them out and put in rivets.

He would like to add to one of Mr Fuller's remarks and ask the Authors whether they could say whether any of the competing designs looked like cantilever bridges.

He understood that the $\frac{1}{2}$-inch concrete wearing surface was very common in bridges in the U.S.A. He did not know what the Americans did when it finally wore off. On the George Washington bridge it had not worn off yet, and he expected that when it did they would put asphalt on.

Mr P. S. A. Berridge said far be it from him to criticize such fine designs. However, he did want to ask the Authors whether they had considered the advantages obtainable through turning the flanges of the bottom chords inwards instead of outwards. With such an arrangement, the gussets would come on the outsides of the lower chord, and an unobstructed surface would be offered to the cross-girders, avoiding the need

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[1] F. G. Thomas and Andrew Short, "A Laboratory Investigation of Some Bridge-Deck Systems." Proc. Instn Civ. Engrs, Part I, vol. 2, p. 125 (March 1952).

[2] P. B. Morice and G. Little, "Load Distribution in Prestressed Concrete Bridge Systems." *Structural Engineer*, vol. 32, p. 83 (March 1954).

for notching the web of the cross-girders, and using two sets of angle cleats in the connexion. At the same time, the connexions of the cross-girders would then have angle cleats running uninterrupted for the depth of the cross-girder webs. He admitted that the inward flanges of the chord would prevent the vertical web members from entering between the webs of the chord, but there was ample room for riveting butt-straps between diaphragms and the webs of the verticals, and the depth of the cross-girders ensured ample stiffness at the joints.

At LO (the joint between the end raker and the lower chord), the inward-turned flanges would enable the webs of the end rakers to be brought down over the webs of the lower chord, making, he maintained, a neater and simpler connexion.

From experience, he had found that where the flanges of the bottom chord were turned inwards, erection in the field was greatly simplified. He showed two slides of a railway bridge in Pakistan to illustrate his point.

Mr K. C. Burden said he had been very interested in the Paper, having been concerned with the reconstruction of damaged railway bridges in the neighbouring country of Burma just after the 1939–1945 war. He had been particularly reminded of a 250-foot double-track span near Rangoon, where, when the debris had been cleared, they found that one bottom-chord member near the centre had left less than 1 square inch of material ; the bridge was being held up by the floor system, which was of the conventional design of the 1920's, with rigid stringers, hefty cross-girders, and many brackets. There had actually been a bow of about 2 inches in the end cross-girder which confirmed that it was the floor steelwork which had been doing the job.

He admired the scientific way in which the floor system of the Siam bridges was made independent of the main structure, but he could not help feeling that the 250-foot span which was repaired and put into use in 1948 would have been found in 50 feet of water, if it had had a floor system of such modern design.

He had failed to find in the Paper any reference to the measures taken for painting the steelwork. It had been his experience that a long sea journey, a period in a tropical port, usually under incessant rain and in very damp heat, followed by erection of the bridge in a dry climate up-country, had disastrous effects on normal red lead paint applied in the shops of steelwork manufacturers, and in many instances great sheets had peeled off. It would be very interesting to know what steps the Authors had taken to protect the steelwork.

Mr O. A. Kerensky, in reply, referred to Mr Shirley Smith's suggestion for the use of a new site for Surat Bridge. A new site had been considered and a design produced for three simple spans of 60 metres each which would have been cheaper than the one used, but it had been decided eventually by other authorities that the old site should be used, because of considerations relating to existing railway facilities.

Mr Pain had referred to high-tensile-steel rivets and Mr Roberts had answered most of the technical points. Mr Kerensky entirely disagreed with Mr Pain and would use high-tensile rivets every time—even in mild steel—if the material were thick enough. There was a saving on gussets and in other directions. He had had considerable experience with high-tensile rivets on Wandsworth bridge, where there had been no difficulty whatever. In the case of the Baghdad bridges, recently constructed by Holloway Bros. with native labour, long H.T.S. rivets were driven without any trouble. They had used oil-fired furnaces and had provided some supervision of heating the rivets.

He was glad that Mr Roberts agreed that high-tensile rivets should be used. In future they would probably use high-tensile rivets in mild-steel design, if weight was saved thereby.

Mr Fuller had asked whether any other length of span had been considered for Surat Bridge. The answer was no ; the conditions laid down were that the river had to be cleared in any case, and the spans placed on the existing piers. Undoubtedly the correct solution for cheapness, so far as the bridge itself was concerned, would have been to shift the site.

Mr Fuller had also remarked that all the old bridges looked like cantilevers and the new ones did not. That was true. With modern shops equipped as they were today the cost of fabrication of the old trusses would probably have been much higher than that of the new ones. Parallel chords paid, even if the weight was increased. In addition, the contractors had said that the erection cranes would travel on the top chords. Several changes had taken place before the bridges had been constructed and eventually the cranes had been floated in the river or had travelled on the bottom deck, but the design had been prepared for easy erection, that was to say, for skidding the cranes along the top chords with clear access to the whole of the steelwork.

The ratio of 70/30 for high-tensile steel to mild steel was reasonable, and Mr Kerensky would use something of that kind when estimating in future. The secret lay in the details ; it had been possible to use mild steel to a considerable extent in the bridges described and usually a few main members also could be made in mild steel, for example, diagonals in the central panels of the truss, and possibly end chords. The proportion of 70/30 was about right for medium-size bridges ; in small ones the quantity of mild steel would be higher and in large ones smaller.

With regard to the shear connectors, the cost of drilling the flanges and riveting-on angle cleats with three holes in them was not high. He had been informed by the contractor that it was less than 1 per cent. The saving in weight amounted to 16 per cent (using 18-inch-by-6-inch-by-55-lb. B.S.B. instead of 20-inch-by-6½-inch-by-65-inch).

Regarding the renewal of the ½-inch wearing surface, one could put some asphalt on top when the surface deteriorated. Given the choice Mr Kerensky would allow for ½ inch of asphalt rather than concrete, but the

weight was the same. The provision of the extra thickness of concrete had been laid down by the Thailand State Railways.

Mr Kerensky apologized for a mis-statement in the Paper. All the vertical I-sections had solid webs, but the double channels in diagonals were laced. Mr Fuller had suggested that the loosening of rivets could be avoided by using bolts and that that would avoid structural difficulties. In the case of the roadway stringers there had been no difficulties. The railway stringers were all identical and it had been thought that the contractors would use jigs and then the extra cost of fabrication would be negligible. There was a saving in weight, and the advantage of having a cheap sliding bearing on top of the cross-girders. Mr Kerensky thought that turned bolts, with the unavoidable site reamering would not have paid, but he would not like to be definite about that.

Mr Fuller had also suggested that the pressure of 40 tons per square foot on concrete was too high. On the contrary, Mr Kerensky thought that it was too low. It had been necessary to reconstruct the piers and add a reinforced-concrete pad under the bearings. With modern cement, provided the supervision was reasonable, he thought that the pressure could safely be doubled, but 40 tons per square foot was the pressure permitted by the Thailand State Railways.

He had no significant second thoughts regarding the design of the three bridges. One or two details could be improved; for example, welding could be avoided in the fabrication of bearings, which would result in some saving in cost, but one might safely say that minor improvements could be added to any design or specification almost *ad infinitum.*

Mr Upstone had asked whether a 60-metre cantilever span had been considered as an alternative for Surat Bridge. The 80-metre span used was exactly the same as that in the Rama VI bridge, and was therefore a complete repetition. Any departure from that would have meant a loss of efficiency. Mr Kerensky did not believe that there was much economy in using a cantilever design for a railway bridge of the type in question, particularly when designing to a specification with a severe stress-reversal clause, as in the present instance. The suspension links were expensive and, although weight had been saved, the economy had probably been lost in the non-repetition of members which were not symmetrical about the centre-line, and in the various gadgets which had to be introduced.

The live load on the Rama and Bandara Bridges exceeded that used on the old bridges by 50 per cent and 100 per cent respectively. Little was known about the old bridges at the time of the new design; the designers had been given a line diagram of the superstructure without proper details of the foundations, and had been asked to produce new steel superstructures to go on old piers, keeping the pier loads as near the old ones as possible. Since then Mr Kerensky had done some research into the old stress sheets and other memoranda relating to the design conditions used at the time, and the loading percentages were based on that.

There was no need to worry much about the increase of pier pressures because the weight of the pier itself was so big compared with the load from the superstructure. He thought it was safe to say that the pressures did not go up very much. The Thailand Authorities had clearly given very careful consideration to the matter before deciding that the old foundations would take the new bridges with the increased loadings.

Mr Kerensky agreed that the "Diamond" diagonal systems were theoretically incomplete, but the well-known Bailey Bridge diamond panels had provoked the same comment when they appeared, and they had since been tested and proved satisfactory. In the present instance the main loads were applied at the intersections of the diamonds and so far as those loads were concerned the system was completely stable; it was only when a load was applied at the apex of a diamond that the system became theoretically unstable. In addition, inclined end members with portal bracing attached to diagonals automatically formed the locking link. That configuration had been used succcessfully in the Baghdad Bridges. He thought that adding a locking member at the beginning of a 400-foot span did not make much difference to the behaviour of the system at, say, midspan.

With regard to deflexions, he was interested in the figures which had been given. He agreed that slip must take place during erection and should be allowed for, but it was wrong to omit the effect of gussets and cover material also, if an accurate estimate was required. In the Paper on the Sydney Harbour Bridge [3] complete data were given on how to do the calculations in order to obtain a more accurate answer. Alternatively, one could use a reduced modulus of elasticity and save a lot of labour in obtaining an approximately correct answer.

Mr Austin's contribution had been very valuable. Mr Kerensky's firm had recently been working on the problem of combined decks and had produced a welded-stringer design with a saving in weight of nearly 50 per cent, although the saving in cost would probably be somewhat less.

The ultimate strength of the combined deck was more than three times as great as that of the non-combined one.

Since the bridges described in the Paper had been designed there had been an advance in the design of reinforced-concrete combined-action decks, which had become very popular in Germany and America. Almost every new German bridge with a reinforced-concrete deck was designed for combined action. Although there had been a tendency to say that the best connector was of a loop type they were now coming back to the rigid connector, and he thought that eventually they would finish up with a few rivet or bolt heads protruding into the concrete. Mr Austin's figures for saving of stringer weight by the adoption of combined action design were

[3] J. F. Pain and G. Roberts, "Sydney Harbour Bridge: Calculations for the Steel Superstructure." Min. Proc. Instn Civ. Engrs, vol. 238 (1933-34, Pt 2), p. 256.

low. He would say that savings from 15 per cent to 50 per cent might be expected.

Mr Austin had asked an awkward question in referring to the use of high-tensile steel tension bolts. Mr Kerensky could not answer that; he did not know whether they would pay or not. They were not sufficiently developed to risk using them on a bridge in the wilds, carrying a railway. There was always the difficulty that it was necessary to omit the painting between the surfaces in contact, and there was the problem of nuts being stolen from the bridge. In the case of Wandsworth Bridge almost every nut had been removed from the manholes in the parapets!

Mr Berridge asked whether consideration had been given to turning in the angles in the chords. That was considered every time a design was made, and sometimes the decision went one way and sometimes the other. Mr Kerensky's firm was now preparing a design in which they turned all the angles in, and the contractors had told them that the cost of fabrication was going up, because it was impossible to use hydraulic riveting; nevertheless the total cost should be lower. In the case in question, however, the chords were only 15 inches wide and were too narrow for an efficient turned-in design. It would have necessitated a lot of pompom riveting and there would be very poor access for painting. The chords of the bridge which had been shown on a slide were much wider, and perhaps there it was the right answer. Mr Kerensky thought that the advantages and disadvantages of turned-in chords were about equal, and he did not know which was the right answer, except when weight was definitely saved thereby, as in the case of larger spans.

Mr Burden had raised an interesting point about the modern floor system being useless when the bridge was being blown up; that was correct. The point was, however, that if it was not blown up the bridge should prove a better structure in service. Bridges with continuous deck, designed with proper provision for the interaction of all its members, were definitely more rigid, but the cost of making that provision was excessive. That had been investigated repeatedly. If one did not take all the necessary precautions to avoid interaction or alternatively take all the stresses into consideration one would get service trouble. There would be loosening of the rivets and possibly development of cracks. Those troubles might not have been too serious in mild steel, but they would be much worse in high-tensile steel, with stresses increased by about 40 per cent. Until some other method of design was developed it was safer to articulate and be sure than to overstrain and keep on repairing.

Mr K. E. Hyatt, who also replied, did not hold any brief for the re-location of the bridges, but nobody had thought that the work would be so difficult as it had turned out to be at Surat. He was sure that the foundations of Bandara and Rama VI had turned out cheaper than entirely new constructions would have been. Surat had been a particularly difficult case. Although the approach banks were not very high—18 to

20 feet—they were very long, and the re-location of the track, even on a metre-gauge railway, would have meant a long length of new approach embankment over rather low country.

The vexed question of high-tensile rivets had been ventilated at some length. Speaking as one normally concerned with work in the field, Mr Hyatt confessed that he was still a little half-hearted about them when really good riveters were not available. In Thailand they had found it very difficult to get a good-looking rivet; the inspectors had agreed with that view and had made them cut a great many out. The rivets had been very difficult to cut out, and in fact more difficult to cut out than to put in. They were extremely good rivets so far as doing their job was concerned, tight in the hole and very hard to cut, with no question of brittleness or of the heads popping off. He was satisfied that as rivets doing a job they were efficient, but they seemed to get hard rather quickly when cooling down, and the Chinese riveters had not been able to get a good form of head on them.

Reverting to the first point, the foundations, it should be mentioned that Christiani & Nielsen designed the foundations more or less as they went along, because the conditions were not known until they had opened up the work. A great deal of credit was due to them for the way in which they trimmed their sails, so to speak, to the conditions prevailing on the sites, particularly as communications were not very rapid, and a good deal of going to and fro had been needed to get things settled. New materials especially, had been difficult to obtain.

On the question of parallel chords for cantilevers there was one point which he would like to make, being connected with fabrication as well as erection, and that was that the most economical design from the point of view of material was not always the most economical design for fabrication, and many engineers would do well to bear that in mind. A saving of 5 per cent in the weight of steel might result in an increase of 10 per cent in the cost of fabrication, which from an economic or any other point of view was rather silly. In the particular case under discussion, the question of parallel chords had been fully dealt with by Mr Kerensky, but the point which Mr Hyatt had made applied not only to bridges but to other structures as well.

So far as second thoughts on methods of construction were concerned, he did not think that there was any great change which he would suggest. The cantilever method worked very well, and the general arrangements had been as economical as could be devised. All the little gadgets had been very simple and economical and did their job. The running of cranes on top chords looked very attractive when getting out a scheme, but there were objections to it. There were extra wind loads, for one thing, and it was necessary to erect the crane 30 feet higher in the air than otherwise. There were various practical objections which any steel erection foreman could explain much better than he could, and which such a foreman would probably be willing to explain at some length.

He had perhaps no need to say very much about deflexions, because that had already been dealt with, but in fact the actual deflexions nearly always came out at less than the design ones, because of the stiffness of connexions and the effect of the bracing members.

The use of high-tensile bolts was very attractive, particularly when riveting was difficult, and riveting was becoming much more difficult in all parts of the world than it used to be. Even in the United Kingdom, it was not easy to get good rivet squads for outside erection and bridge work, and he thought that the high-tensile bolt, when more experience had been obtained with it and a proper method of tightening it was available, would be the greatest boon. He was wholly in favour of research in that direction. In some parts of the world it was almost impossible to get riveting done today by local labour, and men had to be brought in from outside, which was very expensive. In present circumstances a high-tensile bolt could be comparatively expensive in a country such as England, where some riveters were available, but there would be a good deal to be said for it in remote situations.

Mr Berridge's suggestion of the turning-in of angles of chord members was one which had often been debated, as Mr Kerensky said. In general Mr Hyatt would vote in favour of turning them outwards, because otherwise there would be trouble with the connexions to posts and diagonals, and riveting and site painting became difficult. Some joints with angles turned in were very awkward where the post and diagonal connexions occurred, and one had to get hold of a small boy to get inside the chord if the joints had not been designed very carefully.

With regard to Mr Austin's query concerning competing designs, the only ones of which he had knowledge were those prepared by his own firm. Those all had parallel chords and very similar outlines to the bridges actually constructed.

On the subject of painting, with which he had dealt previously at a meeting of the Institution,[4] he thought that people had to make up their minds that they had to get the millscale off the steel somehow, and if it was not taken off by shot blasting—which was an expensive method though much the best and most efficient—there was something to be said for putting the initial shop coat on steel which had to go abroad and waiting for it to come off with the millscale, which always seemed to happen in a more wholesale manner if painted.

With regard to red lead, he would ask people not to specify Type 1 red lead, which of course contained a much larger quantity of actual pigment than Type 2, for jobs where the paint had to be kept a long time and shipped overseas, because it was not very good by the time it got to the site. Even with the inhibitors which were supposed to prevent the lead

[4] K. E. Hyatt and G. W. Morley, "The Construction of Kafr el Zayat Railway Bridge." Proc. Instn Civ. Engrs, Part III, vol. 1, p. 101 (Apr. 1952). *Authors' reply to discussion:* p. 138.

falling to the bottom it was not very good, and it would be better to put up with a lower lead content and have the paint put on properly, instead of having paint which had to be put on with a trowel from one part of the container, or wiped on with a rag from another.

The closing date for Correspondence on the foregoing Paper has now passed without the receipt of any communication.—SEC. I.C.E.

Paper No. 5965

"The Lateral Buckling of Tied Arches"

by

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(Ordered by the Council to be published with written discussion)†

SYNOPSIS

The type of arch dealt with in the Paper is that shown in *Fig. 2*, in which the hangers, taken as simple tension members, are assumed to be connected at their lower end to a laterally rigid chord, OP, thus becoming inclined as the arch deflects laterally; the hangers are the only external connexions to the arch rib between abutments. This is referred to as the "wire-hanger" arch, and comes into the class of arch known as the "Stabbogen," in which the arch rib (being slender about a horizontal axis relative to the stiffening girder beneath) may be treated as a pure compression member. The problem analysed here is that of determining the elastic stability of the arch rib against lateral buckling.

A theoretical solution based on Rayleigh's Principle is outlined, and the full results are given covering two boundary conditions, and showing the influence both of rise and of torsional rigidity on the critical load.

An experimental investigation is described, including the design of a testing bench and test procedure, and the results of about 100 tests on model steel arches are given, together with a general discussion of the behaviour of the arch ribs under test.

The results obtained show that the tension in the hanger system exerts a considerable stabilizing influence on the arch, and this effect makes the problem of arch stability quite different from that of the lateral stability of top chords in parallel boom trusses. The various factors influencing stability are discussed, and the possible effect of overhead bracing between adjacent arch ribs, though not specifically dealt with in this investigation, is mentioned in the discussion on the results for unbraced arches.

Notation (See *Figs 1*)

X, Y, Z	denote	rectangular axes.
X', Y'	,,	tangential and normal axes to the arch rib lying in planes parallel to OXY.
L	denotes	arch span.
f	,,	rise/span ratio.
s	,,	co-ordinate measured from O along arch axis.
S	,,	curved length of arch.
λ	,,	shortening of chord OP due to lateral deflexion of arch rib.
B_2	,,	lateral flexural rigidity of arch rib. $B_2 = EI_{Y'}$; $B_1 = EI_Z$.

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C denotes torsional rigidity of arch rib.

$$\gamma = \frac{B_2}{C}$$

T' ,, moment in arch rib about axes parallel to OX.
M' ,, moment in arch rib about axes parallel to OY.
T ,, torque in arch rib (about axis X').
M ,, bending moment in arch rib (about axis Y').
H ,, horizontal component of thrust in arch rib.
H_{CR} ,, critical value of H.

Boundary conditions :—

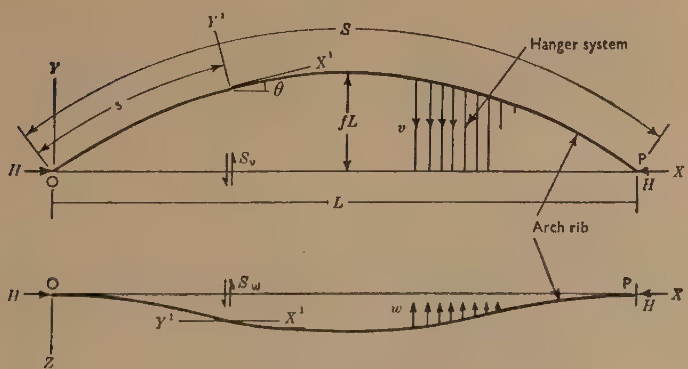
Condition I. The arch rib fixed in position only at O and P.
Condition II. The arch rib fixed both in position and direction (about Y') but torsionally unrestrained at O and P.
Condition III. The arch rib encastré at O and P.
Condition IV. The arch rib encastré at O and P, and fixed in position at mid-span.

Note—The arch profile is taken as parabolic in all cases.

Introduction

The majority of arch bridges constructed on the " Stabbogen " system of *Fig. 2*, in either steel or reinforced concrete, use some overhead bracing

Figs 1



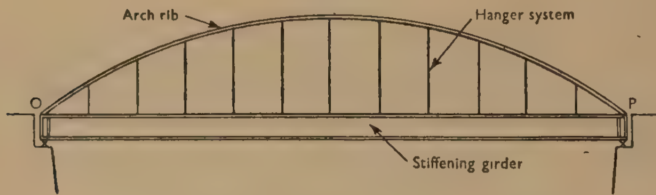
Notation

between arch ribs—a factor that in some cases may have influenced the choice of rise/span ratio on account of the consideration of headroom. Also in many cases the hangers have been designed with a certain degree of stiffness in a direction normal to the plane of the arch, thus forming cross-frames with the transoms. Overhead bracing can at best be included only

over the centre section of the bridge where headroom permits, and in shorter-span bridges it may be more convenient to omit it altogether. When the latter is done, the problem of the lateral stability of the arch rib arises.

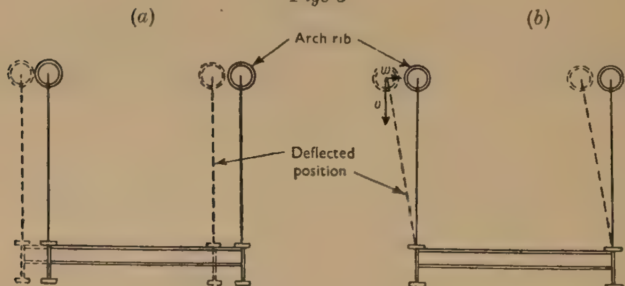
This problem, in the case where the hangers have a degree of stiffness, is one which presents considerable difficulty on account of the number of factors involved—these are, the lateral flexural rigidity (B_2) and torsional rigidity (C) of the arch rib, the rise/span ratio (f), and the cross-frame

Fig. 2



ELEVATION OF A TYPICAL "STABBOGEN" ARCH

Figs 3



DECK SYSTEMS WITH AND WITHOUT LATERAL DEFLEXION

stiffness of the hanger-transom system—and it would clearly be difficult to derive a general solution to such a problem.

Stussi, in a Paper in 1943,[1] dealt with the simplest case of vertical loading on a laterally unsupported arch,*—that is, loading which remains vertical as the arch deflects laterally—a type of loading that would only be realized in "Stabbogen" construction if the decking system presented no resistance to lateral movement (*Fig. 3a*). This Paper brings in one further factor, that of the deck stiffness. It assumes that, relative to the arch ribs, the decking may be considered as completely rigid against lateral movement (chord OP), hence the hangers become inclined as the arch

[1] The references are given on p. 514.
* The results are given in *Fig. 6* for comparison.

deflects laterally, all points on the arch rib moving on a circular arc of radius equal to the hanger length and with chord OP as centre (*Fig. 3b*). The hangers, which are assumed to be inextensible and to have no flexural stiffness, thus apply a horizontal component (w) of hanger tension resisting lateral deflexion in the arch rib. It is reasonable then to expect the stability of this " wire-hanger " arch to be greater than the type analysed by Stussi. Also the results of this problem are readily expressible in general terms as the cross-frame stiffness is zero.

Of the various methods of analysis attempted, that outlined in this Paper, using the energy criterion for stability, was found to provide the simplest solution in conditions I and II (see " Notation ") where the boundary moments are known : it can be extended, though not so simply, to conditions III and IV—these two further conditions are investigated experimentally—but it is not considered likely that the more difficult case with cross-frames would be amenable to this method of analysis. In the energy method, the arch is assumed to be loaded with horizontal loads H at each abutment, and the distance between abutments allowed to shorten as the arch deflects laterally.

Only the problem of elastic stability is considered. Physically this implies that the arch remains in its ideal plane as the thrust is increased until at the first critical value of thrust its form becomes unstable and lateral collapse takes place, the stresses up to that load remaining within the elastic (or more strictly, linear) range.

Theoretical

The energy criterion for stability, as applied to a straight pin-ended strut of length L, is that for any deflected form

$$\frac{1}{2B_2}\int_0^L M^2 \, . \, dx - H\lambda > 0$$

where λ is the axial displacement of H, the axial thrust due to the deformation y.

Hence using any characteristic form of y, the corresponding critical value of H is given by :

$$\frac{1}{2B_2}\int_0^L M^2 \, . \, dx - \frac{H}{2}\int_0^L \left(\frac{dy}{dx}\right)^2 dx = 0 \quad . \quad . \quad . \quad . \quad (1)$$

in which M can be expressed either as $B_2 \cdot \frac{d^2y}{dx^2}$ or as $-Hy$. The equivalent of the latter method is used in the analysis below for the following reasons :—

(1) It provides the more accurate answer for an assumed deformation.[2]

(2) It is most useful in cases where the boundary moments are known (they are zero both in condition I and for symmetrical buckling in condition II).

(3) In the problem of arch stability, an assumed deformation $z = f(x)$ takes into account both torsional and flexural energies; whereas if the equivalent of the former of the above two methods were used, a rotation curve would also be required.

If the arch is loaded by a horizontal thrust H, then, using the fundamental buckling mode, the first critical value of H is given by:

$$H_{CR} \cdot \lambda = \frac{1}{2B_2}\int_o^S M^2 \, . \, ds + \frac{1}{2C}\int_o^S T^2 \, . \, ds. \quad . \quad . \quad . \quad (2)$$

where λ is the shortening of chord OP due to the deformation z.

Rayleigh's method, using a special deformation chosen as the simplest expression satisfying all the boundary conditions, provides a solution for H_{CR} with a high degree of accuracy in the problem under consideration.

Assumptions

The theory is based on the following assumptions peculiar to this problem:—

(1) The web, rather than being taken as a series of individual hangers, is assumed to be a continuous membrane taking only tension in planes parallel to OYZ. This is referred to as "hanger tension."

(2) Only small deflexions (values of z) are considered, and within this limit the following are assumed as the arch deflects laterally:—

(*a*) The vertical component of hanger tension v remains constant.

(*b*) Bending moments and torques in the arch rib are taken about axes Y' and X' respectively.

(*c*) The lateral flexural rigidity of the arch rib remains constant at B_2.

Value of λ

This depends only on the geometry of the arch and on the form of z. Any element δs of the arch rib, the co-ordinates of whose mid-point are x, y, z, only contributes towards λ by virtue of its angular departure from a plane passing through chord OP and the point x, y, z. The component of this angle on the horizontal plane is given by:

$$\frac{\delta z}{\delta x} - \frac{z}{y} \cdot \frac{\delta y}{\delta x}$$

with the condition that this expression becomes zero at $y = 0$.

The true angle is then given by:

$$\left(\frac{\delta z}{\delta x} - \frac{z}{y} \cdot \frac{\delta y}{\delta x}\right) \frac{y}{\sqrt{y^2 - z^2}} \cdot \frac{\delta x}{\delta s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and within the limits of (3) being small, the effect of this on λ is:

$$\delta\lambda = \tfrac{1}{2}\left(\frac{\delta z}{\delta x} - \frac{z}{y}\cdot\frac{\delta y}{\delta x}\right)^2 \frac{y^2}{y^2 - z^2}\cdot\delta x$$

an expression that can be further simplified to:

$$\delta\lambda = \tfrac{1}{2}\left(\frac{\delta z}{\delta x} - \frac{z}{y}\cdot\frac{\delta y}{\delta x}\right)^2 .\,\delta x$$

within the limits of z being small relative to y.

Hence within the limits stated:

$$\lambda = \tfrac{1}{2}\int_o^L \left(\frac{dz}{dx} - \frac{z}{y}\cdot\frac{dy}{dx}\right)^2 .\,dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

As both $\frac{dz}{dx}$ and $\frac{z}{y}\cdot\frac{dy}{dx}$ are proportional to $\frac{1}{L}$ and independent of f, the value of λ, when calculated for any one form of z expressed in terms of x, will apply to all wire-hanger arches irrespective of the value of f and is proportional to $\frac{1}{L}$.

Values of M and T

These are most easily calculated by evaluating S_v, S_w, T', and M' as follows.

For a parabolic arch, the hanger tension per unit x is:

$$v = H\cdot\frac{8f}{L}$$

thus the component of hanger tension at any point, parallel to the z axis, is given by:

$$\omega = -v\cdot\frac{z}{y} = -H\cdot\frac{2z}{x(L-x)}$$

The resultant shear on chord OP at x_1 due to loading v is:

$$Sv_1 = \frac{vL}{2} - vx_1$$

and the resultant shear at the same point due to loading w is:

$$S\omega_1 = -W_o - \int_0^{x_1} \omega\,.\,dx$$

where W_o is the lateral reaction at O due to loading w.

Hence the resultant moment in the arch rib at x_1 about an axis parallel to OX, is given by:

$$T_1' = S\omega_1\,.\,y_1 + Sv_1\,.\,z_1$$

a value, which for any form of z expressed in terms of x, is proportional to H and f, and is independent of L.

The resultant moment in the arch rib at x_1 about an axis parallel to OY is given by:

$$M_1' = -Hz_1 + W_o \,.\, x_1 + \int_o^{x_1} (x_1 - x) \,.\, \omega \,.\, dx$$

all terms being proportional to H, and independent of both f and L.

So far, for a given z curve expressed in terms of x, all of the computation (namely the calculation of λ, T', and M') is essentially common to all parabolic wire-hanger arches, irrespective of rise/span ratio.

The values of bending moment and torque in the arch rib are then:

$$\left.\begin{aligned} M &= M' \cos\theta - T' \sin\theta \\ T &= M' \sin\theta + T' \cos\theta \end{aligned}\right] \quad \ldots\ldots\ldots\ldots \quad (5)$$

and these values must be evaluated separately for each different value of f.

The Buckling Forms

As a first approximation the buckling form was taken as the simplest form of the Fourier series (that is, the fewest possible terms) compatible with all the boundary conditions in each case.

In condition I, z and $\frac{d^2z}{ds^2}$ are each zero for $s = 0$ and S. Also z may be taken as zero for any one other value of s, as for overall equilibrium, the arch should be fixed in position, relative to the supports O and P, at one other point. The simplest curve satisfying these conditions is given by:

$$z = a_2 \sin \frac{2\pi s}{S} \quad \ldots\ldots\ldots\ldots \quad (6)$$

where the value of the parameter a_2 is unimportant and may be taken as unity.

In condition II, provided the buckling form is symmetrical (which it is to give the simplest form), from overall equilibrium we have that $\frac{d^2z}{ds^2} = 0$ at both O and P. Hence in this case the boundary conditions are z, $\frac{dz}{ds}$, and $\frac{d^2z}{ds^2}$ each zero for $s = 0$ and S. (It will be seen that this corresponds to fixing the pin-ended arch in position relative to O and P at any two other points symmetrically placed about mid-span; hence symmetrical buckling in condition II is identical with the form corresponding to the second critical load for pin-ended arches, for which condition I gives the first critical load.) The simplest curve satisfying these conditions is given by:

$$z = a_1 \sin \frac{\pi s}{S} - \frac{a_1}{3} \sin \frac{3\pi s}{S} \quad \ldots\ldots\ldots\ldots \quad (7)$$

where the value of a_1 may be taken as unity.

Expressions (6) and (7) are difficult to handle, the values of z with respect to x varying slightly with f, and z expressed in terms of x rather than s was used throughout, that is :

$$\text{for condition I,} \qquad z = \sin\frac{2\pi x}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\text{for condition II,} \qquad z = \sin\frac{\pi x}{L} - \frac{1}{3}\sin\frac{3\pi x}{L} \quad . \quad . \quad . \quad (9)$$

It was not found necessary to modify this first approximation since expressions (6) and (7) almost exactly agree with the deflected forms found over the entire range of subsequent experimental work (see *Figs 13*), and the errors incurred by using expressions (8) and (9), rather than (6) and (7), in the calculation of H_{CR} were found to be negligible.

Value of H_{CR}

Using the buckling forms of (8) and (9), then, for example, in the case of $f = 0{\cdot}16$, equation (2) becomes :

$$\text{in condition I,} \qquad H_{CR} \cdot \frac{81{\cdot}15}{L} = \frac{H_{CR}^2 \,.\, L}{2B_2}(3{\cdot}651) + \frac{H_{CR}^2 \,.\, L}{2C}(0{\cdot}328)$$

$$\text{Hence} \qquad H_{CR} = \frac{B_2}{L^2}\left(\frac{44{\cdot}46}{1 + 0{\cdot}0898 \,.\, \gamma}\right)$$

$$\text{and similarly in condition II,} \quad H_{CR} = \frac{B_2}{L^2}\left(\frac{81{\cdot}92}{1 + 0{\cdot}0264 \,.\, \gamma}\right)$$

(the three terms in each case being evaluated by numerical integration, using twelve sections per half span).

Coming directly from this method of analysis, the following limiting values of H_{CR} can be determined :—

(*a*) As γ approaches zero, corresponding to an arch of infinite torsional rigidity. In the example given, H_{CR} then becomes $44{\cdot}46\frac{B_2}{L^2}$ and $81{\cdot}92\frac{B_2}{L^2}$ in conditions I and II respectively. The influence of γ on H_{CR} is shown in *Fig. 4*.

(*b*) As f approaches zero, corresponding to a very flat arch. As has already been pointed out, as f approaches zero, λ remains unaltered, values of T' approach zero, and the values of M'^2 which are independent of f approach the values of M throughout. Hence the limiting values for H_{CR} in both conditions are given by :

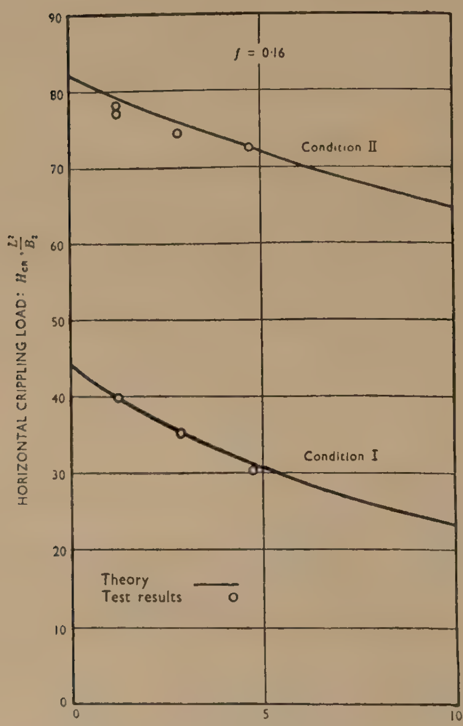
$$H_{CR} \,.\, \lambda = \frac{1}{2B_2}\int_0^L M'^2\, dx$$

and are independent of torsional rigidity. The calculated values of these asymptotes were found to be :

$$\text{condition I,} \qquad H_{CR} = 57{\cdot}71 \frac{B_2}{L^2}$$

$$\text{condition II,} \qquad H_{CR} = 96{\cdot}03 \frac{B_2}{L_2}$$

Fig. 4

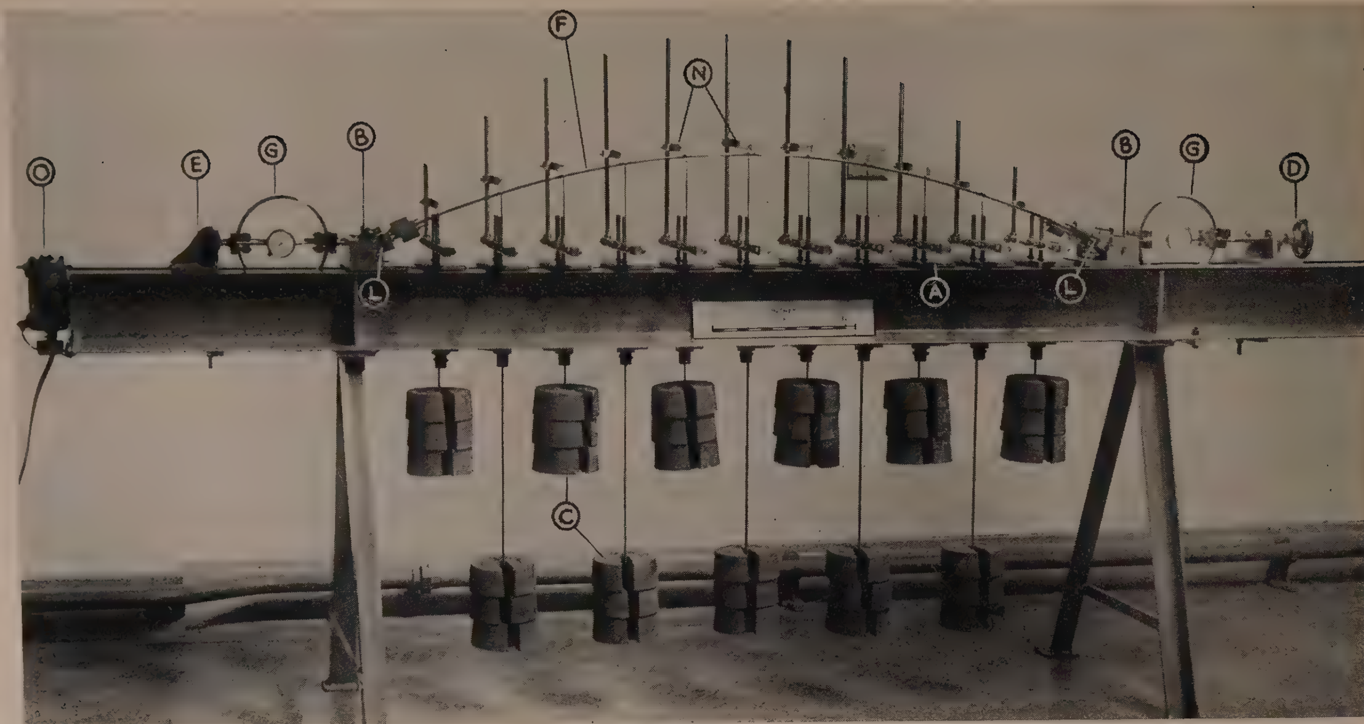


THE INFLUENCE OF γ UPON H_{CR}

The theoretical values of H_{CR} are given in *Fig. 5*, together with the test results.

The small magnitude of error incurred by using expressions (8) and (9) rather than (6) and (7)—which are known from experiment to be correct—can be appreciated in the following comparison at $f = 0{\cdot}24$, the maximum value of f analysed and where any discrepancy should thus be a maximum :

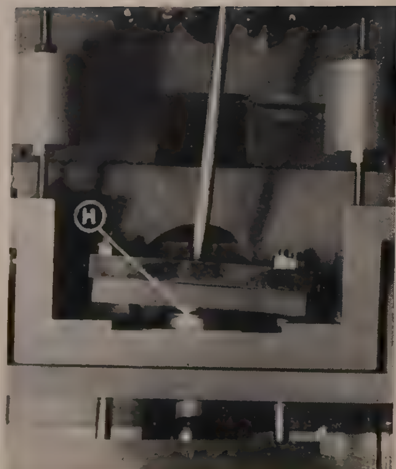
Fig. 7



THE TESTING BENCH

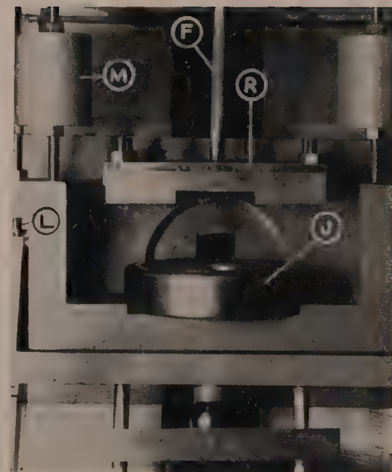
Figs 8

(*a*)



SPHERICAL BEARING FOR CONDITION I TESTS

(*b*)



ANGULAR CONTACT BEARING FOR CONDITION II TESTS

(EXPLODED VIEW)

(*c*)

SOLID DUMMY BEARING FOR CONDITION III AND IV TESTS

(EXPLODED VIEW)

DETAILS OF THRUST BEARINGS AT THE ABUTMENTS

Fig. 9 (a)

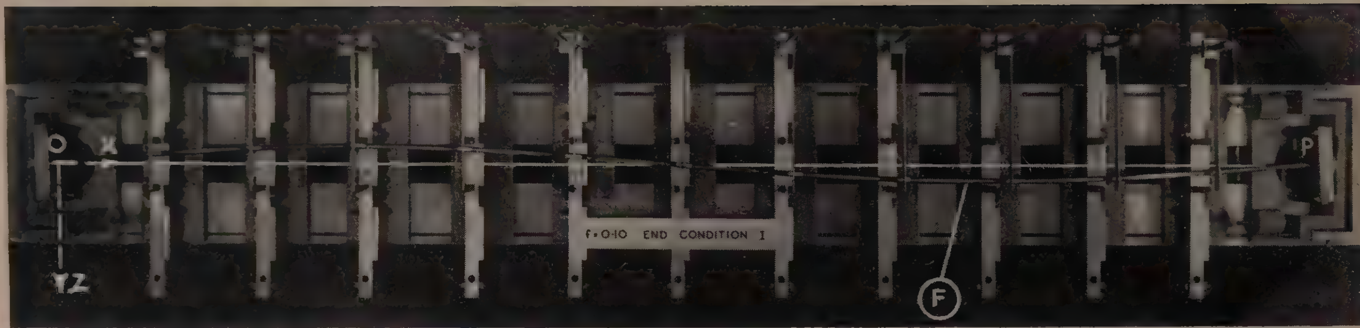
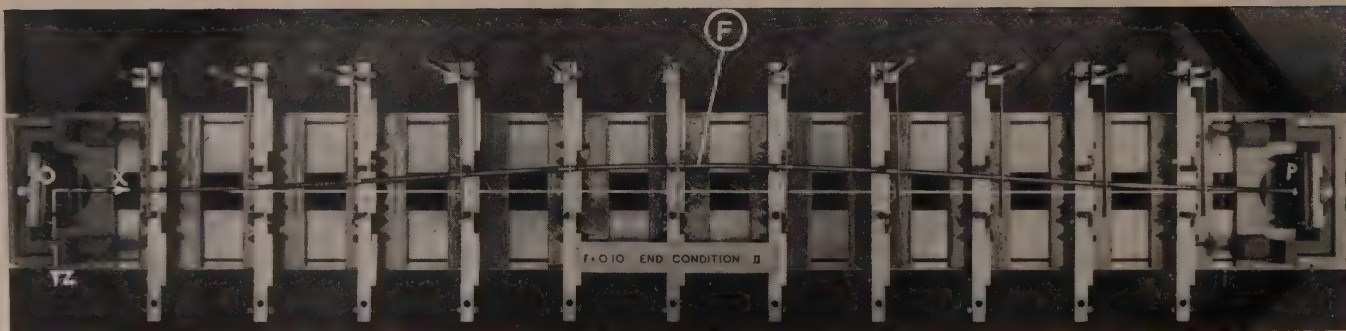


Fig. 9 (b)



Buckling Forms, Conditions I and II ($f = 0{\cdot}10$, $\gamma = 1{\cdot}25$)

Fig. 10 (a)

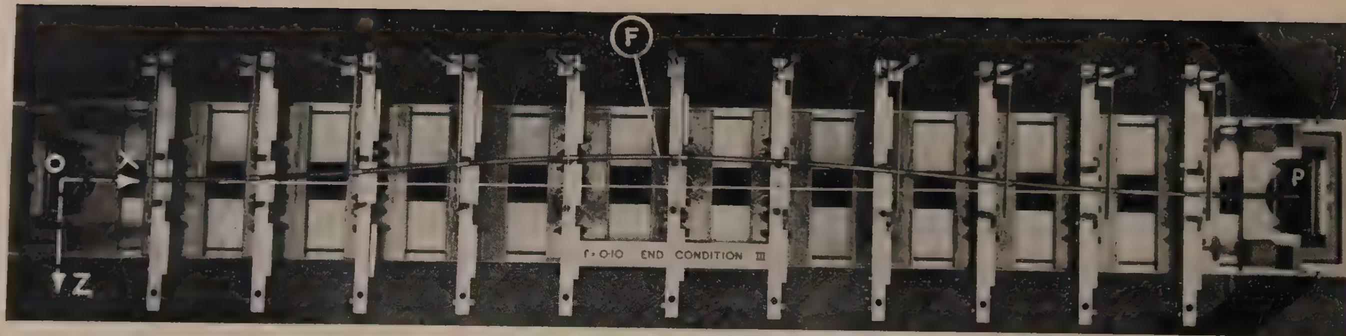


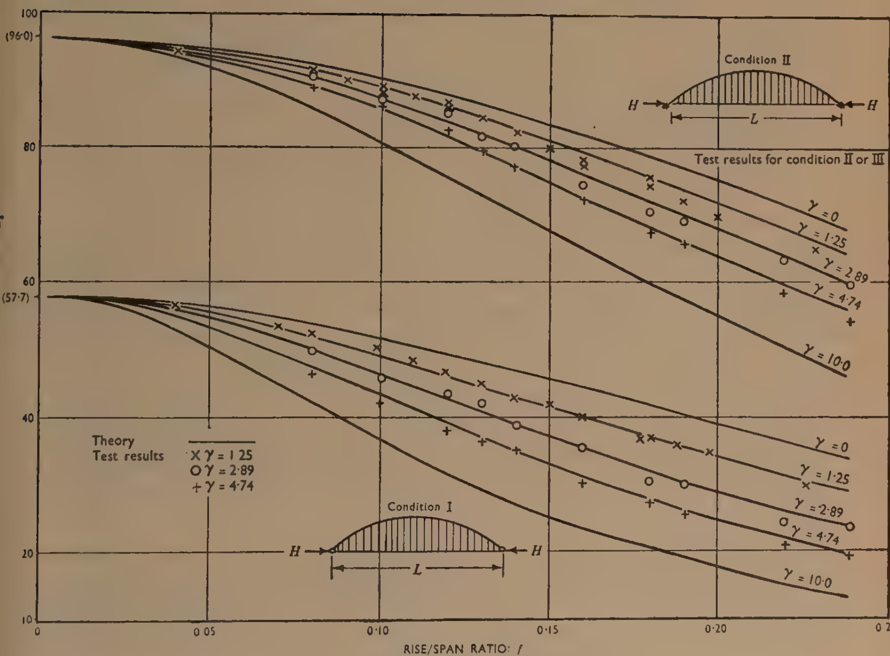
Fig. 10 (b)

BUCKLING FORMS, CONDITIONS III AND IV ($f = 0{\cdot}10$, $\gamma = 1{\cdot}25$)

using (7), $$H_{CR} = \frac{B_2}{L^2}\left(\frac{67{\cdot}80}{1 + 0{\cdot}050\,\gamma}\right)$$

using (9), $$H_{CR} = \frac{B_2}{L^2}\left(\frac{67{\cdot}92}{1 + 0{\cdot}053\,\gamma}\right)$$

Fig. 5



TEST ON 60-INCH-SPAN MODEL

EXPERIMENTAL

Much importance was attached to the experimental work on model steel arches, chiefly on account of the necessity of verifying the validity of the assumptions made in the theory outlined ; apart from those assuming a general linear behaviour of the system, the most important to be checked was the assumption for the buckling form in each of the two boundary conditions analysed. Also the effect of having individual hangers rather than a continuous tension membrane needed to be studied. Hence this work was planned with a view to having the closest possible control over the arch at all stages of loading, and especially when approaching the crippling load.

The bench and the experimental technique described are the outcome

of a preliminary investigation on a smaller and simpler bench, from which was obtained information on the convenient size of model, methods of loading, sources of error, etc.

The Testing Bench

The chief requirements of the testing bench are :—

(*a*) The application of a known horizontal thrust to the arch rib. A reference to the experimental procedure will show that the bench should preferably be one in which the thrust is known for any applied longitudinal strain.

(*b*) The horizontal component of thrust to be constant over the entire span of the arch.

(*c*) The arches to be tested under boundary conditions I and II, and also under two further conditions, III and IV.

The simplest solution to the problem of loading is to apply a longitudinal strain to the arch rib (along OX), rather than to apply the measured load through the hangers, though the latter is nearer to the actual loading conditions of a bridge. In the former method, which was adopted, the hanger bases and abutments must be constrained to move freely along OX on account of the requirement (*b*); this is a relatively simple matter. The one drawback of this method of testing is that the value of hanger tension is not found experimentally, this being one of the only factors over which there is not complete control; but the errors due to the small bending moments taken by the arch in the vertical plane were found to be unimportant.

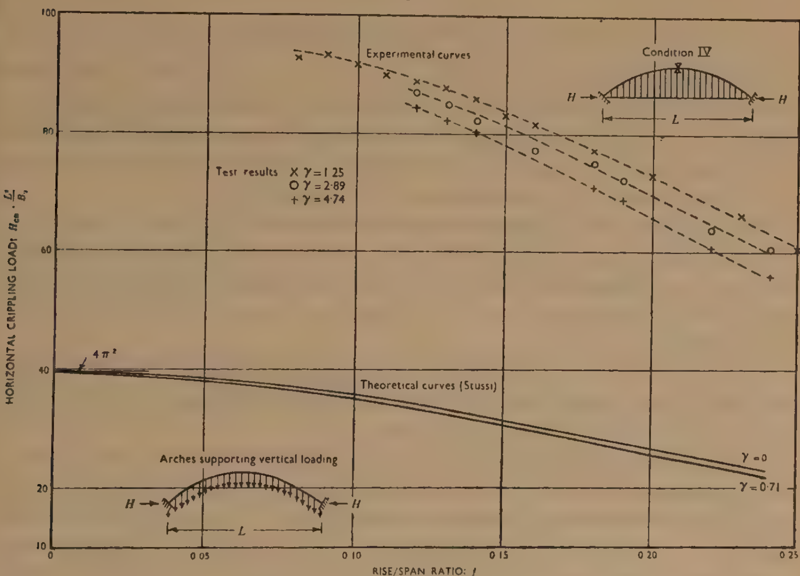
The bench used consists of two 9-foot long 6-inch-by-3-inch channels back-to-back, joined at each end by a steel diaphragm, the upper surfaces being ground after assembly, and two parallel V-notches machined right through to take the ball-bearings. On these ball bearings run the hanger trolleys (A)* and abutments (B), the former being weighted sufficiently by weights (C) to prevent any movement other than axial displacement. The strain is applied manually by a screw (D) passing through a casting which can be clamped in any required position on the bench, and a second casting (E) provides the balancing reaction. The entire arch (F) is thus on ball-bearings, and a proving ring (G) is provided at each end to measure the horizontal thrust and to check against any possible frictional loss in the bearings.

The abutments which provide the different boundary conditions are shown in *Figs 8*. For condition I, a spherical thrust bearing (H) is used, composed of two concentric spherical surfaces separated by balls in an unlocated cage, in which the centre of rotation is coincident with the origin of the arch. For condition II, this bearing is replaced by an angular contact thrust bearing (J). For conditions III and IV, the bearing is replaced by a

* Letters shown thus are the part references shown in *Figs 7* to *10*, between pp. 504 and 505.

solid dummy bearing (K) which prevents all movement by means of taper pins. As the thrust is applied to the arch through the two support screws (L) which pivot about an axis coincident in elevation with the arch origin, no moments M_z are applied to the end of the arch rib in any boundary condition (this movement is locked in condition I where the spherical seating provides all the freedom required). The adjustable counterweights (M) are provided so that the part of the abutment attached to the arch rib is balanced about the points of the support screws.

Fig. 6



TEST ON 60-INCH-SPAN MODEL

The lateral deflexion of the arch rib is measured on horizontal scales (N), adjustable in height on vertical arms fixed to the hanger trolleys.

An electric motor (O) fitted with an eccentric was bolted to the bench in order to provide a vibration just sufficient to eliminate any frictional loss in the ball bearings.

The Model Arches

The results given in *Figs 5* and *6* are all for tests carried out on steel arch-ribs of approximately 60-inches span. As shown, the only two variables to be taken into account were :—

(1) Rise/span ratio (f)

The range covered in detail was from 0·08 to 0·24, there being some

difficulty in getting reliable results from tests on very flat arches (*Fig. 5* does include results of tests on an arch at $f = 0{\cdot}04$). Each arch was used over a small range of f, the rib being shaped to its correct profile for each individual test rather than being sprung in the bench by altering the hanger lengths.

(2) Value of $\gamma\left(=\frac{B_2}{C}\right)$

Three values of γ were tested, the lowest value 1·25 being that of a solid circular section in steel, and the other two values 2·89 and 4·74 being for hollow seamless steel tubes of an approximately elliptical section. The value of γ in any design will of course depend on the value of $\frac{B_2}{B_1}$ and on the type of construction used. In all cases the values of B_2 and C were found experimentally by simple bending and torsion tests on straight specimens. (In the case of the hollow tubing where bending causes a slight alteration in the cross-section, these tests were carried out on a straight specimen altered to precisely the same amount between rollers.)

The hangers, which had to be as flexible as possible, were in most cases of hard-drawn high-carbon steel wire of 0·01-inch diameter, threaded through fine vertical holes in the arch rib, and soldered in position. In all cases eleven hangers were used, and these were spaced equidistant on the horizontal axis.

A steel plate (R), made to a standard size, was attached to each end of the arch rib by bronze-welding, and in all test conditions the arch origin was then on the inner face of this plate.

The arch ribs were bent to the required parabola over a line drawing, and in the case of the hollow sections, where every effort was made to maintain their cross-section, specially constructed rollers were used for this operation. All the work of bending and alignment was carried out on a plane table before the arch rib was erected in the testing bench, though the final checks were made with the arch in position. Any error in alignment is, of course, equivalent to initial eccentricity causing lateral deflexions which increase with axial load.

Erecting the arch in the bench only consisted of bolting the end plates rigidly to the abutments, and fixing the base of each hanger in its hanger trolley, the hanger clamps being adjusted to the same level as the arch origin and support screws.

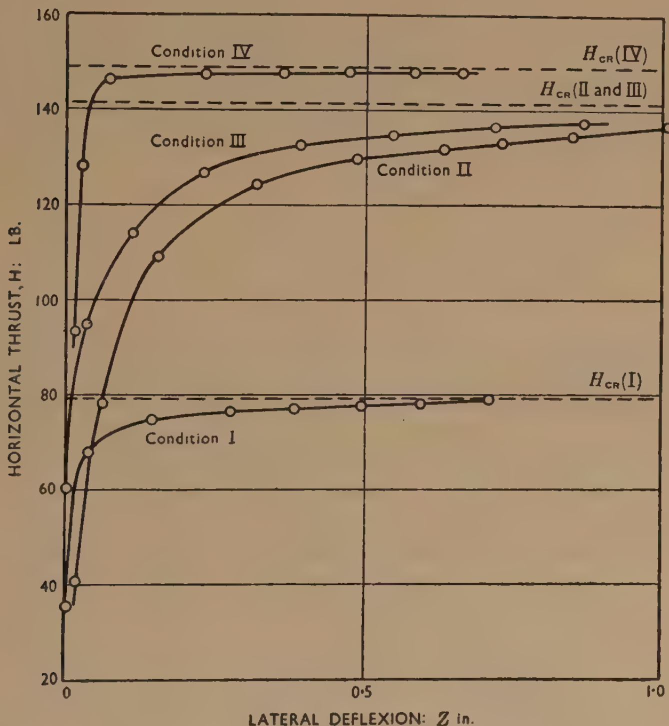
Experimental Procedure

In each test the two matters investigated were the horizontal crippling load H_{CR}, and the deflected form.

The value of H_{CR} was determined by the method first applied by Southwell[3] to the case of the pin-ended strut and which gives the first critical load without the necessity of applying its full value experimentally.

Fig. 12 shows the method in a modified form proposed by Lundquist and Donnell[4] applied to the tests on one arch in all four boundary conditions. This method is valuable as not only does it check against all non-linear behaviour (effect of large displacements, local instability or yield, frictional effects in the bench, slip in the clamps, etc.) and observational errors, but provides the value of the critical load independent of the effect of small

Fig. 11



CURVES OF DISTORTION FOR $f = 0{\cdot}10$, $\gamma = 1{\cdot}25$

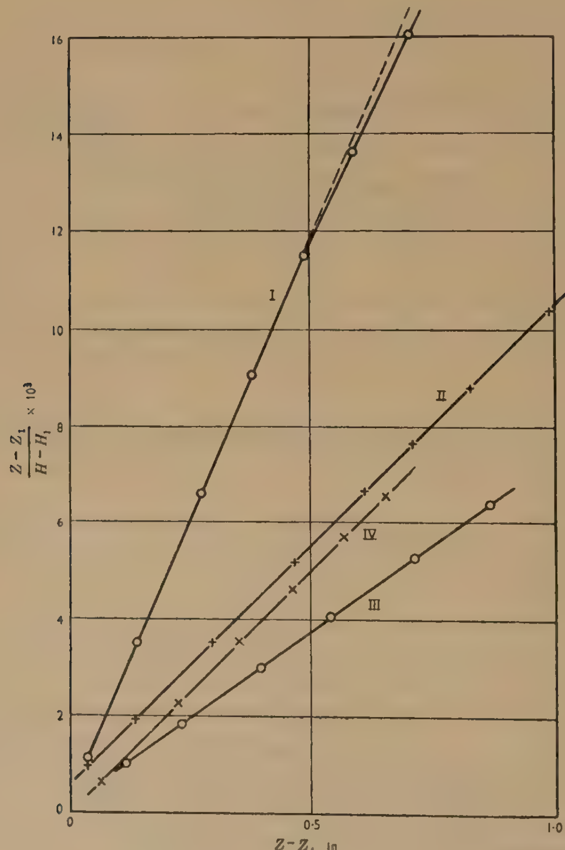
initial eccentricities—and in spite of the care taken in construction and erection, sudden buckling from the straight (accompanied by an audible thud) occurred only twice during the course of experimental work extending over a period of 2 years.

The test procedure was to observe lateral deflexions with the corresponding horizontal thrust measured on both proving rings :—

(*a*) At all hanger points, for "no load" (or at H_1, being a small value of H), and for the maximum H applied. These values provide the deflected form.

(*b*) At the hanger showing maximum deflexion, for approximately equal deflexion intervals at that point. These provide the most suitable readings for calculating the value of the critical load. (The method of straining provides the control necessary to adjust the deflexion to any required value).

Fig. 12



"SOUTHWELL PLOT." $f = 0{\cdot}10$, $\gamma = 1{\cdot}25$

From the observations the following were plotted :—

(1) The curve of distortion *Fig. 11*, being the graph of z plotted against H for the hanger point showing maximum displacement. The plotting of this graph, though providing a useful picture of the behaviour of the arch under load, is not necessary for determining the value of the critical load.

(2) The "Southwell Plot" or the curve of $\frac{z - z_1}{H - H_1}$ plotted against $z - z_1$ (z_1 being the deflexion reading corresponding to H_1). With a hyperbolic curve of distortion this graph will approximate to a straight line, with an inverse slope providing the value of $H_{CR} - H_1$. Any non-linear behaviour in the structure will cause this graph to depart from the ideal straight line. Apart from the scatter at small values of load, and the influence of second order effects at large deflexions, the points obtained lie so nearly on a perfectly straight line in all tests that there is little possibility of a personal error, or need to use the method of least squares as originally done by Southwell to determine the best fitting line.

(3) The buckling form. This was plotted primarily to check with the form predicted and used in the theoretical solution. The buckling forms for one series of tests can be seen in the "aerial" photographs of *Figs 9* and *10*.

General Results

Buckling forms.—Over the entire range of f tested, the buckling forms corresponding to conditions I and II are one wave anti-symmetrical and one wave symmetrical respectively. In each case the form plotted on a development of the arch axis remains sensibly constant, being apparently independent of either f or γ within the investigated range of both these quantities; thus the variation in plan is slight. Also the similarity between the experimental buckling forms and those of expressions (6) and (7) seems to be almost perfect when the deflexions about mid-span are appropriately averaged to overcome the slight irregularities in deflected form encountered. *Fig. 13* shows a comparison between the curves of expressions (6) and (7) and the experimental buckling forms for tests on arches $f = 0{\cdot}04$, $\gamma = 1{\cdot}25$ and $f = 0{\cdot}24$, $\gamma = 4{\cdot}74$, these being the two extremes of the investigated range. It can be seen that even in the case of $f = 0{\cdot}24$, $\gamma = 4{\cdot}74$ where the torques are a maximum and the torsional rigidity a minimum, the buckling form appears to be unchanged.

Condition III tests produced results identical with those for condition II, the end rotation of the arch in the latter condition being either zero or so small as to be quite unimportant.

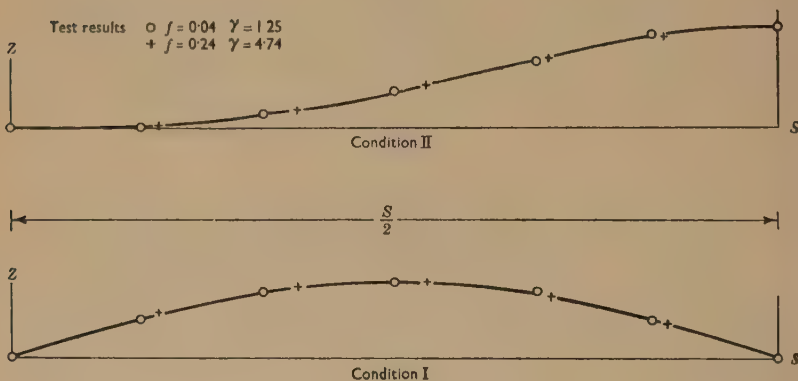
The buckling form in condition IV tests, shown in *Fig. 10* (*b*) displayed the same lack of variation over the range of f investigated.

In the tests on conditions II (or III) perfectly symmetrical buckling was seldom obtained in spite of the care taken in construction; but even in the extreme cases where the maximum displacement occurred at the hanger adjacent to that at mid-span, this lack of symmetry had no apparent effect on the value of crippling load obtained. This tendency to an indefinite buckling form in these tests, as well as being due to the effect of unavoidable

eccentricities and slight inequality of hanger tensions, is probably because of the proximity of the first and second critical loads for encastré arches (condition III and IV tests respectively). The effect of even small eccentricities in a condition III test did, on one or two occasions, cause the arch to buckle anti-symmetrically, as in a condition IV test, supporting momentarily a load in excess of its first critical load.

Values of crippling load.—Fig. 5 gives the test results of experimental work carried out on the three different values of γ, and under boundary conditions I and III. Twelve tests covering the complete range of f,

Figs 13



Buckling Forms

$$\text{Upper curve: } z = \sin\frac{\pi s}{S} - \frac{1}{3}\sin\frac{3\pi s}{S}$$

$$\text{Lower curve: } z = \sin\frac{2\pi s}{S}$$

and the three different values of γ, showed no apparent difference in values of H_{CR} between condition II and condition III tests (any small differences were irregular and within the normal experimental scatter encountered between similar tests). A detailed investigation into scatter in condition III tests for $f = 0{\cdot}16$, $\gamma = 1{\cdot}25$, covering all the possible sources of variation, showed that tests done on different settings of one arch produced results within $\pm$ 1 per cent of the mean value, and if several similar arches were used the results lie within $\pm$ 2 per cent; the latter figure represents the maximum permissible departure from the theoretical value. Similar tests at other values of f and γ showed that the variation in load remained fairly constant, and thus the percentage scatter varied according to the value of H_{CR}. It can thus be seen that the comparison between test values and theoretical results is satisfactory all through. The results of condition IV tests are given in *Fig. 6*.

Effect of large displacements.—It was found that in all tests there is a considerable range of displacement where the "small deflexion" assumption holds good. This range is a minimum in condition I tests, and from *Fig. 12* it is seen to be approximately 0·5 inch in the case of a 60-inch-span arch, $f = 0{\cdot}10$, $\gamma = 1{\cdot}25$. In all four test conditions, if this limiting value of deflexion is exceeded, the load taken by the arch slowly increases; this effect was observed to some degree in all tests where yielding in the steel did not occur first, as in the majority of tests on solid circular sections.

Effect of the number of hangers.—Tests made on one arch at $f = 0{\cdot}16$, $\gamma = 1{\cdot}25$, having first eleven and then five equally spaced hangers, showed that the latter test gave a reduction in the value of H_{CR} by the following amounts: condition I, 1·7 per cent; condition III, 0·8 per cent. These values are representative. It will be noticed that in the condition I test with five equally spaced hangers there are only two hangers per half-wave deflexion, and even in this extreme case the value of H_{CR} is only slightly affected. In the normal type of construction the "tension membrane" assumption is well justified and gives rise to no appreciable error in the value of H_{CR}.

Conclusions

The large difference in crippling loads between condition III arches and encastré arches supporting vertical point loads (*Figs 5* and *6*) is a measure of the stabilizing effect of hanger tension when the hangers are connected to a laterally rigid chord. This effect of hanger tension, which of course only applies to arches or trusses with a curved top chord, does not appear to have been taken into account in design specifications giving the effective length of the top chord in resisting lateral buckling.

A comparison of the values of H_{CR} in boundary conditions I and II (*Fig. 5*) shows the importance of the directional fixing of the arch at each end; and within the full range of tests carried out, the fact that conditions II and III produce identical results indicates that the torsional fixing at each end of the arch rib is quite unimportant.

The close proximity of the first and second critical loads for the fully encastré "wire-hanger" arch (conditions III and IV respectively) shows that there is very little increase in stability to be gained by fixing the crown of the arch in position. This would suggest that from the point of view of elastic stability there may be little advantage in raking adjacent arches to meet overhead at mid-span, as was suggested in a recent bridge design competition.[5]

Fig. 4 shows the comparatively small effect of γ on H_{CR}, and since H_{CR} is proportional to B_2, effective design calls for bracing (if required) to increase the effective stiffness of the rib against lateral bending rather than against torsion. The limiting case of $\gamma = 0$, corresponding to a section infinitely rigid in torsion, is derived from theoretical work, and is included as an

indication of the doubtful value of increasing the torsional stiffness by external means. The torsional stiffness must, of course, be adequate as the value of H_{CR} does drop considerably as γ becomes large.

Most specifications and design recommendations in not mentioning the specific problem of arch ribs presumably intend it to be covered by the general ruling for unsupported top chords ; but from the data presented in this Paper this would appear to be irrational, and it is clear that this type of arch has a considerably greater resistance to lateral buckling than has been allowed for up to the present.

Acknowledgements

Acknowledgement is due to Mr S. R. Cochrane who carried out a preliminary investigation; to Mr T. W. Willington who applied other methods of analysis to this and more complex arch stability problems; to Mr N. G. Derby whose experimental results are given in *Figs 5* and *6*, and to Professor A. H. Naylor of the Department of Engineering, Queen's University, Belfast, where the work described in the Paper was carried out.

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The Paper is accompanied by eight photographs and nine sheets of drawings from which the half-tone page plates and the Figures in the text have been prepared.

Paper No. 5966

"A Study of the Bowstring Arch having Extensible Suspension Rods and Different Ratios of Tie-Beam to Arch-Rib Stiffness"

by

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and

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(*Ordered by the Council to be published with written discussion*) †

SYNOPSIS

The structural system of the bowstring arch is highly indeterminate. The ends of the arch are fixed to the tie-girder so that both rotate together, and the tie-girder is continuous over several extensible suspension rods. An accurate analysis must take account of these factors, although in design they are frequently ignored.

In this Paper, the method of influence coefficients has been used to analyse a bowstring arch having six suspension rods, and the results are compared with those obtained by a simplified design method. A second theoretical method, in which the suspension rods are assumed to be replaced by a membrane, is also described, and although this method finds its most accurate application when the number of suspension rods is large, very good results are obtained in cases which are just within the practical limits of calculation by the method of influence coefficients.

An experimental method using direct strain measurements on a loaded celluloid model has been used to check the results of the calculations for three bowstring arches having different ratios of arch-to-tie-beam-stiffness.

INTRODUCTION

A BOWSTRING-ARCH bridge is used when maximum headroom is required and the abutments cannot be relied upon to take large thrusts. Horizontal thrust in the arch is taken by the tie-girder which also serves as the deck to carry load. The tie-girder is connected to the arch at various points along its length by vertical suspension rods, and the complete structure is therefore highly indeterminate.

In design offices the problem is often simplified by assuming that the ends of the arch and tie-girder are pinned together, that the suspension rods are inextensible, and that the second moment of area of the arch is

† Correspondence on this Paper should be received at the Institution by the 15th December, 1954, and will be published in Part III of the Proceedings. Contributions should be limited to about 1,200 words.—SEC. I.C.E.

small compared with that of the tie-girder, so that the structure has only one redundancy no matter how many suspension rods are used.

When account is taken of the rigidity of the arch, an extra redundancy is introduced by each suspension rod, and even with six rods a bowstring arch has nine redundancies including two due to the end moments on the arch.

In this Paper, the method of influence coefficients is first described in general terms and then applied to a bowstring arch having six suspension rods and three different ratios of arch-to-beam stiffness. Making the usual simplifying design assumptions, this problem is readily solved, but when the structure having nine redundancies is analysed, extreme arithmetical accuracy is necessary in forming and solving the equations derived from strain-energy calculations. Increment transformations have been used to facilitate the computation.

A more elegant approach to the problem can be made by a method which has been used by Professor Pippard on other structures. The arch and tie-girder are assumed to be connected by an elastic membrane, and the vertical force in the membrane is expressed as a single continuous function. The force in any suspension rod is then obtained by integrating the membrane force function between appropriate mid-panel limits. The accuracy of the assumption in this method increases with the number of suspension rods, but it is shown that very good results are obtained when there are only six rods. The computation is not complicated by the number of suspension rods, and charts are given which make the method suitable for use in the design office.

To check the various theoretical results, an experimental method of measuring strains on loaded celluloid models has been developed.

The work was carried out in the Civil Engineering Department of Imperial College, London.

Part I.—The Method of Influence Coefficients

In order to analyse a statically indeterminate framed structure by the method of influence coefficients, " cuts " and " hinges " are imagined to be inserted at suitable places in the structure to make the frame determinate, and forces and moments are then applied to both sides of the " cuts " and " hinges " to restore the structure to its original condition. By expressing the relative displacements of the sides of every " cut " or " hinge " in terms of each force or moment applied, as many elastic equations as there are unknowns can then be formed.

The method will be described by reference to the bowstring arch shown in *Fig. 1*.

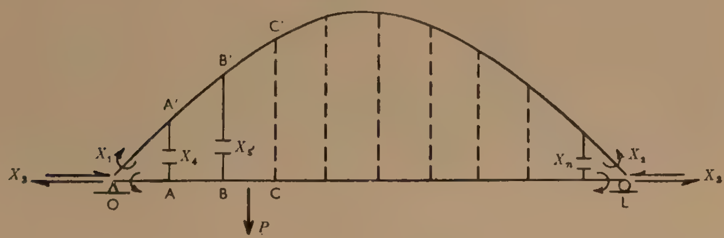
Unknown forces and bending moments applied at the cut sections to restore the original condition of the structure are $X_1, X_2, X_3, \dots X_n$. At a cut K, δ_{0k} denotes the relative displacement of the sides of the cut in the

direction of the force X_k due only to the external applied load P, and $\delta_{1k}, \delta_{2k}, \delta_{3k} \ldots \delta_{nk}$ are the relative displacements of the sides of the cut in the direction of the force X_k, due only to a unit force or moment applied to the structure instead of X_1, X_2, X_2, . . . or X_n, so that the corresponding relative displacements are $X_1\delta_{1k}$, $X_2\delta_{2k}$, . . . $X_n\delta_{nk}$. Hence the net relative displacement of the sides of the cut in the direction of the force X_k when all the forces are acting together is, by the Principle of Superposition :

$$\delta_k = \delta_{0k} + X_1\delta_{1k} + X_2\delta_{2k} + \ldots X_i\delta_{ik} + \ldots X_n\delta_{nk} \quad . \quad (1)$$

If there is no initial lack of fit of members, the original condition of the structure at this " cut " is that obtaining when the sides of the " cut " do

Fig. 1



not move apart, that is, when $\delta_k = 0$. If now every " cut " or " hinge " is considered in turn, the following set of equations is obtained :

$$\left.\begin{aligned}
\delta_1 &= \delta_{01} + X_1\delta_{11} + X_2\delta_{21} + \ldots X_i\delta_{i1} + \ldots X_n\delta_{n1} \\
\delta_2 &= \delta_{02} + X_1\delta_{12} + X_2\delta_{22} + \ldots X_i\delta_{i2} + \ldots X_n\delta_{n2} \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\delta_k &= \delta_{0k} + X_1\delta_{1k} + X_2\delta_{2k} + \ldots X_i\delta_{ik} + \ldots X_n\delta_{nk} \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\delta_n &= \delta_{0n} + X_1\delta_{1n} + X_2\delta_{2n} + \ldots X_i\delta_{in} + \ldots X_n\delta_{nn}
\end{aligned}\right\} \quad . \quad (2)$$

All the coefficients ($\delta_{11}, \delta_{22}, \delta_{ik}$, etc.) influence the values of the unknowns X_1, X_2, X_3 . . . X_n, so they are called " influence coefficients." They can be calculated by the Principle of Virtual Work, for if M_0 denotes the bending moment caused by the external load P ; M_i and M_k denote the bending moments caused by the unit forces or moments $X_i = 1$ and $X_k = 1$ respectively in the reduced determinate system ; F_0 denotes the force produced by the external load P, and F_i and F_k denote the forces produced in the system by the unit forces or moments $X_i = 1$ and $X_k = 1$; then, neglecting the work expended in shear and torsion :

$$\delta_{0k} = \int_0^l M_0 M_k \frac{ds}{EI} + \int_0^l F_0 F_k \frac{ds}{EA} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\delta_{ik} = \int_0^l M_i M_k \frac{ds}{EI} + \int_0^l F_i F_k \frac{ds}{EA} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The integration is made through the entire length of every member in the structure.

Having solved the equation (2) and obtained the values of $X_1, X_2, \ldots X_n$, the bending moments and the forces acting on any section are :

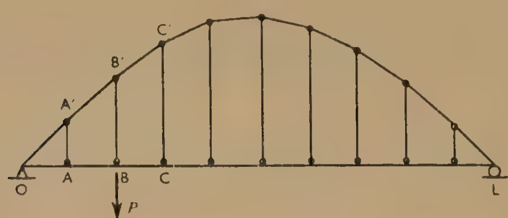
$$M = M_0 + \sum_{i=1}^{n} X_i M_i \quad \ldots \ldots \quad (5)$$

$$\text{and} \quad F = F_0 + \sum_{i=1}^{n} X_i F_i \quad \ldots \ldots \quad (6)$$

Derivation of Equations for Bowstring Arch having Any Number of Suspension Rods

If X_1 and X_2 denote the bending moments at the ends of the arch and girder, X_3 denotes the horizontal force in the girder, and $X_4, X_5, \ldots X_n$ denote the forces in the suspension rods, the moment produced in the arch

Fig. 2.

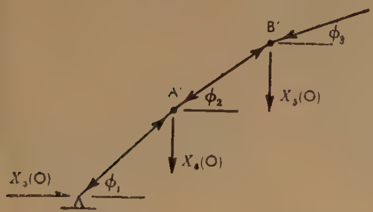


due to X_3 is opposite in sign to that produced by $X_4, X_5, \ldots X_n$, and inaccuracies in the calculation of the forces may have a serious effect on the accuracy of the moment calculations. It is desirable, therefore, to transform the equations so that the accuracy of their solution is not so critical. This is done by introducing a " null "-system bowstring arch in which the arch rib takes no bending moment, that is, the forces in the suspension rods produce only axial forces in the rib. Then, in a bowstring arch in which the arch rib resists bending moment, the bending moment in it is caused mainly by the differences between the internal forces produced by the applied load and those in the " null " system. If, therefore, the elastic equations are transformed and the differences calculated, bending moments can be obtained directly.

As an example of this transformation, *Fig. 2* shows a " null " system comprising a linear bowstring arch with inextensible pinned suspension rods. In this system the arch rib carries only axial force, and the three

concurrent forces at each joint are in equilibrium. If $X_4(0)$, $X_5(0)$, . . . $X_n(0)$ denote the forces in the suspension rods (*Fig. 3*), then :

Fig. 3



$$\left.\begin{aligned} X_4(0) &= (\tan \phi_1 - \tan \phi_2)X_3(0) \\ X_5(0) &= (\tan \phi_2 - \tan \phi_3)X_3(0) \\ &\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \quad (7)$$

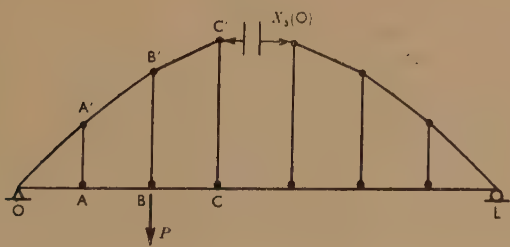
If $(\tan \phi_1 - \tan \phi_2) = C_4$
and $(\tan \phi_2 - \tan \phi_3) = C_5$ etc.

$$\left.\begin{aligned} \text{then } X_4(0) &= C_4X_3(0) \\ X_5(0) &= C_5X_3(0) \\ &\ldots\ldots\ldots\ldots \\ X_n(0) &= C_nX_3(0) \end{aligned}\right\} \quad . \quad . \quad . \quad (8)$$

In the " null " system it is also assumed that there is no deformation of the rib under load, so that C_4, C_5, . . . C_n, the suspension-rod forces produced by unit horizontal force $X_3(0)$ in the arch, are all constant.

The " null " system for a bowstring arch shown in *Fig. 4* can be analysed

Fig. 4



by " influence coefficients " by inserting a cut at the crown of the arch and applying the horizontal force $X_3(0)$ as the redundant on either side of the cut.

If δ'_{03} is taken to denote the relative horizontal movement of the sides of the cut due to the external applied load P, and δ'_{33} denotes the relative horizontal movement of the sides of the cut due only to unit force applied instead of $X_3(0)$, then :

$$X_3(0) = -\frac{\delta'_{03}}{\delta'_{33}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

When calculating the coefficients δ'_{03} and δ'_{33}, it must be remembered that it is assumed there is no deformation of the form of the rib, and the suspension rods are inextensible. In other words, the strain energy in the arch, the suspension rods, and the girder due to the direct force is ignored.

Having obtained $X_3(0)$ the values of the suspension-rod forces $X_4(0)$, $X_5(0)$, . . . $X_n(0)$ can be obtained from equations (7) and (8).

If, now, $\varDelta X_3, \varDelta X_4, \varDelta X_5, \ldots \varDelta X_n$ denote respectively the amounts by which the forces in the arch differ from those in the " null " system, that is :

$$\left.\begin{array}{l} X_3 = X_3(0) + \varDelta X_3 \\ X_4 = X_4(0) + \varDelta X_4 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ X_n = X_n(0) + \varDelta X_n \end{array}\right\} \quad \ldots \ldots \quad (10)$$

equations (2) become :

$$\left.\begin{array}{l} X_1\delta_{11} + X_2\delta_{21} + \varDelta X_3\delta_{31} + \ldots \varDelta X_n\delta_{n1} \\ \qquad = -[\delta_{01} + X_3(0)\delta_{31} + \ldots X_n(0)\delta_{n1}] \\ X_1\delta_{12} + X_2\delta_{22} + \varDelta X_3\delta_{32} + \ldots \varDelta X_n\delta_{n2} \\ \qquad = -[\delta_{02} + X_3(0)\delta_{32} + \ldots X_n(0)\delta_{n2}] \\ X_1\delta_{13} + X_2\delta_{23} + \varDelta X_3\delta_{33} + \ldots \varDelta X_n\delta_{n3} \\ \qquad = -[\delta_{03} + X_3(0)\delta_{33} + \ldots X_n(0)\delta_{n3}] \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ X_1\delta_{1k} + X_2\delta_{2k} + \varDelta X_3\delta_{3k} + \ldots \varDelta X_n\delta_{nk} \\ \qquad = -[\delta_{0k} + X_3(0)\delta_{3k} + \ldots X_n(0)\delta_{nk}] \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ X_1\delta_{1n} + X_2\delta_{2n} + \varDelta X_3\delta_{3n} + \ldots \varDelta X_n\delta_{nn} \\ \qquad = -[\delta_{0n} + X_3(0)\delta_{3n} + \ldots X_n(0)\delta_{nn}] \end{array}\right\} \quad . \quad (11)$$

Hence the unknowns, $X_3, X_4, \ldots X_n$ of the original equations (2) are transformed to the increment unknowns$\varDelta X_3, \varDelta X_4, \ldots X\varDelta_n$ in equations (11).

Since the bending moment in the arch due to P is zero, and the horizontal force $X_3(0)$ together with the suspension-rod forces $X_4(0) \ldots X_n(0)$ in the " null " system produce no bending moment in the arch :

$$M^a = X_1M_1^a + X_2M_2^a + \sum_{i=3}^{n} \varDelta X_iM_i^a \quad \ldots \quad (12)$$

Since $X_3(0)$ and the force increments $\varDelta X_3 \varDelta X_4 \ldots \varDelta X_n$ are known, all the unknowns $X_3, X_4, \ldots X_n$ can be obtained by equations (10).

Solution of Bowstring Arch having Six Suspension Rods

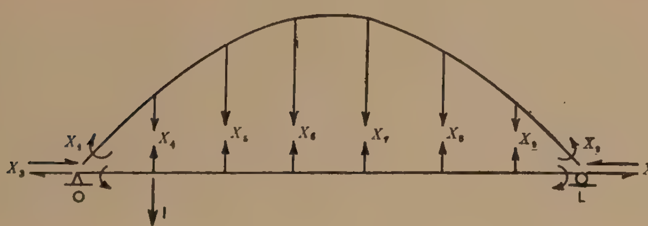
To illustrate the method, it is proposed to consider the symmetrical bowstring arch having six suspension rods (*Fig. 5*) and carrying a unit concentrated load as shown. The modified elastic equations for this system written according to equations (11) are as given in Table 1.

TABLE 1.—EQUATIONS (13)

	X_1	X_2	ΔX_3	ΔX_4	ΔX_5		ΔX_8	ΔX_9	
i	δ_{11}	δ_{21}	δ_{31}	δ_{41}	δ_{51}		δ_{81}	δ_{91}	$-\delta_{01}-\sum_{i=3}^{9} X_i(0)\delta_{i1}$
ii	δ_{12}	δ_{22}	δ_{32}	δ_{42}	δ_{52}		δ_{82}	δ_{92}	$-\delta_{02}-\sum_{i=3}^{9} X_i(0)\delta_{i2}$
iii	δ_{13}	δ_{23}	δ_{33}	δ_{43}	δ_{53}		δ_{83}	δ_{93}	$-\delta_{03}-\sum_{i=3}^{9} X_i(0)\delta_{i3}$
iv	δ_{14}	δ_{24}	δ_{34}	δ_{44}	δ_{54}		δ_{84}	δ_{94}	$-\delta_{04}-\sum_{i=3}^{9} X_i(0)\delta_{i4}$
v	δ_{15}	δ_{25}	δ_{35}	δ_{45}	δ_{55}		δ_{85}	δ_{95}	$-\delta_{05}-\sum_{i=3}^{9} X_i(0)\delta_{i5}$
viii	δ_{18}	δ_{28}	δ_{38}	δ_{48}	δ_{58}		δ_{88}	δ_{98}	$-\delta_{08}-\sum_{i=3}^{9} X_i(0)\delta_{i8}$
ix	δ_{19}	δ_{29}	δ_{39}	δ_{49}	δ_{59}		δ_{89}	δ_{99}	$-\delta_{09}-\sum_{i=3}^{9} X_i(0)\delta_{i9}$

Since the structure is symmetrical ($\delta_{11}=\delta_{22}$; $\delta_{44}=\delta_{99}$; $\delta_{55}=\delta_{88}$; etc.), the work can be simplified by splitting the equations into two groups by

Fig. 5



addition and subtraction, and solving the two groups separately. Thus by addition and $\delta_{ik}=\delta_{ki}$, the first group of equations are formed as shown in Table 2 ; and by subtraction, the second group of equations are formed as shown in Table 3.

TABLE 2.—EQUATIONS (14)

	(13)	X_1+X_2	ΔX_3	$\Delta X_4+\Delta X_9$	$\Delta X_5+\Delta X_8$	$\Delta X_6+\Delta X_7$	
i	i+ii	$\delta_{11}+\delta_{12}$	$\delta_{31}+\delta_{32}$	$\delta_{41}+\delta_{42}$	$\delta_{51}+\delta_{52}$	$\delta_{61}+\delta_{62}$	$-(\delta_{01}+\delta_{02})$ $-\sum_{i=3}^{9} X_i(0)[\delta_{i1}+\delta_{i2}]$
ii	iii	δ_{13}	δ_{33}	δ_{43}	δ_{53}	δ_{63}	$-\delta_{03}-\sum_{i=3}^{9} X_i(0)\delta_{i3}$
iii	iv + ix	$\delta_{14}+\delta_{19}$	$\delta_{34}+\delta_{39}$	$\delta_{44}+\delta_{49}$	$\delta_{54}+\delta_{59}$	$\delta_{64}+\delta_{69}$	$-(\delta_{04}+\delta_{09})$ $-\sum_{i=3}^{9} X_i(0)[\delta_{i4}+\delta_{i9}]$
iv	v+viii	$\delta_{15}+\delta_{18}$	$\delta_{35}+\delta_{38}$	$\delta_{45}+\delta_{48}$	$\delta_{55}+\delta_{58}$	$\delta_{65}+\delta_{68}$	$-(\delta_{05}+\delta_{08})$ $-\sum_{i=3}^{9} X_i(0)[\delta_{i5}+\delta_{i8}]$
v	vi+vii	$\delta_{16}+\delta_{17}$	$\delta_{36}+\delta_{37}$	$\delta_{46}+\delta_{47}$	$\delta_{56}+\delta_{57}$	$\delta_{66}+\delta_{67}$	$-(\delta_{06}+\delta_{07})$ $-\sum_{i=3}^{9} X_i(0)[\delta_{i6}+\delta_{i7}]$

TABLE 3.—EQUATIONS (15)

	(13)	X_1-X_2	$\Delta X_4-\Delta X_9$	$\Delta X_5-\Delta X_8$	$\Delta X_6-\Delta X_7$	
i	i − ii	$\delta_{11}-\delta_{12}$	$\delta_{41}-\delta_{42}$	$\delta_{51}-\delta_{52}$	$\delta_{61}-\delta_{62}$	$-(\delta_{01}-\delta_{02})$
ii	iv − ix	$\delta_{14}-\delta_{19}$	$\delta_{44}-\delta_{49}$	$\delta_{54}-\delta_{59}$	$\delta_{64}-\delta_{69}$	$-(\delta_{04}-\delta_{09})$
iii	v − viii	$\delta_{15}-\delta_{18}$	$\delta_{45}-\delta_{48}$	$\delta_{55}-\delta_{58}$	$\delta_{65}-\delta_{68}$	$-(\delta_{05}-\delta_{08})$
iv	vi − vii	$\delta_{16}-\delta_{17}$	$\delta_{46}-\delta_{47}$	$\delta_{56}-\delta_{57}$	$\delta_{66}-\delta_{67}$	$-(\delta_{06}-\delta_{07})$

Then (X_1+X_2), ΔX_3, $(\Delta X_4+\Delta X_9)$, $(\Delta X_5+\Delta X_8)$, $(\Delta X_6+\Delta X_7)$, the solution of the first group of equations, together with (X_1-X_2), $(\Delta X_4-\Delta X_9)$, $(\Delta X_5-\Delta X_8)$ $(\Delta X_6-\Delta X_7)$, the solution of the second group, give the values of X_1, X_2, $\Delta X_3, \ldots \Delta X_9$, the unknowns of the modified equations (11).

Assuming the suspension rods to be inextensible, and that the second moments of area of the girder and the arch are given respectively by :

$$I_g = mI_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$\text{and } I_a = I_c \sec\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

it can be shown that the coefficients $\delta_{01}, \delta_{02}, \ldots \delta_{09}$, calculated from the virtual work due to this external load, are $\dfrac{1}{m+1}$ times the corresponding

coefficients δ_{41}, δ_{42}, . . . δ_{49}, which are the work done by unit suspension-rod forces in the whole system of the structure.

In such cases, equations (15) have the simple solution :

$$\varDelta X_4 - \varDelta X_9 = \frac{1}{m+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

all the rest = 0.

Similarly, when unit load is applied to the girder at the joint of the second suspension rod from the left :

$$\varDelta X_5 - \varDelta X_8 = \frac{1}{m+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

all the rest = 0.

And if unit load is applied to the girder at the joint of the third suspension rod :

$$\varDelta X_6 - \varDelta X_7 = \frac{1}{m+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

all the rest = 0.

Extension of Suspension Rods

The energy stored in the suspension rods is small compared with that due to bending moments and axial forces in the remainder of the structure. In equations (14) (Table 2), the coefficients δ_{44}, δ_{49}, δ_{55}, δ_{58}, δ_{66}, δ_{67} are all large, and only δ_{44}, δ_{55}, and δ_{66} are affected by extension of the suspension rods. This effect is small and equations (14) therefore give approximately the same solution for rigid and extensible suspension rods. In equations (15) (Table 3), however, differences of the coefficients are used and a small change in δ_{44}, δ_{55}, and δ_{66} can make appreciable difference in the solution. Since, as already described, equations (15) give a simple solution for the case of rigid suspension rods, this solution can be used as the " null " system and increment transformations applied to the equations as follows.

The expressions $(X_1 - X_2)$, $(\varDelta X_4 - \varDelta X_9)$, $(\varDelta X_5 - \varDelta X_8)$, and $(\varDelta X_6 - \varDelta X_7)$ in the equations (15) can be denoted for brevity by T_1, T_2, T_3, and T_4 respectively, and $T_1(0)$, $T_2(0)$, $T_3(0)$, and $T_4(0)$ can denote the solution for the case of rigid rods. Now let :

$$\left.\begin{aligned} T_1 &= T_1(0) + \varDelta T_1 \\ T_2 &= T_2(0) + \varDelta T_2 \\ T_3 &= T_3(0) + \varDelta T_3 \\ T_4 &= T_4(0) + \varDelta T_4 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where $\varDelta T_1$, $\varDelta T_2$, $\varDelta T_3$, and $\varDelta T_4$ represent the influence of the suspension rods on the unknowns.

Also let $\delta^R_{44}, \delta^R_{55}, \ldots \delta^R_{99}$ denote the coefficients for the case of rigid rods, $\delta^R_{44} + \epsilon_{44}, \delta^R_{55} + \epsilon_{55}, \ldots \delta^R_{99} + \epsilon_{99}$ the coefficients, including the energy in the rods. Then for the case of unit load applied to the girder at the joint A of the first suspension rod from the left, equations (15) after substitution become those shown in Table 4. The solution of these transformed equations gives directly the influence of the elongation of the suspension rods.

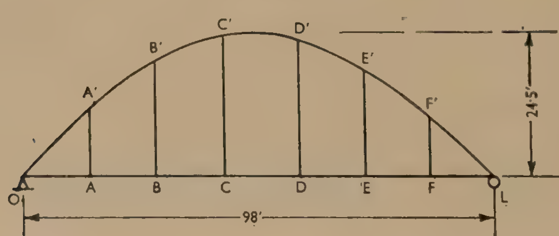
TABLE 4.—EQUATIONS (22)

	ΔT_1	ΔT_2	ΔT_3	ΔT_4	
i	$\delta_{11}-\delta_{12}$	$\delta_{41}-\delta_{42}$	$\delta_{51}-\delta_{52}$	$\delta_{61}-\delta_{62}$	0
ii	$\delta_{14}-\delta_{19}$	$(\delta^R_{44}-\delta_{49})-\epsilon_{44}$	$\delta_{54}-\delta_{59}$	$\delta_{64}-\delta_{69}$	$-\epsilon_{44}\left(\frac{1}{m+1}\right)$
iii	$\delta_{15}-\delta_{18}$	$\delta_{45}-\delta_{48}$	$(\delta^R_{55}-\delta_{58})+\epsilon_{55}$	$\delta_{65}-\delta_{68}$	0
iv	$\delta_{16}-\delta_{17}$	$\delta_{46}-\delta_{47}$	$\delta_{56}-\delta_{57}$	$(\delta^R_{66}-\delta_{67})+\epsilon_{66}$	0

Numerical Solution of a Particular Bowstring Arch

The effect of extension of the suspension rods cannot be seen from equations (14) and (15), and the following arithmetical solution is worked out as an illustration.

Fig. 6



The arch is of reinforced concrete, parabolic in form, and has the dimensions shown in *Fig. 6*. The second moment of area of the arch I_a and the cross-sectional area A_a at any section vary according to the equations $I_a = I_c \sec \phi$ and $A_a = A_c \sec \phi$, where I_c and A_c denote the second moment of area of the arch and the cross-sectional area of the arch at the crown, and are respectively $\frac{4}{3}$ ft^4 and 4 ft^2. The reinforced-concrete tie-girder has a cross-sectional area A_g of 10 ft^2 (equivalent

concrete section). Suspension rods are each of two rods of $1\frac{1}{2}$-inch diameter at 14-foot centres. Their lengths are 12, 20, 24, 24, 20, and 12 feet.

Calculations are given for a ratio of second moment of area of girder to arch of 5, but results are also given for ratios of 10 and 15. The steel-concrete modular ratio is 15 and the modulus of elasticity of steel is 30,000,000 lb. per square inch. Assuming no lack of fit in the structure, and that the deflexion of the girder is so small in comparison with the span length that the bending effect of the horizontal force in the girder can be neglected, it can be written :

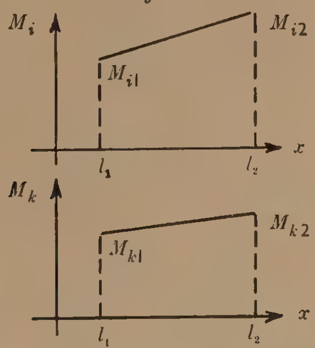
$$\delta_{ik} = \int_0^l M_i M_k \frac{ds}{EI} + \int_0^l F_i F_k \frac{ds}{EA} \quad . \quad . \quad . \quad . \quad (23)$$

Now let $\bar{\delta}_{ik} = E_c I_c \delta_{ik}$

$$\text{gives } \bar{\delta}'_{ik} = \int_0^l M_i M_k \frac{E_c I_c}{EI}\, ds + \int_0^l F_i F_k \frac{E_c I_c}{EI}\, ds$$

It is also assumed that the arch is straight and of constant second

Figs 7



moment of area between consecutive suspension rods, so that the coefficient can be calculated by applying the trapezoidal rule of integration.

If M_{i1} M_{i2} (*Figs 7*) is the diagram of the bending moment M_i in the part of the arch $l_1 l_2$ between consecutive suspension rods, and $M_{k1} M_{k2}$ is the corresponding diagram of bending moment M_k in $l_1 l_2$, then :

$$\int_{l_1}^{l_2} M_i M_k dx = \left(\frac{l_2 - l_1}{6}\right)[M_{i1}(2M_{k1} + M_{k2}) + M_{i2}(2M_{k2} + M_{k1})] \quad (24)$$

Table 5 shows the internal forces and bending moments successively produced in the arch and in the girder by all the moments and redundant forces $X_1, X_2 \ldots X_n$.

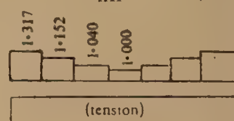
The influence coefficients are calculated from the $M_i M_k$ and $F_i F_k$ diagram in Table 5, using equation (24).

$$\bar{\delta}_{11} = + \left(1 + \frac{1}{m}\right)\frac{98}{3} = 32{\cdot}667\,\frac{m+1}{m}$$

$$\bar{\delta}_{33} = \left\{ + \frac{14}{3} [12(24 + 44) + 20(52 + 64) + 24(68 + 72)] \right.$$
$$\left. + \frac{4}{3} \times \frac{1}{4}[28(1{\cdot}317^2 + 1{\cdot}152^2 + 1{\cdot}040^2) + 14] + \frac{4}{3} \times \frac{1}{10} \times 98 \right\}$$
$$= 30{,}315 + 43 + 13$$
$$= + 30{,}371$$

The other coefficients are calculated similarly and are shown in Table 6. All values of δ_{ik} should be multiplied by $(m + 1)$. Where two coefficients are shown, the smaller is the correction for extensible suspension rods.

TABLE 5

Values of X in *Fig. 5*	Bending moments (M_i or M_k diagrams)	Axial force (F_i or F_k diagrams)
$X_1 = 1$ lb.-ft : arch		nil
girder		nil
$X_2 = 1$ lb.-ft : arch		nil
girder		nil
$X_3 = 1$ lb. : arch		
girder	nil	
$X_4 = 1$ lb. : arch		small and neglected
girder		small and neglected
rods	nil	$X_4 = -1$; others $= 0$
$X_5 = 1$ lb. : arch		small and neglected
girder		small and neglected
rods	nil	$X_5 = -1$; others $= 0$
$X_6 = 1$ lb. : arch		small and neglected
girder		small and neglected
rods	nil	$X_6 = -1$; others $= 0$

Coefficient index k	$\bar{\delta}_{1k}$	$\bar{\delta}_{2k}$	$\bar{\delta}_{3k}$	$\bar{\delta}_{4k}$	$\bar{\delta}_{5k}$	$\bar{\delta}_{6k}$	$\bar{\delta}_{7k}$	$\bar{\delta}_{8k}$	$\bar{\delta}_{9k}$
1	$+32{\cdot}667$	$+16{\cdot}333$	$-\frac{784m}{m+1}$	$+364$	$+560$	$+616$	$+560$	$+420$	$+224$
2	$+16{\cdot}333$	$+32{\cdot}667$	$-\frac{784m}{m+1}$	$+224$	$+420$	$+560$	$+616$	$+560$	$+364$
3	$-\frac{784m}{m+1}$	$-\frac{784m}{m+1}$	$+\frac{30{,}371m}{m+1}$	$-\frac{10{,}584m}{m+1}$	$-\frac{18{,}947m}{m+1}$	$-\frac{23{,}520m}{m+1}$	$-\frac{23{,}520m}{m+1}$	$-\frac{18{,}947m}{m+1}$	$-\frac{10{,}584m}{m+1}$
4	$+364$	$+224$	$-\frac{10{,}584m}{m+1}$	$+4{,}704 + \frac{65m}{m+1}$	$+7{,}513$	$+8{,}363$	$+7{,}644$	$+5{,}749$	$+3{,}071$
5	$+560$	$+420$	$-\frac{18{,}947m}{m+1}$	$+7{,}513$	$+13{,}067 + \frac{108m}{m+1}$	$+15{,}157$	$+14{,}112$	$+10{,}715$	$+5{,}749$
6	$+616$	$+560$	$-\frac{23{,}520m}{m+1}$	$+8{,}363$	$+15{,}157$	$+18{,}816 + \frac{130m}{m+1}$	$+18{,}228$	$+14{,}112$	$+7{,}644$
7	$+560$	$+616$	$-\frac{23{,}520m}{m+1}$	$+7{,}644$	$+14{,}112$	$+18{,}228$	$+18{,}816 + \frac{130m}{m+1}$	$+15{,}157$	$+8{,}363$
8	$+420$	$+560$	$-\frac{18{,}947m}{m+1}$	$+5{,}749$	$+10{,}715$	$+14{,}112$	$+15{,}157$	$+13{,}067 + \frac{108m}{m+1}$	$+7{,}513$
9	$+224$	$+364$	$-\frac{10{,}584m}{m+1}$	$+3{,}071$	$+5{,}749$	$+7{,}644$	$+8{,}363$	$+7{,}513$	$+4{,}704 + \frac{65m}{m+1}$

To obtain the load coefficients for a concentrated load on the girder, a unit load is applied successively at the joint of the suspension rods, A to F (*Fig. 8*), and the coefficients are calculated from the M_0M_k and F_0F_k diagrams and equations (3), giving the values shown in Table 7.

Fig. 8

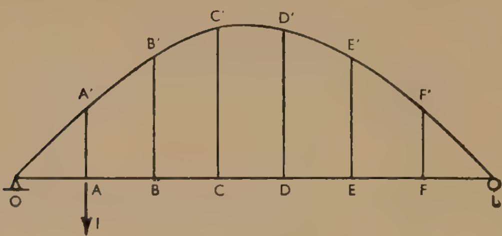


TABLE 7

Unit load at	$m\bar{\delta}_{01}$	$m\bar{\delta}_{02}$	$m\bar{\delta}_{03}$	$m\bar{\delta}_{04}$	$m\bar{\delta}_{05}$	$m\bar{\delta}_{06}$
A	364	224	0	4,704	7,513	8,363

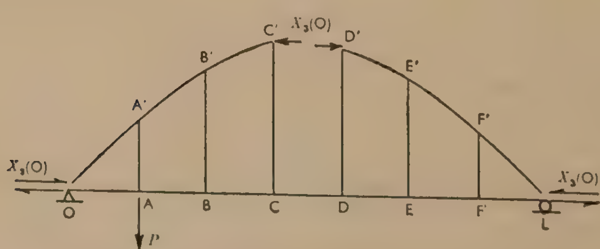
Calculation of " Null " System

The relation between the horizontal force $X_3(0)$ and the suspension-rod forces $X_4(0) \ldots X_9(0)$ in the " null " system (the bowstring arch having no bending moment in the arch rib) is given by equations (7) and (8). For the arch of parabolic form, the horizontal force produces equal suspension-rod forces, that is :

$$X_4(0) = X_5(0) = \ldots X_9(0) = aX_3(0)$$

where $$a = \tan \phi_1 - \tan \phi_2 = \frac{2}{7}$$

Fig. 9

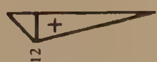


The value of $X_3(0)$ (see *Fig. 9*), given by equation (9), is :

$$X_3(0) = -\frac{\delta'_{03}}{\delta'_{33}}$$

For the calculation of $X_3(0)$, the diagrams of the bending moments and forces produced in the arch and the girder by $X_3(0)$ and the external load P at A are as shown in Table 8.

TABLE 8

	Bending moment	Axial force
$X_3(O) = 1$ produces in girder		− 1·000 (tension)
" " arch	nil	
" " rods	nil	neglected
unit load at A " " girder		nil
" " arch	nil	nil
" " rods	nil	nil

$\delta'_{33} = +\dfrac{30{,}315}{m}$ (disregarding the terms due to direct forces, on the assumption that there is no displacement in the arch).

For unit load at A $\quad \delta'_{03} = -\dfrac{10{,}584}{m}$

$$X_3(0) = 0{\cdot}34913$$

Equations (14) become as shown in Table 9, for this case with $m = 5$.

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TABLE 9

$X_1 + X_2$	ΔX_3	$\Delta X_4 + \Delta X_9$	$\Delta X_5 + \Delta X_8$	$\Delta X_6 + \Delta X_7$	
+ 49	− 1,307	+ 588	+ 980	+ 1,176	+ 6·761
− 784	+ 30,371	− 10,584	− 18,947	− 23,520	− 19·551
+ 588	− 17,640	+ 7,829	+ 13,263	+ 16,007	+ 53·234
+ 980	− 31,578	+ 13,263	+ 23,872	+ 29,269	− 12·504
+ 1,176	− 39,200	+ 16,007	+ 29,269	+ 37,153	− 81·021

The solutions are :

$$\left.\begin{aligned} 1{,}000\,(X_1 + X_2) &= +\ 330{\cdot}01 \\ 1{,}000\ \ \Delta X_3 &= -\ \ \ \ 6{\cdot}00 \\ 1{,}000\,(\Delta X_4 + \Delta X_9) &= +\ \ 80{\cdot}84 \\ 1{,}000\,(\Delta X_5 + \Delta X_8) &= -\ \ 28{\cdot}64 \\ 1{,}000\,(\Delta X_6 + \Delta X_7) &= -\ \ 31{\cdot}22 \end{aligned}\right\} \quad . \quad . \quad . \quad (25)$$

By a similar procedure, equations (15), for inextensible suspension rods, take the form shown in Table 10.

TABLE 10

$X_1 - X_2$	$\Delta X_4 - \Delta X_9$	$\Delta X_5 - \Delta X_8$	$\Delta X_6 - \Delta X_7$	
+ 16·33	+ 140	+ 140	+ 56	$+\dfrac{140}{m+1}$
+ 140	+ 1,633	+ 1,764	+ 719	$+\dfrac{1{,}633}{m+1}$
+ 140	+ 1,764	+ 2,352	+ 1,045	$+\dfrac{1{,}764}{m+1}$
+ 56	+ 719	+ 1,045	+ 588	$+\dfrac{719}{m+1}$

The solution is :

$$\Delta X_4 - \Delta X_9 = \frac{1}{m+1},\ \text{the rest} = 0 \quad . \quad . \quad . \quad (26)$$

For extensible suspension rods, equations (15) become as shown in Table 11.

TABLE 11

$X_1 - X_2$	$\Delta X_4 - \Delta X_9$	$\Delta X_5 - \Delta X_8$	$\Delta X_6 - \Delta X_7$	
+ 16·33	+ 140	+ 140	+ 56	$+\frac{140}{m+1}$
+ 140·00	$+1{,}633+65\frac{m}{m+1}$	+ 1,764	+ 719	$+\frac{1{,}633}{m+1}$
+ 140·00	+ 1,764	$+2{,}352+108\frac{m}{m+1}$	+ 1,045	$+\frac{1{,}764}{m+1}$
+ 56·00	+ 719	+ 1,045	$+588+130\frac{m}{m+1}$	$+\frac{719}{m+1}$

This transformed in terms of ΔT, according to equations (22 i) to (22 iv) (Table 4) takes the form shown in Table 12.

TABLE 12

ΔT_1	ΔT_2	ΔT_3	ΔT_4	
$+16{\cdot}33\frac{m+1}{m}$	$+140\frac{m+1}{m}$	$+140\frac{m+1}{m}$	$+56\frac{m+1}{m}$	0
$+140\frac{m+1}{m}$	$+1{,}633\frac{m+1}{m}$ $+65$	$+1{,}764\frac{m+1}{m}$	$+719\frac{m+1}{m}$	$-\frac{65}{m+1}$
$+140\frac{m+1}{m}$	$+1{,}764\frac{m+1}{m}$	$+2{,}352\frac{m+1}{m}$ $+108$	$+1{,}045\frac{m+1}{m}$	0
$+56\frac{m+1}{m}$	$+719\frac{m+1}{m}$	$+1{,}045\frac{m+1}{m}$	$+588\frac{m+1}{m}$ $+130$	0

As before, these equations solved for $m = 5$ become as shown in Table 13.

TABLE 13

ΔT_1	ΔT_2	ΔT_3	ΔT_4	
+ 16·33	+ 140	+ 140	+ 56	0
+ 140	+ 1,687	+ 1,764	+ 719	$-\frac{5}{36}\times 65$
+ 140	+ 1,764	+ 2,442	+ 1,045	0
+ 56	+ 719	+ 1,045	+ 697	0

The solution is :

$$\begin{aligned} 1{,}000\,\Delta T_1 &= +\ 182{\cdot}65 \\ 1{,}000\,\Delta T_2 &= -\ \ 39{\cdot}31 \\ 1{,}000\,\Delta T_3 &= +\ \ 19{\cdot}12 \\ 1{,}000\,\Delta T_4 &= -\ \ \ 2{\cdot}79 \end{aligned}$$

The solutions of equations (15) for extensible rods are obtained by superposing the solutions for the system with inextensible rods on the solutions of the ΔT equations given above, and give, for $m = 5$:

$$\left.\begin{aligned} 1{,}000\,(X_1 - X_2) &= +\ 182{\cdot}65 \\ 1{,}000\,(\Delta X_4 - \Delta X_9) &= +\ 127{\cdot}36 \\ 1{,}000\,(\Delta X_5 - \Delta X_8) &= +\ \ 19{\cdot}12 \\ 1{,}000\,(\Delta X_6 - \Delta X_7) &= -\ \ \ 2{\cdot}79 \end{aligned}\right\} \quad . \quad . \quad (27)$$

Solutions of the Transformed Equations of Force Increments

By successive addition and subtraction of the results (25), (26), and (27) from the first and second groups of equations, the values of the force increments for cases of extensible and inextensible suspension rods are found, as shown below for load at position A.

TABLE 14

	Inextensible rods	Extensible rods
$1{,}000\,X_1$	+ 165·01	+ 256·33
$1{,}000\,X_2$	+ 165·01	+ 73·68
$1{,}000\,\Delta X_3$	− 6·00	− 6·00
$1{,}000\,\Delta X_4$	+ 123·76	+ 104·10
$1{,}000\,\Delta X_5$	− 14·32	− 4·76
$1{,}000\,\Delta X_6$	− 15·61	− 17·01
$1{,}000\,\Delta X_7$	− 15·61	− 14·22
$1{,}000\,\Delta X_8$	− 14·32	− 23·88
$1{,}000\,\Delta X_9$	− 42·92	− 23·26

Forces and Moments in the Structure

The horizontal and suspension-rod forces are calculated according to equation (10), as follows :

$$X_3 = X_3(0) + \Delta X_3,\ X_4 = X_4(0) + \Delta X_4, \text{ etc.}$$

and for load case A and $m = 5$ give the values shown in Table 15.

TABLE 15

	Horizontal force	Suspension-rod forces					
Inextensible suspension rods	0·3431	0·2235	0·0854	0·0841	0·0841	0·0854	0·0568
Extensible suspension rods	0·3431	0·2038	0·0950	0·0827	0·0855	0·0759	0·0765

The axial forces in the arch are calculated from the expression (*Fig. 3*, p. 519).

$$Q = X_3 \sec \phi = [X_3(0) + \Delta X_3] \sec \phi$$

and are as shown in Table 16.

TABLE 16

Section	O	A′	B′	C′
Axial force Q	0·4851	0·4217	0·3733	0·3465

Axial forces are practically unaffected by the extensibility of suspension rods.

The bending moments in the arch are given by equation (12), as follows :

$$M^{(a)} = X_1 M_1^{(a)} + X_2 M_2^{(a)} + \sum_{i=3}^{9} \Delta X_i M_i^{(a)} \quad . \quad . \quad . \quad . \quad (28)$$

The bending moments in the tie-girder are given by the equation :

$$M^{(g)} = M_0^{(g)} + X_1 M_1^{(g)} + X_2 M_2^{(g)} + X_3(0) M_3^{(a)} + \sum_{i=4}^{9} \Delta X_i M_i^{(g)} \quad (29)$$

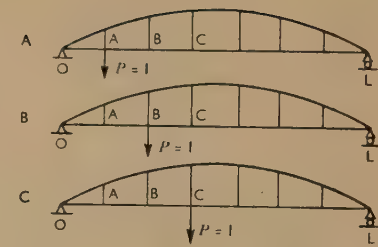
The bending moments in the arch and girder, and the effect of extensibility of suspension rods are shown in *Figs 10*.

Part 2.—The Simplified Method of Analysis

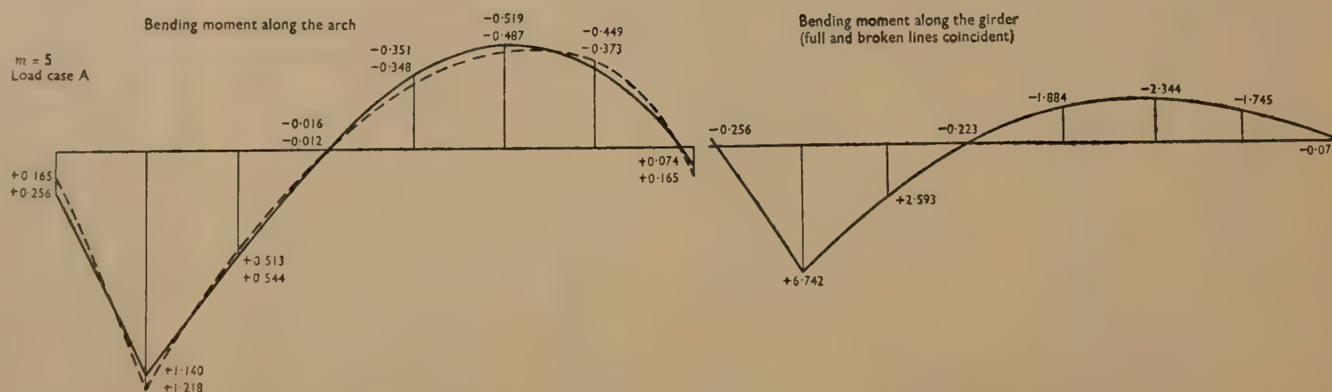
In practice, the bowstring arch problem is usually simplified by first assuming that the second moment of area of the arch is so small compared to that of the girder that it can be ignored, and that the suspension rods are inextensible. Hence, only axial force exists in the arch and every joint of the arch can be assumed pin-connected. By making a cut

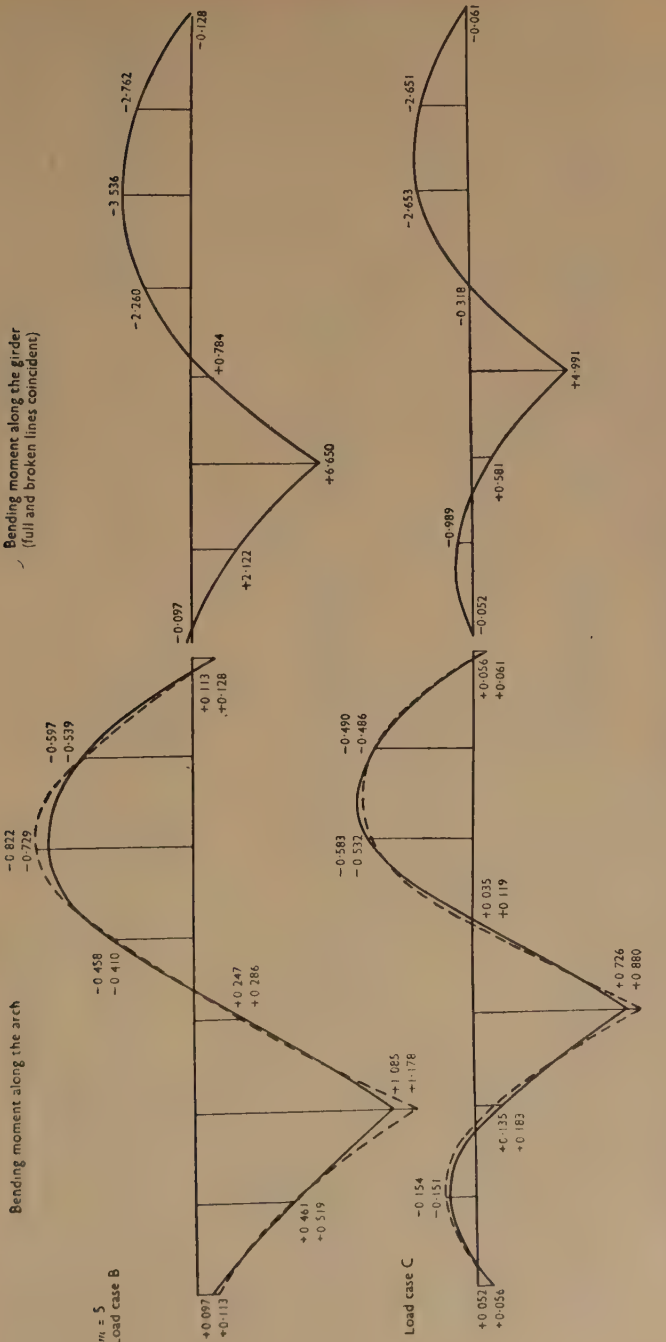
Figs 10

Broken lines show bending moments in the system with inextensible rods.
Full lines show bending moments in the system with extensible rods.
Linear scale : 1 inch = 30 feet
Moment scales : 1 inch = 1 lb.-ft in the arch
1 inch = 10 lb.-ft in the girder
$m = I_g/I_c$ = ratio between second moments of area of girder and arch sections



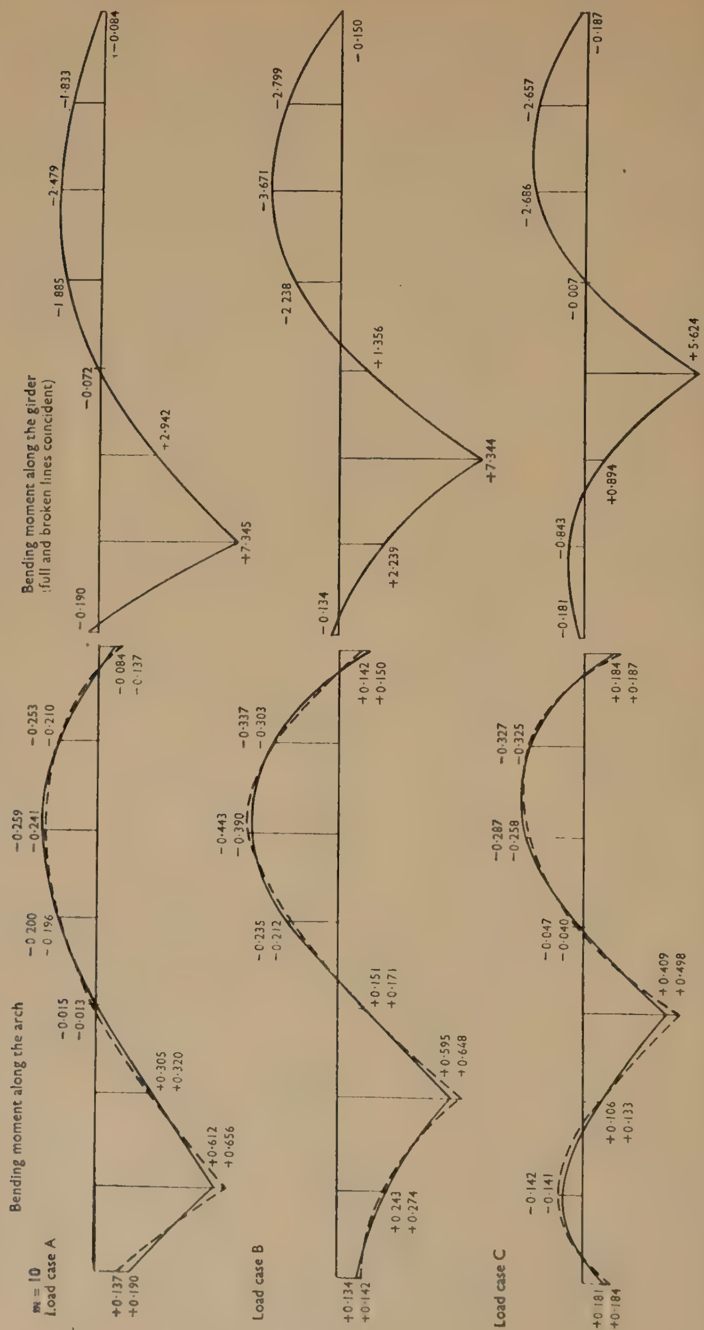
LOAD CASES

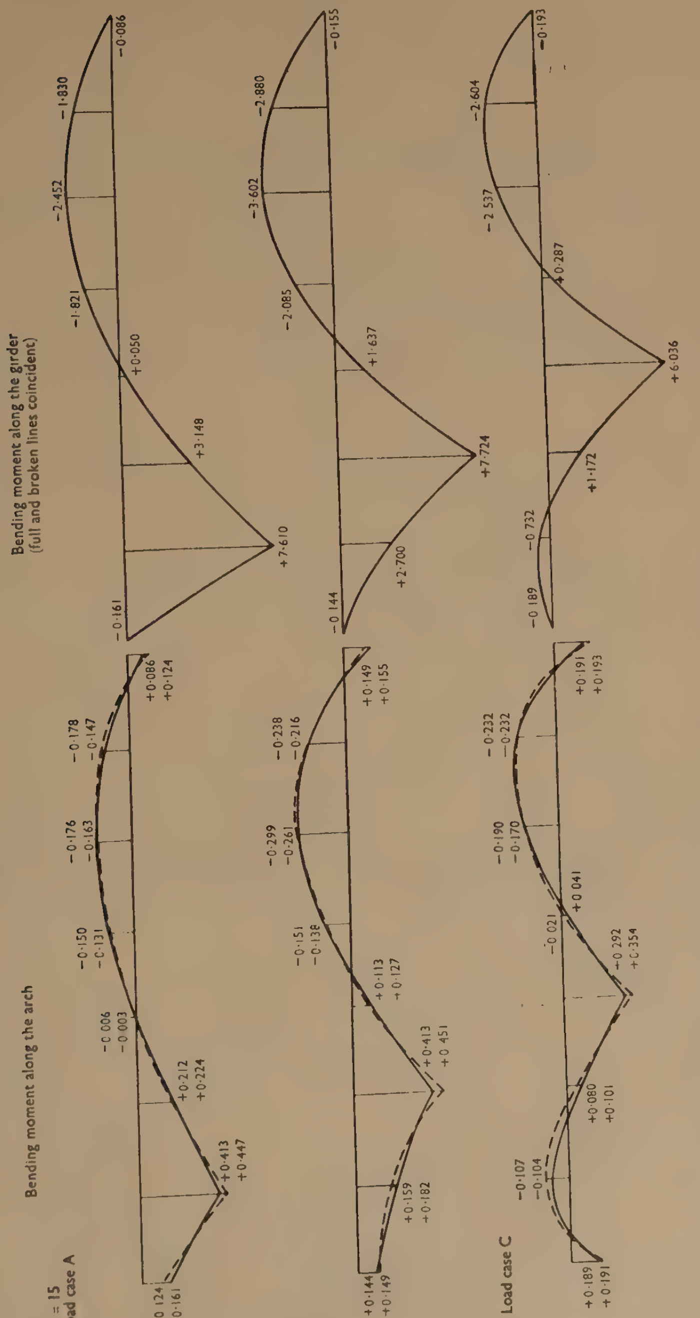




Bending Moments due to Concentrated Load, by Method of Influence Coefficients in the System with Nine Unknowns
(continued on next two pages)

Figs 10 (continued)





BENDING MOMENTS DUE TO CONCENTRATED LOAD, BY METHOD OF INFLUENCE COEFFICIENTS IN THE SYSTEM WITH NINE UNKNOWNS

at the crown and inserting the horizontal force X_3 (*Fig. 11*) as the only unknown on both sides of the cut, it is found that :

$$X_3 = -\frac{\delta_{03}}{\delta_{33}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

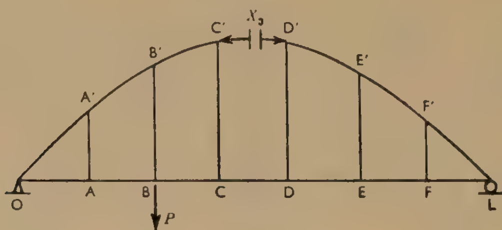
The process is thus similar to that in the analysis of the "null" system, but in this simplified system the energy due to direct forces is also included.

The total bending moment in the bridge is given by :

$$M = M_0^{(g)} - X_3 M_3^{(g)} . \quad . \; . \; . \; . \; . \; . \quad (31)$$

and it is assumed that this moment is taken by the girder and the arch in proportion to their second moments of area.

Fig. 11

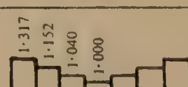


Thus
$$M^{(a)} = \frac{1}{m+1} M \quad . \; . \; . \; . \; . \; . \; . \quad (32)$$

and
$$M^{(g)} = \frac{m}{m+1} M \quad . \; . \; . \; . \; . \; . \; . \quad (33)$$

The coefficients δ_{03} and δ_{33} for the calculation of X_3 from equation (30) can be calculated by the trapezoidal rule of integration as before. The diagrams of M_iM_k and F_iF_k are as shown in Table 17.

TABLE 17

	Bending moments	forces
$X_3 = 1$ produces in the arch	nil	
$X_3 = 1$ „ „ „ girder		+ 1·000 (tension)
Unit load at A „ „ „ „		nil

$$\bar{\delta}_{03} = -\frac{10{,}581}{m}$$

$$\bar{\delta}_{33} = \frac{30{,}315}{m} + 56$$

$$\text{and } X_3 = -\frac{\delta_{03}}{\delta_{33}} = \frac{E_cI_c}{E_cI_c}\frac{\bar{\delta}_{03}}{\bar{\delta}_{33}}$$

and for unit load in position A and $m = 5$:

$$X_3 = \frac{10{,}581}{30{,}595} = 0{\cdot}34584$$

The total bending moment in the bridge is given as before by:

$$M = M_0^{(g)} - X_3M_3^{(g)}$$

and the moments in the arch and girder, being assumed proportional to the second moments of area of the arch and girder respectively, are:

$$M^{(a)} = \frac{1}{m+1}M \quad \text{and} \quad M^{(g)} = \frac{m}{m+1}M$$

and are plotted in *Figs 12*, which comprise diagrams showing values of bending moment at the ends of the arch ignored in the analysis by the orthodox method (system with one unknown). Bending-moment diagrams for point loads and uniformly distributed loading are shown.

Part 3.—Analysis of the Membrane Analogy

From the exact analysis of a bowstring arch having six suspension rods it can be seen in *Figs 10* that elongation of the suspension rods reduces the bending moment in the arch by about 10 per cent. Also, if the suspension rods are inextensible the bending moments at the ends of the arch are equal.

In order to simplify the analysis of the arch by the membrane theory, therefore, the membrane has been assumed to be inextensible so that the deflexions of the arch rib and tie-girder are the same, and end moments in the rib are equal. The charts produced on these assumptions are accurate enough for design purposes.

The arch is assumed to be of parabolic form (*Fig. 13*), having the equation of:

$$y = 4h\left[\frac{x}{l} - \left(\frac{x}{l}\right)^2\right] \qquad \qquad (34)$$

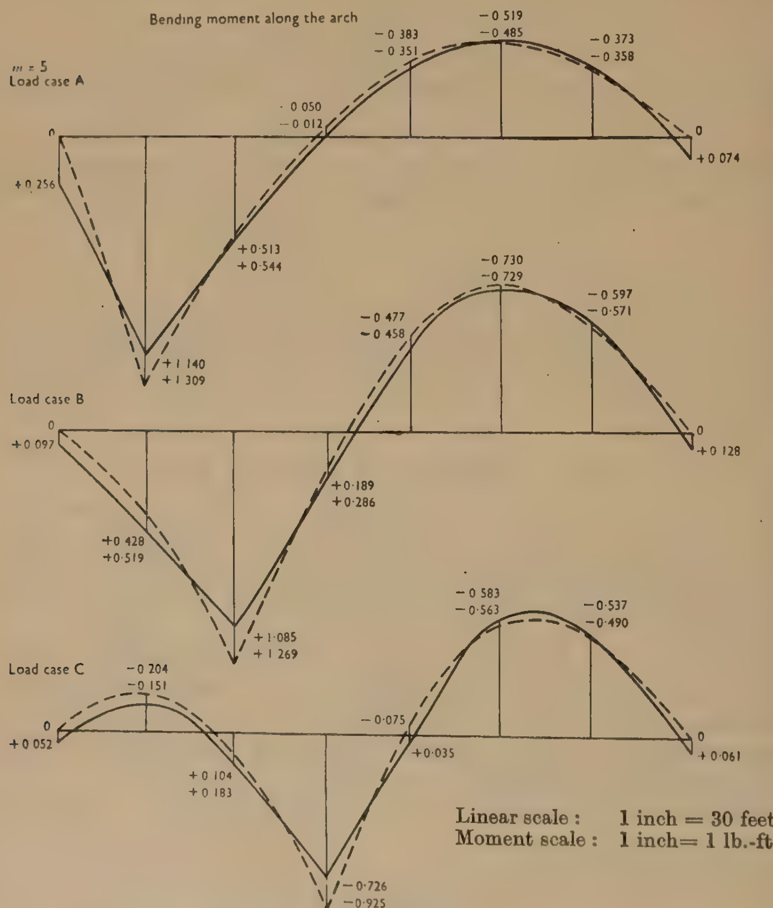
As before, the second moments of area and cross-sectional area of the arch vary according to the equations:

$$I_a = I_c \sec\phi \qquad \qquad (35)$$

$$A_a = A_c \sec\phi \qquad \qquad (36)$$

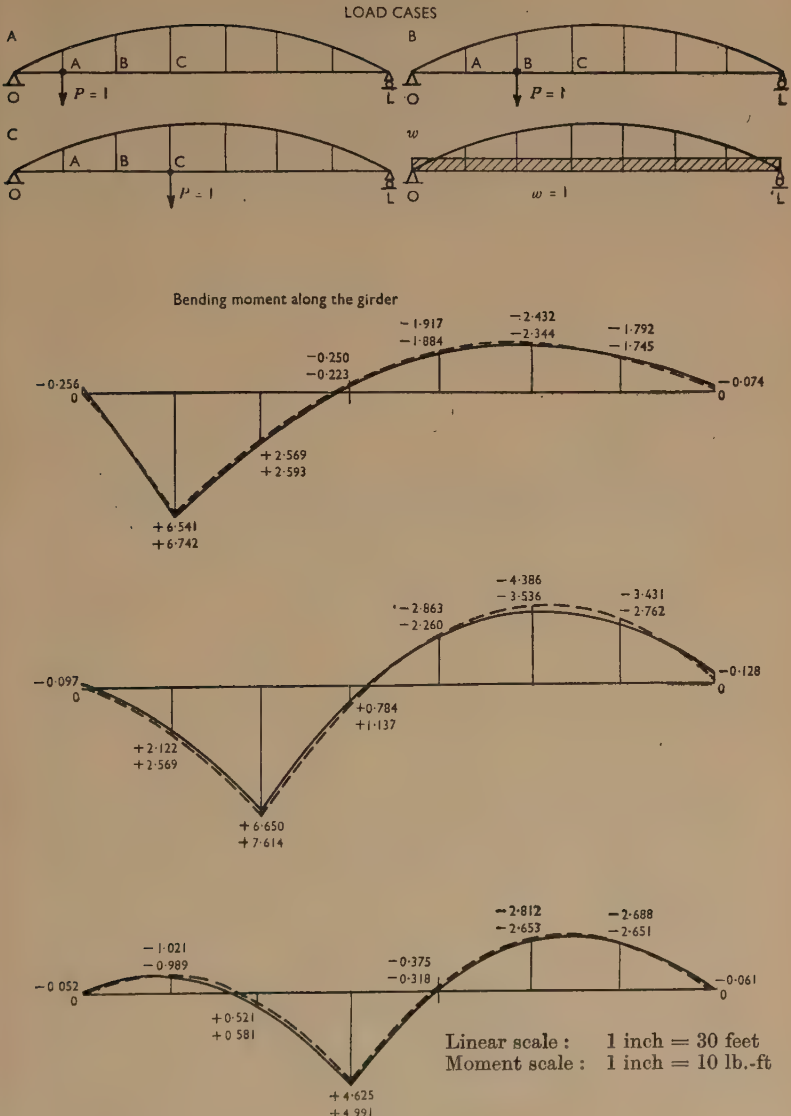
Figs 12

Full lines show bending moments calculated in the system with nine unknowns.
Broken lines show bending moments calculated in the system with one unknown.
$m = I_g/I_c$ = ratio between second moments of area at girder and arch sections



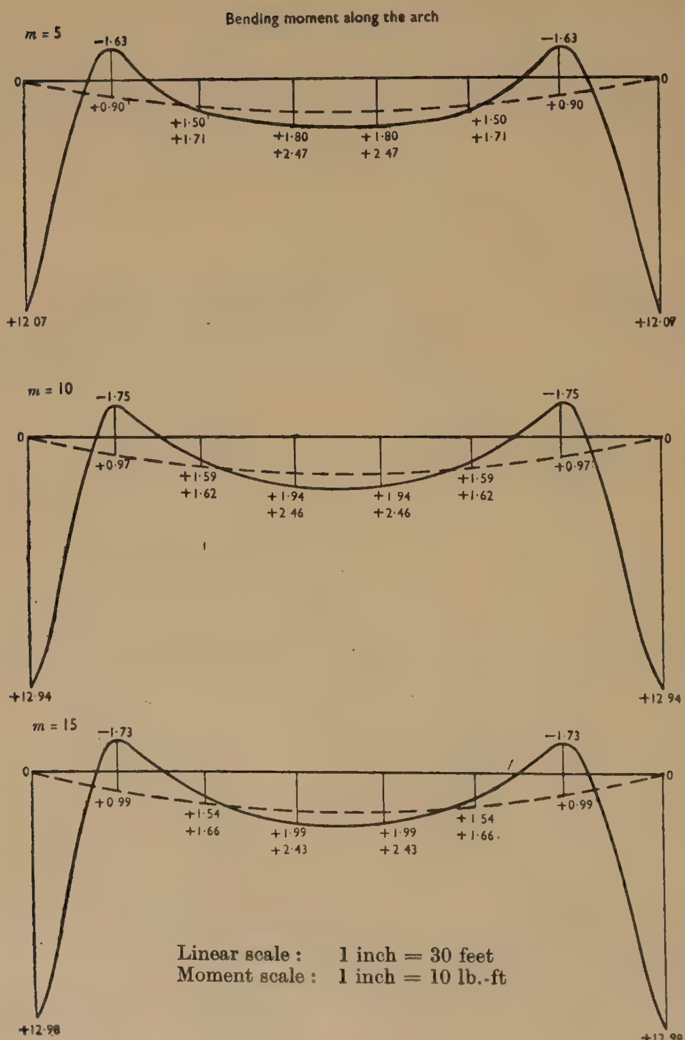
VALUES OF BENDING MOMENT (ARCH) FOR CONCENTRATED LOAD

Figs 12 (continued)



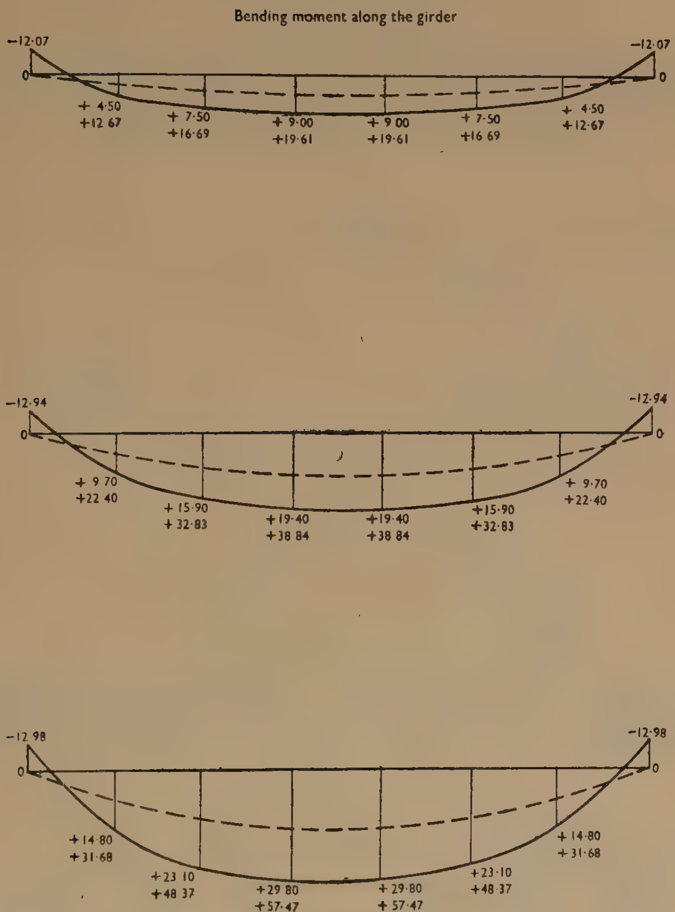
Values of Bending Moment (Girder) for Concentrated Load
(Values for uniform load are shown on the following two pages)

Figs 12 (continued)



VALUES OF BENDING MOMENT FOR UNIFORM LOAD, $w = 1$ LB. PER FOOT

Figs 12 (continued)



Linear scale : 1 inch = 30 feet
Moment scale : 1 inch = 100 lb.-ft

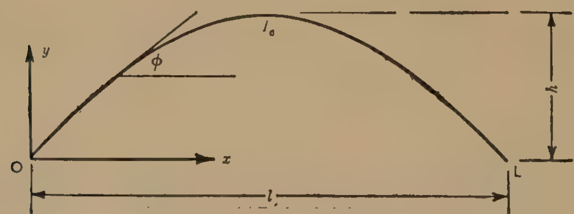
VALUES OF BENDING MOMENT FOR UNIFORM LOAD, $w = 1$ LB. PER FOOT

Now $$\phi = \frac{dy}{dx} = \frac{4h}{l}\left(1 - \frac{2x}{l}\right) \quad . \quad . \quad . \quad . \quad . \qquad (37)$$

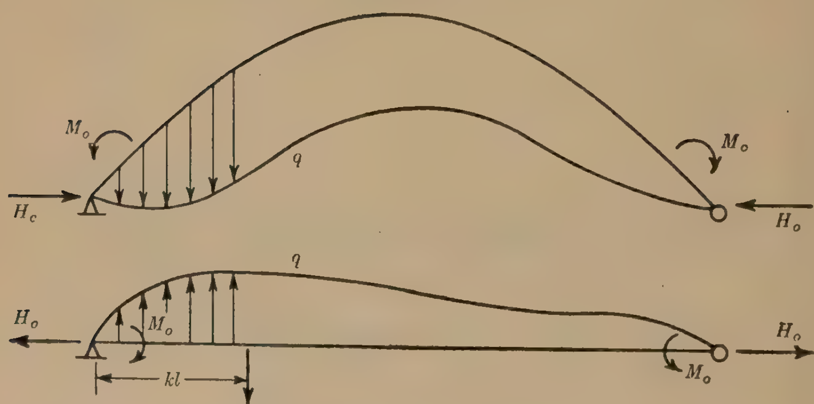
and $$\sec\phi = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{16h^2}{l^2} - \frac{64h^2}{l^2}\frac{x}{l} + \frac{64h^2}{l^2}\left(\frac{x}{l}\right)^2} \qquad (38)$$

In order to analyse the moments and forces in the arch and tie-girder due to a load P at any distance kl from the left end, the arch is separated from the girder by inserting hinges at its ends and making a cut across

Fig. 13



Figs 14



the membrane. Forces and moments are then applied to the hinges and cut as shown in *Figs 14* to restore the equilibrium conditions of the complete structure. The membrane connecting the arch and girder is assumed to function in the same way as an infinite number of suspension rods and carries a vertical force q of varying intensity along the span length.

It is necessary first to consider the effect of q on the two-hinged arch.

The vertical deflexion of the arch shown in *Fig. 15* is given by :

$$q = EI \cos\phi \frac{d^4\Delta_q}{dx^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

or

$$M_q^a = EI \cos\phi \frac{d^2\Delta_q}{dx^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

where Δ_q denotes the vertical deflexion due only to the membrane force q, and $M_q{}^a$ denotes the bending moment due only to q.

Fig. 15

The deflexion produced by the membrane force q can be represented by the Fourier series :

$$\Delta_q = \frac{a_0}{EI_c} + \sum_{n=1}^{\infty} \frac{a_n}{EI_c} \sin \frac{n\pi x}{l} + \sum_{n=1}^{\infty} \frac{b_n}{EI_c} \cos \frac{n\pi x}{l} \quad . \quad (41)$$

where a_0, a_n, and b_n denote constants to be determined.

Considering the end conditions of the arch, $\Delta_q = 0$ at $x = 0$ and $x = l$; also $\frac{d^2}{dx^2}\Delta_q = 0$ at $x = 0$ and $x = l$; the above series reduces to :

$$\Delta_q = \sum_{n=1}^{\infty} \frac{a_n}{EI_c} \sin \frac{n\pi x}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

By successive differentiation of Δ_q and noticing that $I_a = I_c \sec\phi$, the following expressions for the membrane force q and the bending moment M_q are obtained :

$$M_q = -\sum_{n=1}^{\infty} a_n \left(\frac{n\pi}{l}\right)^2 \sin \frac{n\pi x}{l} \quad . \quad . \quad . \quad . \quad . \quad (43)$$

and

$$q = \sum_{n=1}^{\infty} a_n \left(\frac{n\pi}{l}\right)^4 \sin \frac{n\pi x}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Equation (44) indicates that the membrane force can be regarded as being composed of an infinite number of distributed vertical loads whose magnitudes vary sinusoidally along the span length, as shown in *Figs 16* :

$$q = \left(\frac{\pi}{l}\right)^4 a_1 \sin\frac{\pi x}{l} + \left(\frac{2\pi}{l}\right)^4 a_2 \sin\frac{2\pi x}{l} + \left(\frac{3\pi}{l}\right)^4 a_3 \sin\frac{3\pi x}{l} + \ldots \quad (45)$$

The membrane force can be represented by equation (45) to any desired degree of accuracy depending upon the number of terms considered.

Figs 16

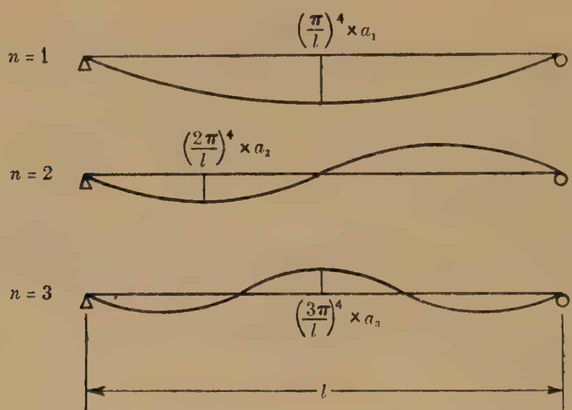
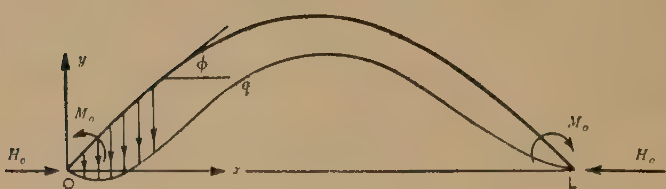


Fig. 17



Since the membrane is assumed to be inextensible, the vertical deflexion of any point on the arch is equal to that of the point on the girder immediately below it, and, in the bowstring-arch rib (*Fig. 17*), is the sum of the deflexions produced by the horizontal force H_0, the end moments, M_0, and the membrane force q, that is :

$$\Delta = \Delta_H + \Delta_M + \Delta_q \quad \ldots \ldots \quad (46)$$

The deflexion due to H_0 is given by :

$$\frac{d^2\Delta_H}{dx^2} = \frac{H_0 y}{EI_c} = \frac{4H_0 h}{EI_c}\left[\frac{x}{l} - \frac{x^2}{l^2}\right]$$

integrating and satisfying the end conditions that $\Delta_H = 0$ at $x = 0$ and $x = l$:

$$\Delta_H = -\frac{H_0 h l^2}{3EI_c}\left[\left(\frac{x}{l}\right)^4 - 2\left(\frac{x}{l}\right)^3 + \left(\frac{x}{l}\right)\right] \quad . \quad . \quad (47)$$

The deflexion due to M_0 is given by:

$$\frac{d^2\Delta_M}{dx^2} = \frac{M_0}{EI_c}$$

which, since there is no deflexion at $x = 0$ and $x = l$ gives, similarly:

$$\Delta_M = -\frac{M_0}{2EI_c}(l - x)\,x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

Hence:

$$\Delta = -\frac{H_0 h l^2}{3EI_c}\left[\left(\frac{x}{l}\right)^4 - 2\left(\frac{x}{l}\right)^3 + \left(\frac{x}{l}\right)\right] - \frac{M_0}{2EI_c}(l - x)x + \sum_{n=1}^{\infty}\frac{a_n}{EI_c}\sin\frac{n\pi x}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

and when $x = kl$, the deflexion Δ_P under load P is:

$$\Delta_P = -\frac{M_0 l^2 k}{2EI_c}(1 - k) - \frac{H_0 h l^2}{3EI_c}(k^4 - 2k^3 + k) + \sum_{n=1}^{\infty}\frac{a_n}{EI_c}\sin n\pi k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

H_0, M_0, and the constants $a_1 \ldots a_n$ remain to be determined.

Since the relative movement on each side of the cut section is zero:

$$\delta_M = \delta_{0M} + M_0\delta_{MM} + H_0\delta_{HM} = 0 \quad . \quad . \quad . \quad (51)$$

and

$$\delta_H = \delta_{0H} + M_0\delta_{MH} + H_0\delta_{HH} = 0 \quad . \quad . \quad . \quad . \quad (52)$$

from which

$$H_0 = \frac{\delta_{0M}\,\delta_{MH} - \delta_{0H}\,\delta_{MM}}{\delta_{HH}\delta_{MM} - \delta_{HM}\,\delta_{MH}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

and

$$M_0 = \frac{\delta_{0H}\,\delta_{HM} - \delta_{0M}\,\delta_{HH}}{\delta_{HH}\,\delta_{MM} - \delta_{HM}\,\delta_{MH}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

$\delta_{0M}\,\delta_{0H}$ denote load coefficients due to the external load P and the membrane force q, and are determined as follows. The bending moments caused by the redundant forces and moments in the reduced determinate system shown in *Figs 14*, p. 544, are as indicated in Table 18.

Moments are positive for tension outside, that is, on the top side of the arch and the underside of the tie girder. The coefficients in equations (53) and (54) are:

$$\bar{\delta}_{MM} = EI_c\,\delta_{MM} = \frac{m+1}{m}\cdot l$$

TABLE 18

Bending moment produced by :	in the arch :	in the girder :
$M_0 = 1$	$+1$	$+1$
$H_0 = 1$	$+y$	nil
$q = \sum_{n=1}^{\infty} a_n \left(\frac{n\pi}{l}\right)^4 \sin \frac{n\pi x}{l}$	$-\sum_{n=1}^{\infty} a_n \left(\frac{n\pi}{l}\right)^2 \sin \frac{n\pi x}{l}$	$-\sum_{n=1}^{\infty} a_n \left(\frac{n\pi}{l}\right)^2 \sin \frac{n\pi x}{l}$
P	nil	$P(1-k)x$ for $0 \leqslant x \leqslant kl$ $Pk(l-x)$ for $kl \leqslant x \leqslant l$

$$\bar{\delta}_{HM} = \bar{\delta}_{MH} = EI_c \, \delta_{HM} = \tfrac{2}{3} hl$$

$$\bar{\delta}_{HH} = EI_c \, \delta_{HH} = \frac{8}{15} h^2 l$$

$$\bar{\delta}_{0H} = EI_c \, \delta_{0H} = -16 \frac{h}{l} \sum_{1.3.5\ldots}^{\infty} a_n \left(\frac{1}{n\pi}\right)$$

$$\bar{\delta}_{0M} = EI_c \left[\delta_{0M}^{q} + \delta_{0M}^{P}\right] = \frac{Pl^2}{2m}(1-k)k - \frac{2}{l}\,\frac{m+1}{m} \sum_{1.3.5\ldots}^{\infty} a_n (n\pi)$$

Substituting in equations (53) and (54) and reducing :

$$H_0 = \frac{15}{4}\,\frac{k(1-k)}{m+6} \cdot \frac{Pl}{h} + \frac{15(m+1)}{m+6} \cdot \frac{l}{h} \sum_{1.3.5\ldots}^{\infty} \frac{a_n}{l^3}\left(\frac{12}{n\pi} - n\pi\right)$$

$$\text{and } M_0 = -\frac{3k(1-k)}{m+6} Pl - \frac{12(m+1)}{m+6}\, l \sum_{1.3.5\ldots}^{\infty} \frac{a_n}{l^3}\left(\frac{10}{n\pi}\,\frac{m}{m+1} - n\pi\right)$$

Now, putting :

$$\frac{15k(1-k)}{4(m+6)}\,\frac{l}{h} = L_h \quad \ldots \ldots \quad (55)$$

$$15\,\frac{m+1}{m+6}\,\frac{l}{h} = A_h \quad \ldots \ldots \quad (56)$$

$$\left(\frac{12}{n\pi} - n\pi\right) = B_h \quad \ldots \ldots \quad (57)$$

$$\frac{3k(1-k)}{m+6}\,\frac{l}{h} = L_m \quad \ldots \ldots \quad (58)$$

$$\frac{12(1+m)}{m+6}\,\frac{l}{h}=A_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

$$\frac{10}{n\pi}\,\frac{m}{m+1}-n\pi=B_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

H_0 and M_0 can be written :

$$H_0=PL_h+A_h\sum_{1.3.5\,\ldots}^{\infty}\frac{a_n}{l^3}B_h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

and

$$M_0=-h\left[PL_m+A_m\sum_{1.3.5\,\ldots}^{\infty}\frac{a_n}{l^3}.B_m\right] \quad . \quad . \quad . \quad . \quad . \quad (62)$$

Thus H_0 and M_0 can be determined when the Fourier constants are known, and these in turn can be calculated by consideration of the energy in the system. If a small increment da_i is given to a Fourier constant a_i, there will be a small change in the internal stresses and also a small consequent deformation of the whole system including the load point. For equilibrium, the change in potential energy in the system is equal to the change in work done by the external load.

Thus

$$\frac{\partial U}{\partial a_i}.da_i=P\left(\frac{\partial}{\partial a_i}\Delta_P\right)da_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

The total potential energy in the system is :

$$U=\int_0^l(M^a)^2\,\frac{dx}{2EI_c}+\int_0^l(M^g)^2\,\frac{dx}{2EI_g} \quad . \quad . \quad . \quad . \quad (64)$$

that is,

$$U=\frac{m+1}{2EI_c}\int_0^l\left(EI_c\,\frac{d^2\Delta}{dx^2}\right)^2dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

Now

$$EI_c\,\frac{d^2\Delta}{dx^2}=M_0+H_0y-\sum_1^{\infty}a_n\left(\frac{n\pi}{l}\right)^2\sin\frac{n\pi x}{l}$$

Substituting in equation (65) and integrating gives :

$$U=\frac{m+1}{2EI_c}\left[M_0^2\,l+\frac{8}{15}H_0^2\,h^2l+\sum_1^{\infty}\frac{a_n{}^2}{2l^3}(n\pi)^4+\frac{4}{3}\,M_0H_0hl\right.$$
$$\left.-\,4M_0\sum_{1.3.5\,\ldots}^{\infty}a_n\left(\frac{n\pi}{l}\right)-32H_0h\sum_{1.3.5\,\ldots}^{\infty}\frac{a_n}{l}\left(\frac{1}{n\pi}\right)\right] \quad . \quad (66)$$

Substituting for M_0 and H_o and differentiating :

$$\frac{\partial U}{\partial a_i}=\frac{m+1}{2EI_c}\left[\frac{a_i}{l^3}\left\{(i\pi)^4+\frac{2h^2}{l^2}\left(A_m{}^2B_m{}^2-\frac{4}{3}A_hB_hA_mB_m+\frac{8}{15}A_h{}^2B_h{}^2\right)\right.\right.$$
$$\left.+\,\frac{8h}{l}\,A_mB_m\left(i\pi-\frac{8}{i\pi}\,\frac{A_hB_h}{A_mB_m}\right)\right\}$$

$$+ 2P\frac{h^2}{l^2}\left(L_mA_mB_m + \frac{8}{15}L_hA_hB_h - \frac{2}{3}L_mA_hB_h - \frac{2}{3}L_hA_mB_m\right)$$

$$+ \frac{4Ph}{l}L_m\left(i\pi - \frac{8L_h}{i\pi L_m}\right)\Bigg]$$

Letting $$\frac{2h^2}{l^2}\left(A_m{}^2B_m{}^2 - \frac{4}{3}A_hB_hA_mB_m + \frac{8}{15}A_h{}^2B_h{}^2\right)$$

$$+ \frac{8f}{l}A_mB_m\left(i\pi - \frac{8}{i\pi}\cdot\frac{A_hB_h}{A_mB_m}\right) = \alpha_i \quad . \quad . \quad . \quad . \quad (67)$$

and $$\frac{2h^2}{l^2}\left(L_mA_mB_m + \frac{8}{15}L_hA_hB_h - \frac{2}{3}L_mA_hB_h - \frac{2}{3}L_hA_mB_m\right)$$

$$+ \frac{4h}{l}L_m\left(i\pi - \frac{8}{i\pi}\frac{L_h}{L_m}\right) = \beta_i \quad . \quad . \quad . \quad . \quad . \quad (68)$$

Then $$\frac{\partial U}{\partial a_i} = \frac{m+1}{2EI_c}\left[\frac{a_i}{l^3}\left\{(i\pi)^4 + \alpha_i\right\} + P\beta_i\right] \quad . \quad . \quad . \quad (69)$$

The change in potential energy in the bowstring arch due to the increment da_i is :

$$\frac{\partial U}{\partial a_i}da_i = \frac{m+1}{2EI_c}\left[\frac{a_i}{l^3}\left\{(i\pi)^4 + \alpha_i\right\} + P\beta_i\right]da_i \quad . \quad . \quad . \quad (70)$$

The change in the work done by P is :

$$P\frac{\partial}{\partial a_i}\Delta_P da_i = \frac{P}{EI_c}\left[\sin i\pi k + \frac{h}{l}\cdot A_mB_m\left\{\frac{k}{2}(1-k) - \frac{B_hA_h}{B_mA_m}\frac{(k^4 - 2k^3 + k)}{3}\right\}\right]da_i$$

Letting $$\frac{h}{l}A_mB_m\left\{\frac{k}{2}(1-k) - \frac{A_hB_h}{A_mB_m}\frac{k^4 - 2k^3 + k}{3}\right\} = \gamma_i \quad . \quad (71)$$

Then $$\frac{P\partial}{\partial a_i}\Delta_P da_i = \frac{P}{EI_c}\left[\sin i\pi k + \gamma_i\right] \quad . \quad . \quad . \quad . \quad . \quad (72)$$

The change in work done by P is equal to the change in the potential energy of the system, so :

$$\frac{m+1}{2EI_c}\left[\frac{a_i}{l^3}\left\{(i\pi)^4 + \alpha_i\right\} + P\beta_i\right] = \frac{P}{EI_c}\left[\sin i\pi k + \gamma_i\right]$$

Hence $$a_i = Pl^3\left[\frac{2(\sin i\pi k + \gamma_i) - \beta_i(m+1)}{(m+1)[(i\pi)^4 + \alpha_i]}\right] \quad . \quad . \quad . \quad . \quad (73)$$

where i is odd.

For even values of i, the change da_i has no effect on H_0 and M_0. Hence, from equation (70) :

$$\frac{\partial U}{\partial a_i}da_i = \frac{m+1}{2EI_c}\frac{a_i}{l^3}(i\pi)^4 da_i \, . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

And from equation (72):

$$\frac{P\partial \Delta_P}{\partial a_i} d_{ai} = \frac{P}{EI_c} \sin i\pi k . d_{ai} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

Since equation (74) is equal to equation (75):

$$a_i = \frac{2Pl^3 \sin i\pi k}{(m+1)(i\pi)^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

The Fourier constants can now be substituted in equations (61) and (62) to give H_0 and M_0, equations (44) and (45) can be used to give M_q and q, and the bending moments in the arch and girder are then given by (*Figs 14*, p. 544):

$$M^a = M_0 + H_0 y - \sum_1^\infty a_n \left(\frac{n\pi}{l}\right)^2 \sin \frac{n\pi x}{l} \quad . \quad . \quad . \quad . \quad (77)$$

and

$$M^g = M_P + M_0 - \sum_1^\infty a_n \left(\frac{n\pi}{l}\right)^2 \sin \frac{n\pi x}{l} \quad . \quad . \quad . \quad . \quad (78)$$

The suspension-rod forces are obtained by integrating equation (45) between appropriate limits.

To facilitate the evaluation of the Fourier constants, *Figs 18* to *21* have been prepared for the first four terms of the series, and *Fig. 22* gives values of L_n and L_m.

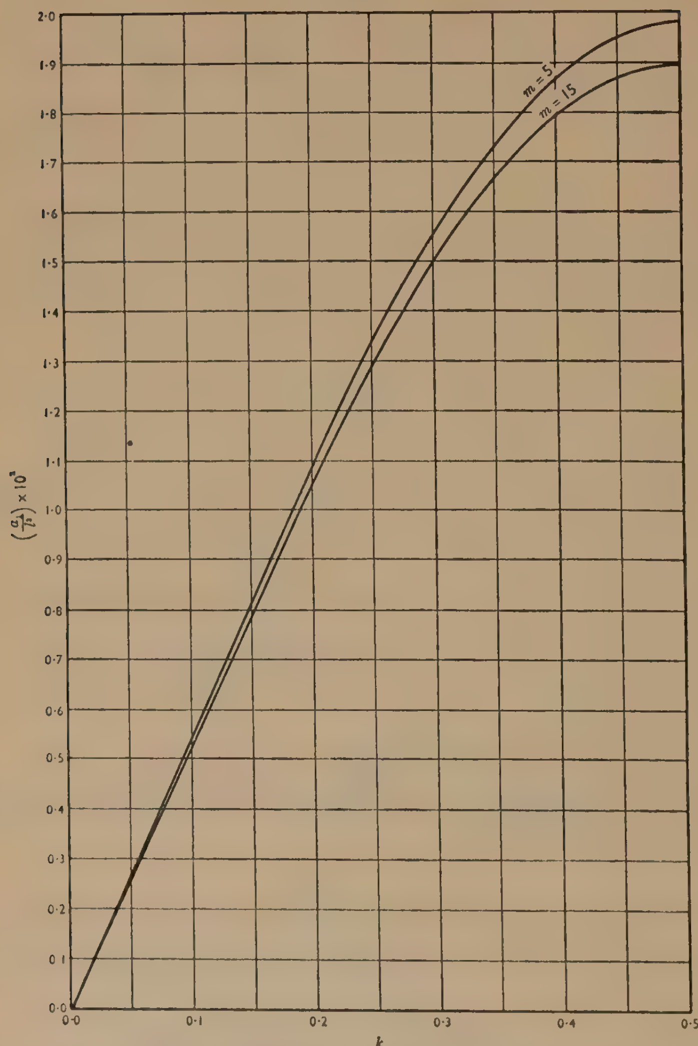
Table 19 gives values of A_n, A_m, B_n, and B_m.

Using *Figs 18* to *22* and Table 19, the curve of bending moment in the arch can be calculated, and is shown in *Figs 23* for a load applied at A in the bridge shown in *Fig. 6*.

TABLE 19

	m	5	10	15
	A_h	+ 32·73	+ 41·25	+ 45·71
	A_m	+ 26·18	+ 33·00	+ 36·57
$n = 1$	B_h	+ 0·678	+ 0·678	+ 0·678
	B_m	− 0·489	− 0·248	− 0·157
$n = 3$	B_h	− 1·868	− 1·868	− 1·868
	B_m	− 2·257	− 2·177	− 2·147

Fig. 18



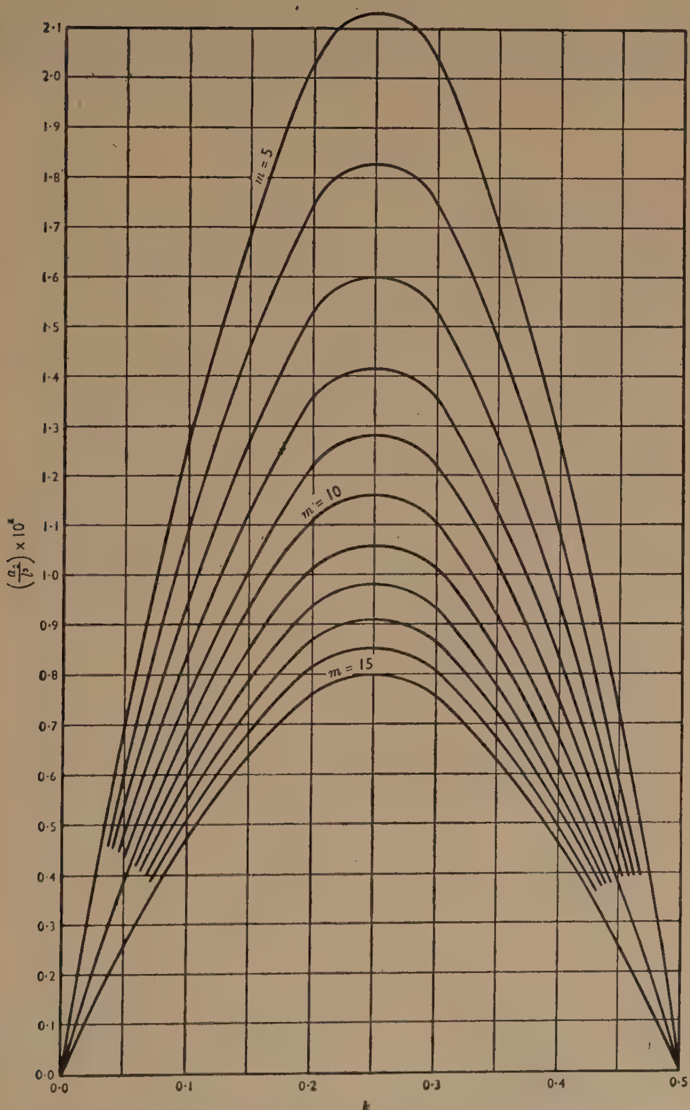
VALUES OF THE CONSTANT a_n FOR THE FIRST TERM IN THE SERIES:

$$q = \sum_{1}^{\infty} a_n \left(\frac{n\pi}{l}\right)^4 \sin \frac{n\pi x}{l}$$

CALCULATED FOR CONCENTRATED LOAD.

$$\text{Arch } \frac{\text{span}}{\text{rise}} = 4$$

Fig. 19

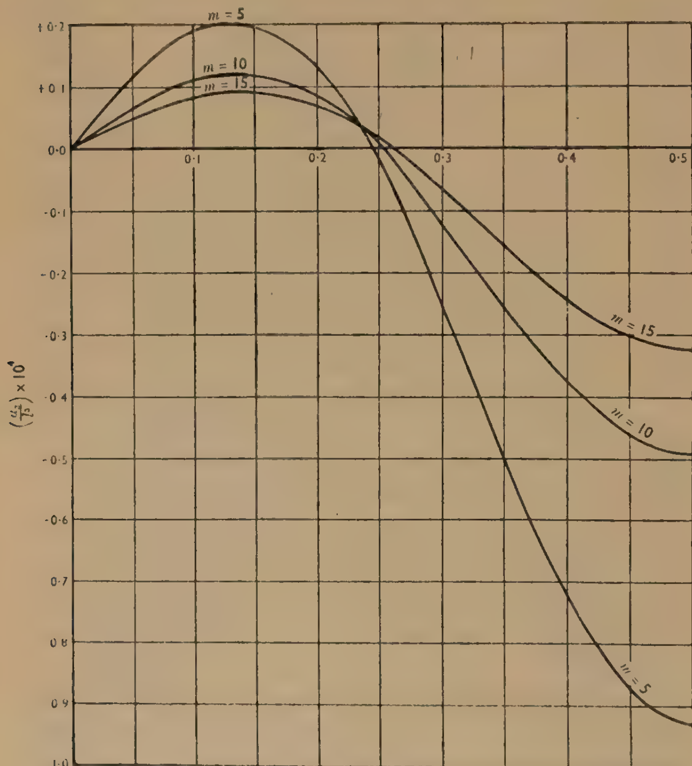


VALUES OF THE CONSTANT a_n FOR THE SECOND TERM OF THE SERIES

Part 4.—Experimental Work

Preliminary experiments were carried out to determine the suitability of the indirect method of analysis, using celluloid models and a modified form of Beggs's apparatus. The relatively large stiffness of the arch and beam compared with the suspension rods, however, gave a model in which

Fig. 20



Values of the Constant a_n for the Third Term of the Series

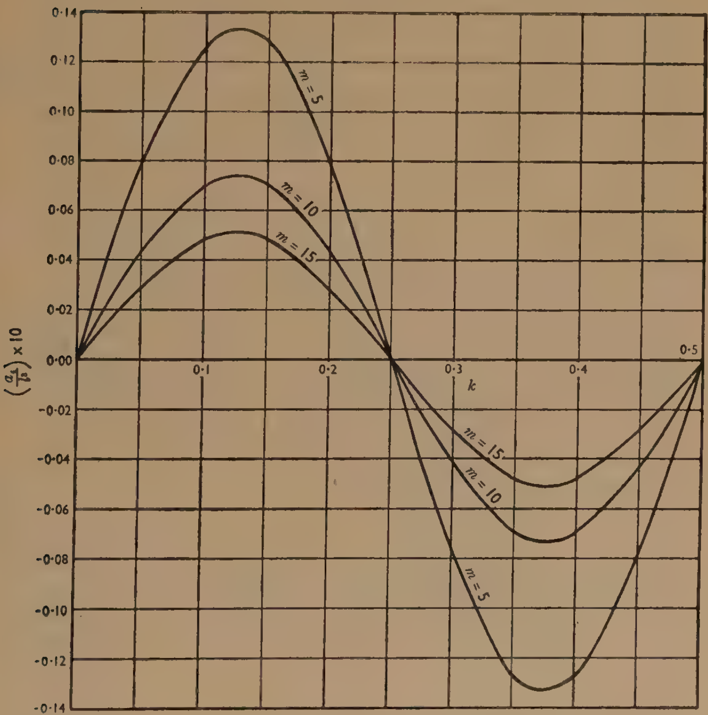
large forces were required to cause quite small displacements, and the results obtained were not reliable.

When celluloid is loaded the strain is a function of time as well as of load, and this property of "creep" complicates the use of the material in model analysis. Fortunately, if strains are measured at fixed time-intervals after the application of load, it is found that, within normal

limits of experimental accuracy, load/strain curves for any time interval are linear. In other words, the strain measured at a time interval t after application of a load W is half the strain measured at a time interval t after application of a load $2W$. Temperature effects, which are usually (and erroneously) thought to be serious when using the Beggs apparatus, are negligible when readings are taken over a reasonably short period.

One of the Authors had previously analysed successfully a tall framed

Fig. 21

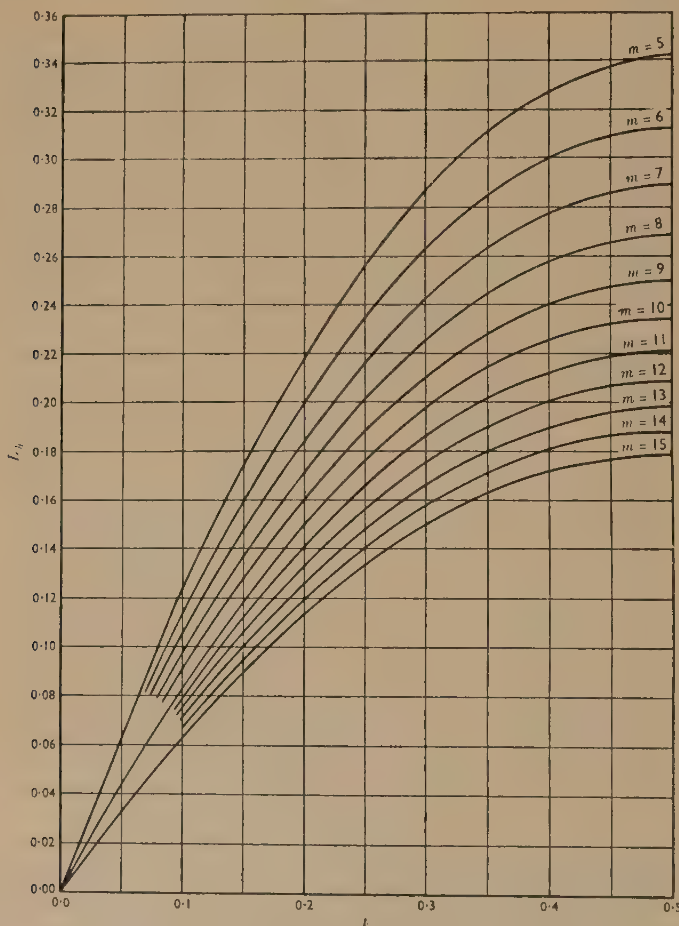


VALUES OF THE CONSTANT a_n FOR THE FOURTH TERM OF THE SERIES

structure by measuring slopes and deflexions produced in a loaded celluloid model, all measurements being taken 3 minutes after application of the load, and it was decided to try a similar technique in the experimental analysis of the bowstring arch. Because of the shape of the model, however, slope-deflexion measurements could not readily be interpreted in terms of moments and forces in the structure, and the standard laboratory practice of measuring strains in the model were therefore adopted, but all strains were measured at a fixed time interval after application of load.

Preliminary tests were carried out on a simply supported celluloid beam which was loaded centrally, and Huggenberger tensometers were used for measuring strains. Readings were taken 1 minute after application of load, and although the load/deflexion curves shown in *Fig. 24* are not the same for increasing and decreasing load, mean readings could be repeated with satisfactory consistency. In these tests, the beam was

Fig. 22



VALUES OF THE CONSTANT L_h (CONCENTRATED LOAD)

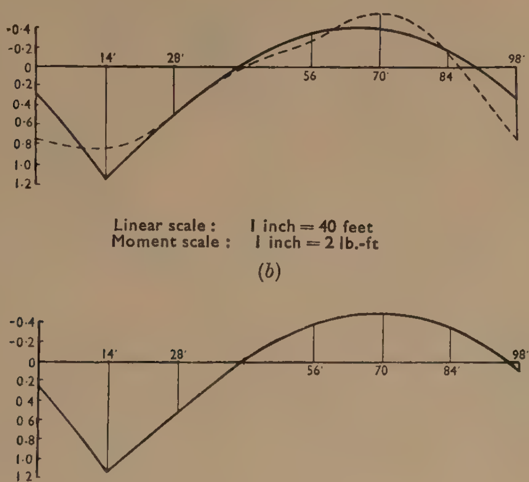
$$L_h = \frac{15}{4} \cdot \frac{k(1-k)}{6+m} \cdot \frac{l}{h} \qquad H_o = PL_h + A_h \sum_{1\cdot3}^{\infty}\left(\frac{a_n}{l^3}\right)B_h$$

$$L_m = 0{\cdot}8L_h \qquad M_o = -h\left[L_mP + A_m \sum_{1\cdot3}^{\infty}\left(\frac{a_n}{l^3}\right)B_m\right]$$

loaded horizontally and the weight of the tensometer was taken on ball-bearing castors in the same manner as shown in *Fig. 25*. By this method, a simply supported celluloid beam of 0·18-inch-by-1·10-inch cross-section and 20-inch span deflected 0·0198 inch per lb. of central load, giving a value of Young's Modulus of 423,000 lb. per square inch. The tensometer reading at a section 9 inches from one support was 0·355 per lb. of applied load. Hence one division on the tensometer represented a strain in the celluloid of $0{\cdot}83 \times 10^{-3}$.

Figs 23

(*a*)



Bending Moment in the Arch: (*a*) by Method of Membrane Analogy; (*b*) by Method of Influence Coefficients

(In (*a*) the broken line indicates bending moments when only four terms in the series are considered. The full line shows expected bending moments if all terms are considered.)

A celluloid model of the bowstring arch analysed theoretically was made to a scale of 1 inch to 4 feet, in celluloid 0·18 inch thick. The arch rib was 0·400 inch wide at the crown and varied according to $I_\phi = I_c \sec \phi$ towards the abutments. The tie-girder was initially 0·985 inch thick and was later cut to 0·860 inch and 0·685 inch thick to give the three ratios of second moment of area of girder to arch 15, 10, and 5 respectively. To be strictly to scale, the suspension rods should be about 0·04 inch wide, but since the effect of their extension is very small, the suspension rods were made of a practicable size 0·1 inch wide.

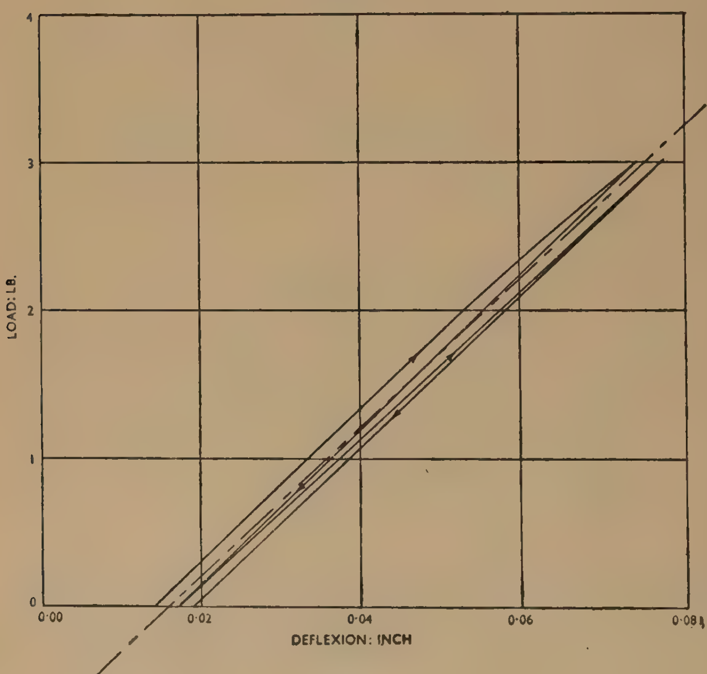
The model was loaded in five increments of 2 lb. to a maximum load of 10 lb. at each suspension-rod position, and strains on two sides of the

tie-girder and the arch rib were measured by tensometer 1 minute after the application of each increment of load.

Warping of the model was prevented by supporting it on ball-bearing castors as shown in *Fig. 25.*

The reduction of experimental readings was carried out as follows : $e_1 f_1$ and $e_2 f_2$ were taken to denote the strains and stresses respectively

Fig. 24



at positions 1 and 2 on opposite edges of the celluloid member. Also $e_q' f_q'$ and $e_m f_m$ were taken to denote the strains and stresses produced respectively by the axial force q' and the bending moment M^{model} at the positions 1 and 2.

Then :

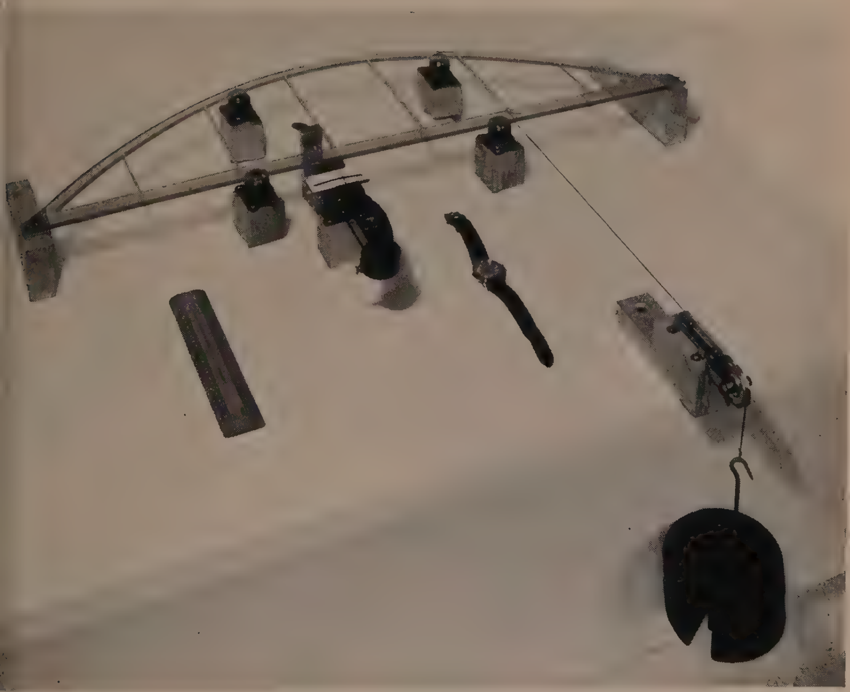
$$e_1 = e_q' + e_m \qquad e_2 = e_q' - e_m$$

$$f_1 = f_q' + f_m \qquad f_2 = f_q' - f_m$$

that is, $$e_1 E = f_q' + f_m \quad \text{and} \quad e_2 E = f_q' - f_m$$

f_q' and f_m can thus be determined from the readings e_1 and e_2 taken by the tensometer.

Fig. 25



ARRANGEMENT OF EXPERIMENT

Now $$q' = f_q' \, A \text{ and } M^{model} = \frac{2I_m}{d} \cdot f_m$$

where d denotes the width of the member and A denotes the area of cross-section of the member.

Since the model represents the actual structure to a linear scale factor s, the corresponding force and moment in the section of the actual structure are :

$$Q = q' \text{ and } M = sM^{model}$$

The relation between the axial force and horizontal force in the arch is

$$Q = X_3 \sec \phi$$

that is, $$X_3 = \frac{e_q' EA}{\sec \phi} = e_q' \frac{EA_c \, d}{\sec \phi \, d_c}$$

where d and d_c denote respectively the depths of the arch rib at the section considered and at the crown.

Since $I = I_c \sec \phi$, the model was made such that

$$d = d_c \sqrt[3]{\sec \phi}$$

Therefore $$X_3 = e_q'.EA_c \left(\frac{1}{\sec \phi}\right)^{\frac{2}{3}}$$

And $$M^a = 2se_m \frac{EI_c}{d_c} (\sec \phi)^{\frac{2}{3}}$$

Putting $E = 423{,}000$ lb. per square inch, $s = 48$,

$$\frac{2I_c}{d_c} = 0{\cdot}0048 \text{ in.}^3 \text{; and } A_c = 0{\cdot}072 \text{ in.}^2$$

gives $$X_3 = 30{\cdot}40 \times 10^3 e_q \left(\frac{1}{\sec \phi}\right)^{\frac{2}{3}} \text{ lb.}$$

and $$M^a = 8{\cdot}1 \times 10^3 e_m (\sec \phi)^{\frac{2}{3}} \text{ lb.ft.}$$

Similarly, the expressions for horizontal force and moment for the girder can be obtained :

for $\dfrac{I_g}{I_c} = 5$ $\quad X_3 = 70{\cdot}5 \times 10^3 e_q$ lb.

$\quad M^g = 49{\cdot}2 \times 10^3 e_m$ lb.ft

for $\dfrac{I_g}{I_c} = 10$ $\quad X_3 = 63{\cdot}6 \times 10^3 e_q$ lb.

$\quad M^g = 37{\cdot}7 \times 10^3 e_m$ lb.ft

and for $\dfrac{I_g}{I_c} = 15$ $\quad X_3 = 51{\cdot}8 \times 10^3 e_q$ lb.

$\quad M^g = 23{\cdot}7 \times 10^3 e_m$ lb.ft.

TABLE 20

Load position	m	Horizontal force: By influence coefficients	Horizontal force: By membrane analogy	By experiment
A	5	0·343	0·343	0·356
B		0·613		0·632
C		0·762		0·778
A	10	0·337	0·327	0·352
B		0·603		0·619
C		0·749		0·763
A	15	0·331	0·326	0·356
B		0·593		0·588
C		0·736		0·750

Comparison of the theoretical and experimental values of bending moments in the arch and girder are shown in *Figs 26*, and the theoretical and experimental values of the axial force are compared in Table 20.

Part 5.—Discussion and Conclusions

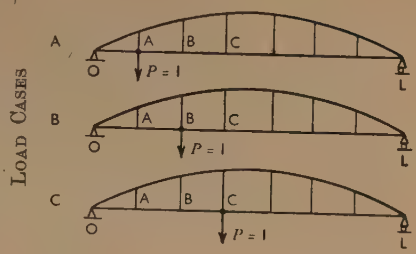
(1) The assumptions ordinarily made in the analysis of bowstring arches (namely, that the suspension rods are inextensible, the arch rib and tie-girder are pinned together, and the second moment of area of the arch is small by comparison with that of the tie-girder) are seen to be partly justified by more exact analysis. Extension of the tie-rods is seen (*Figs 10*, pp. 534 to 537) to have very little influence on bending moments in the arch rib when the structure carries a single concentrated load, and has no measurable effect on the bending moment in the tie-girder.

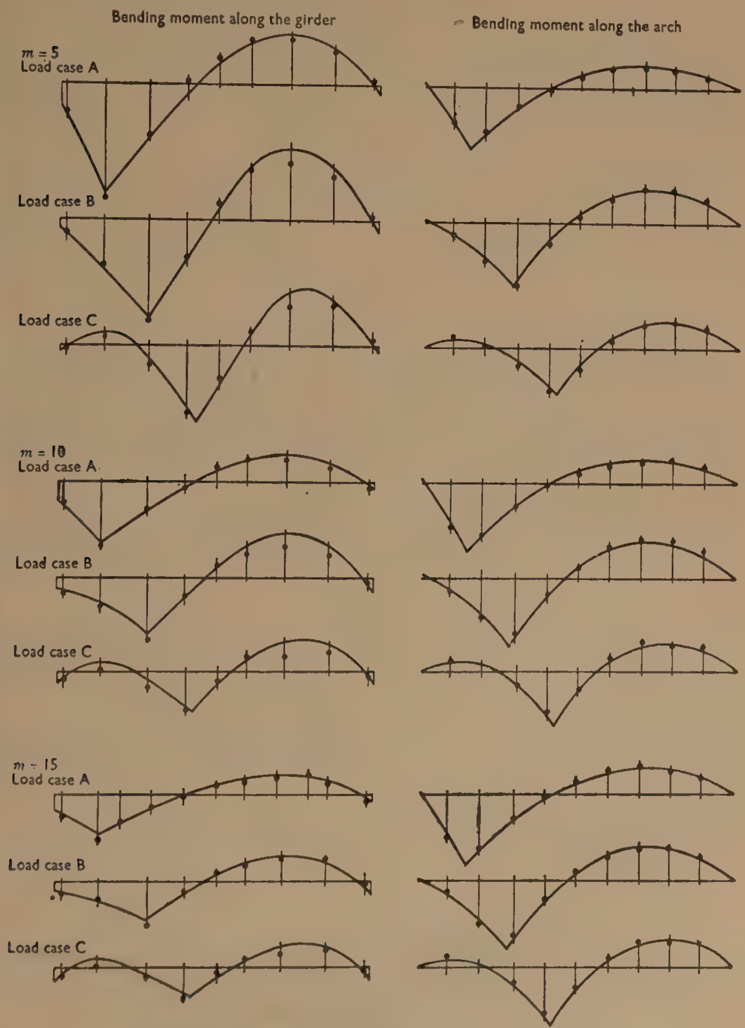
Comparison of the results obtained by the simplified and exact methods of analysis are shown in *Figs 12*, pp. 540 to 543. For a concentrated load anywhere on the structure there is seen to be little difference in the bending moments calculated by the two methods. For a uniformly distributed load, however, bending moments towards the ends of the arch are very high, being several times those at the centre, and very serious errors are introduced by assuming the tie-girder and arch rib to be pinned together.

(2) Very good agreement was obtained between the exact analysis of a bowstring arch having nine redundancies and experimental results (*Fig. 24*, p. 558, and Table 20). The bending moments along the girder are almost the same, but in the arch the experimental curve of bending moment is below the theoretical curve at some points, owing to the change in form of the arch caused by its deflexion under load. The model

Figs 26

Full lines show bending moments calculated by the method of influence coefficients in the system with extensible suspension rods.
Points indicate experimental values
Linear scale : 1 inch = 60 feet.
Moment scales : 1 inch = 2 lb.-ft in the arch.
1 inch = 20 lb.-ft in the girder.
$m = I_g/I_c$ = ratio of second moments of area of girder and arch sections.





BENDING MOMENTS IN THE BOWSTRING ARCH

was of 24 inches span and under 10 lb. load its deflexion was about $\frac{1}{4}$ inch. The behaviour was elastic and linear within the limits of experimental accuracy, but a small change in the rise-to-span ratio of an arch causes an appreciable change in bending moments.

Horizontal forces determined by influence coefficients, by the membrane analogy, and by experiment all agree within about 3 per cent.

(3) Extension of the suspension rods causes an increase in the suspension-rod forces of about 10 per cent near the load, and there is a much smaller difference away from the load (Table 15, p. 533).

(4) Although the use of the Beggs apparatus would have eliminated difficulties due to creep, it was impracticable to make a scale model of sufficient flexibility to obtain readings with this apparatus. The direct method of analysis used, however, has given very satisfactory results and appears to be suitable for use in a wider range of experimental work.

(5) It is of interest to note that when suspension rods are inextensible the bending moments at the ends of the arch rib are equal, regardless of the type and position of the load (*Figs 10*, pp. 534 to 537). Also, variation in m from 5 to 15 only causes about 25 per cent difference in the bending moment in the rib due to concentrated load. For uniform loading, the ratio of the second moments of area has practically no effect on the moment in the rib.

(6) Analysis of a bowstring arch having six suspension rods probably represents the limit of practicability of ordinary methods of calculation based on strain energy considerations. The analysis, involving as it does the formation and solution of nine algebraic equations, presents considerable computational difficulty and would not ordinarily give reliable answers unless increment transformations were used.

The membrane analogy enables a bowstring arch having any number of suspension rods to be tackled with confidence in the design office. Charts can be prepared from which arches of different proportions can be studied.

The accuracy obtained by the membrane analogy depends on the number of terms considered in the Fourier series. *Figs 18* to *21* (pp. 552, 555) and the curve shown in *Figs 23* (p. 557) take only four terms into account, but nevertheless give an accuracy within about 25 per cent of the maximum value. Charts for design purposes should, of course, take five or six terms into consideration. The example is given in sufficient detail, however, to illustrate the method.

Notation

I_g denotes second moment of area of girder
I_a ,, second moment of area of arch
I_c ,, second moment of area of arch at crown
M ,, bending moment

M^a denotes bending moment in arch
M^g ,, bending moment in girder
X ,, unknown force or moment
Q ,, axial force in arch rib
P ,, applied point load
ϕ ,, slope of arch axis to horizontal
A ,, cross-sectional area
A_a ,, cross-sectional area of arch
A_c ,, cross-sectional area at centre
Δ ,, increment or deflexion
a_n, b_n denote Fourier constants
K_m, L_m, A_m, B_m, ,, constants for determination of end moments in arch
K_h, L_h, A_h, B_h ,, constants for determination of horizontal thrust in arch
U denotes strain energy
α, β, γ denote constants for determination of a_n
f denotes stress
e ,, strain
E ,, Young's modulus

Other notation is explained in the text as it is introduced.

The Paper is accompanied by one photograph and nine sheets of diagrams, together with seventeen diagrams in the MS., from which the half-tone page plate and the Figures in the text have been prepared.

Paper 5979

"The Drawdown of the Water-Table under Non-Steady Conditions near a Pumped Well in an Unconfined Formation"

by

Norman Savage Boulton, M.Sc., M.I.C.E.

(*Ordered by the Council to be published with written discussion*)†

SYNOPSIS

A new equation is obtained for the drawdown of the water-table near a pumped well in an aquifer of any thickness and permeability, under non-equilibrium conditions. Tables of the mathematical functions involved make the equation simple to use in practice. Approximations in deriving the equation are unavoidable, but it is shown that allowance for them can be made in certain cases and, in other cases, their effect on the drawdown is unimportant provided that the variables involved are restricted to a specified range of values. For a thin and very permeable aquifer, the well-known formula involving the exponential integral, originally deduced for artesian conditions, is shown to be accurate provided that a correction is applied to it when necessary.

For an aquifer in which the conditions approximate to those assumed in the Paper (p. 568), it should be possible to use the equation to obtain reliable values of the coefficients of permeability and storage from the results of field pumping tests. The application of the new equation is illustrated by numerical examples, but its application to the results of actual field pumping tests is left for a subsequent Paper.

Simple modifications of the equation are indicated, for a case of anisotropic permeability and for the recovery of the free surface after pumping has ceased. The case when the aquifer is of unlimited depth is considered in Appendix I.

INTRODUCTION

THE main purpose of the Paper is to introduce a new equation for the drawdown of the water-table (free surface) near a gravity well before the flow has attained a state of equilibrium. The exponential integral [1] (Theis's formula) has been used for some time to represent the piezometric surface under artesian flow conditions, and its usefulness in appropriate cases for determining formation constants from the results of field pumping tests has been demonstrated by Wenzel [2] and others in the United States of America and, recently, by Ineson [3, 4] in the United Kingdom.

When equilibrium exists, the drawdown of the water-table in an unconfined formation can be calculated with sufficient accuracy for practical

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† Correspondence on this Paper should be received at the Institution by the 15th December, 1954, and will be published in Part III of the Proceedings. Contributions should be limited to about 1,200 words.—SEC. I.C.E.

[1] The references are given on p. 578.

purposes, as the Author has shown in a recent Paper.[5] For non-equilibrium conditions, however, no satisfactory theory has yet been published, and Ineson [6] has recently drawn attention to the need for such a theory.

As applied to water-table conditions, the exponential integral is theoretically unsound, since it takes no account of the vertical velocity-component of the pore-water approaching the well. This criticism applies particularly to the early stages of a pumping test in a deep well, when the motion of the water in the upper part of the aquifer is more vertical than horizontal. However, a time factor $\tau = \frac{kt}{Sh_e}$,* and not merely the duration of pumping t, determines whether the exponential integral gives a good approximation to the drawdown of the free surface. This time factor involves the coefficients of permeability and storage, and the depth of the saturated part of the aquifer, as well as the duration of pumping.

It is shown on p. 571 that the exponential integral gives a good approximation to the drawdown only when $\tau > 5$. From the definition of τ it is evident that, for a thin and very permeable aquifer, this condition may be satisfied after quite a short duration of pumping. In fact, for a well penetrating coarse sand and gravel, 27 feet thick, described by Wenzel,[7] the time factor at the end of 48 hours pumping is found to be 122. On the other hand, for a typical pumped well in the Bunter sandstone of the Midlands, about 900 feet thick, the time factor for the same duration of pumping is of the order of 0·04. Whereas the exponential integral can justifiably be applied to a 48-hour pumping test in the first case, it would be very inaccurate in the second case. It follows from these two examples that, under practical conditions, the time factor may vary between a small fraction and many hundreds.

The question as to which water-bearing formations the theory described in the Paper is applicable can only be answered by comparing predictions from the theory with the results of actual pumping tests. The agreement obtained would depend upon the extent to which the assumptions (p. 567) underlying the theoretical analysis are satisfied under field conditions.

By means of the equation, average values of the formation constants can be determined from a pumping test of any duration from a few hours upwards, in a formation of any thickness and permeability.

Theoretically, the solution of the flow problem is difficult, particularly because of the changing position of the free surface. A large sand model or an electro-analogy model might provide the most satisfactory solution, especially when the water level in the well is lowered to a considerable depth; although, owing to the very small ratio of diameter to length of well in many pumped wells, it might be impossible to reproduce accurately the actual boundary conditions at the well surface.

The method adopted is to obtain a solution by potential theory, which

* The notation is defined on pp. 566 and 567.

only strictly satisfies the given boundary conditions at the free surface and at entrance to the well when the water level in the well is lowered by a small amount as compared with the thickness of the aquifer. Corrections are then applied to this solution, where possible, in cases when it is known to be seriously in error.

For the case $\tau > 5$, when the exponential integral is applicable, the Author has obtained a simple method, which is described later in the Paper, for correcting the drawdown, based on his previous free-surface curves for steady flow. This correction is necessary because the exponential integral does not satisfy the correct boundary conditions at the free surface and at the well surface. The correction enables the integral to be used with readings from observation wells, which are quite close to the pumped well, and for pumping levels near the base of the aquifer.

NOTATION

h denotes height of free surface above impermeable stratum during pumping.
h_e ,, height of undisturbed free surface above impermeable stratum.
h_s ,, value of h when $r = r_w$.
h_w ,, height of water level in well above impermeable stratum during pumping.
k ,, coefficient of permeability of aquifer.
k_r ,, coefficient of permeability in horizontal direction.
k_z ,, coefficient of permeability in vertical direction.
l ,, length of well below water-table for an aquifer of unlimited depth.
m ,, symbol defined by equation (18).
p ,, pressure intensity in pore-water (atmospheric pressure being taken as zero).
Q ,, constant rate of discharge of well during pumping.
r ,, horizontal distance from well-axis to any point.
r_w ,, radius of well.
s ,, drawdown of free surface at distance r.
s_c ,, drawdown given by equation (15).
S ,, coefficient of storage.
t ,, time since pumping commenced.
T ,, coefficient of transmissibility.
$u = \frac{\rho^2}{4\tau}$
v_r denotes "discharge velocity" component in horizontal direction away from well.
v_z ,, discharge velocity component in upward vertical direction.
V ,, integral in equation (10).
X_0 ,, correction term defined by equation (13).

X_1 denotes correction term defined by equation (12).
y ,, depth of any point below initial water-table.
z ,, height of any point above impermeable stratum.
γ_w ,, density of water.

$$\mu = \sqrt{\frac{k_z}{k_r}}$$

$$\rho = \frac{r}{h_e} \text{ and } \rho_w = \frac{r_w}{h_e}$$

σ denotes a correction factor.

$\tau = \frac{kt}{Sh_e}$, a dimensionless time factor.

ϕ denotes pressure head plus potential head at any point of saturated

$$\text{aquifer}: \ \phi = \frac{p}{g\gamma_w} + z$$

Statement of Problem and Assumptions

The conditions assumed in the main problem investigated are :—

(1) The aquifer is homogeneous and, except where otherwise stated, isotropic ; it extends to the ground surface, is of infinite lateral extent, and is underlain by a horizontal impermeable bed.

(2) The well completely penetrates the aquifer and is unlined.

(3) The coefficient of storage (S) is constant.

(4) The flow in the aquifer obeys Darcy's law (k = constant).

(5) The water-table is initially horizontal. The replenishment of the water-table near the well by the rainfall is not considered, since the flow system due to the rainfall may be regarded as an independent system in equilibrium ; its effect on the drawdown of the free surface due to pumping is likely to be small.

(6) The well is pumped at a constant rate from the instant $t = 0$.

Basic Equations

Darcy's law is expressed by the equations :

$$\left.\begin{aligned} v_r &= -k\frac{\partial\phi}{\partial r} \\ v_z &= -k\frac{\partial\phi}{\partial z} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (1)$$

where ϕ denotes the sum of the pressure and potential heads at the point in the flow considered.

The well-known equation which results from Darcy's law and the equation of continuity of flow of incompressible fluids is :

$$\nabla^2(\phi) = \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad . \quad . \quad . \quad . \quad (2)$$

which must be satisfied at all points of the saturated aquifer.

The differential equation to be satisfied at the variable free-surface boundary may be obtained in the following way. Since the pressure on this surface is atmospheric (assumed zero), the equation to the surface is :

$$\phi(r, z, t) - z = 0.$$

Also, since a particle of fluid, which is once in the free surface, never leaves it,

$$\frac{D}{Dt}(\phi - z) = 0$$

where D denotes differentiation following the motion of the particle. Therefore, since,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{v_r}{S}\frac{\partial}{\partial r} + \frac{v_z}{S}\frac{\partial}{\partial z}$$

it follows that $$\frac{\partial\phi}{\partial t} + \frac{v_r}{S}\cdot\frac{\partial\phi}{\partial r} + \frac{v_z}{S}\left(\frac{\partial\phi}{\partial z} - 1\right) = 0$$

where $\frac{v_r}{S}$ and $\frac{v_z}{S}$ are "seepage velocity" components.

Substituting for v_r and v_z from equations (1), the free-surface boundary equation obtained is :

$$\frac{\partial\phi}{\partial t} = \frac{k}{S}\left\{\left(\frac{\partial\phi}{\partial r}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2 - \frac{\partial\phi}{\partial z}\right\} \quad . \quad . \quad . \quad . \quad (3)$$

Since equation (3) is non-linear, a solution satisfying it cannot easily be found. However, if the ϕ-gradients are small, their squares may be neglected and equation (3) then reduces to the linear equation :

$$\frac{\partial\phi}{\partial t} + \frac{k}{S}\frac{\partial\phi}{\partial z} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which is to be satisfied when $z = h$

Since the undisturbed water-table is assumed to be a horizontal plane, $z = h_e$, it follows that :

$$\phi = h_e \text{ when } t = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

At the surface of the well :

$$\left.\begin{array}{ll} \phi = h_w ; & 0 \leqslant z \leqslant h_w \\ \phi = z ; & h_w \leqslant z \leqslant h_s \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

At the impermeable layer

$$\frac{\partial \phi}{\partial z} = 0; \quad z = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

Also, as $r \to \infty$, $\phi \to h_e$; $0 \leqslant z \leqslant h_e$.

Equation (4) should, strictly speaking, be satisfied at all points of the free surface, but this is impossible except by using a laborious step-by-step method of calculation. Advantage may, however, be taken of the fact that the drawdown of the free surface is usually a small fraction of h_e.* Thus, subject to an error of the order already neglected, equation (4) may be written:

$$\frac{\partial \phi}{\partial t} + \frac{k}{S} \frac{\partial \phi}{\partial z} = 0; \quad z = h_e \quad . \; . \; . \; . \; . \; . \quad (4a)$$

A solution can be obtained, satisfying equations (2), (5), (6), (7), and (4a), assuming that the water level in the well is constant. The solution, however, is difficult to evaluate numerically. Also, a constant rate of pumping instead of a constant water level is the usual requirement in practice. To satisfy the former requirement, a simplification of the boundary condition at the well surface is necessary. In place of equations (6), it is assumed that the discharge, per unit length of well of vanishingly small radius, is constant. That is:

$$Q = 2\pi k h_e r \frac{\partial \phi}{\partial r}; \quad r \to 0; \quad 0 \leqslant z \leqslant h_e . \; . \; . \; . \quad (8)$$

The correction to be applied, when the error due to this approximation is important, is considered later in the Paper.

The Drawdown Equation

The required solution is:

$$h_e - \phi = \frac{Q}{2\pi k h_e} \int_0^\infty \frac{J_0(\beta r)}{\beta} \left\{ 1 - \frac{\cosh \beta z}{\cosh \beta h_e} \exp\left(-\frac{k}{S} t\beta \tanh \beta h_e \right) \right\} d\beta \quad (9)$$

where J_0 denotes the Bessel function of the first kind of zero order.

It can be verified that equation (9) satisfies equations (2), (5), (7), and (4a), using the properties of the Bessel function. It is also easy to show that the exponential term, involving the time, makes no contribution to the discharge into a well of vanishingly small radius, and hence that equation (8) is satisfied.

Assuming, as previously stated, that the drawdown of the water-table is small, a first approximation for the drawdown is obtained by putting $z = h_e$ in equation (9). Since the effect of this approximation is to make the calculated drawdown too small, whereas the effect of using equation (4a)

* This fact is illustrated for the case when $h_w < 0{\cdot}5 h_e$ in reference 5, p. 538 (*Fig. 2*).

instead of equation (3) is to make the drawdown too large, the errors due to these two approximations tend to cancel each other.

For numerical computation, it is convenient to introduce the non-dimensional quantities $\rho = \frac{r}{h_e}$ and $\tau = \frac{kt}{Sh_e}$. Then, making these substitutions and also changing the variable of integration by writing $\lambda = \beta h_e$, equation (9) gives for the drawdown :

$$s = \frac{Q}{2\pi k h_e}\int_0^\infty \frac{J_0(\lambda\rho)}{\lambda}\left\{1 - \exp(-\tau\lambda\tanh\lambda)\right\}d\lambda \quad . \quad . \quad (10)$$

Denoting the definite integral by $V(\rho, \tau)$ and introducing the "transmissibility coefficient," $T = kh_e$, equation (10) becomes :

$$s = \frac{Q}{2\pi T}V(\rho, \tau) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

which is the proposed general equation for the drawdown of the water-table.

When the time factor τ is sufficiently large, $\tanh\lambda$ may be replaced by λ in equation (10) without appreciably changing the value of the integral. By using Weber's first integral,[8] it can then be shown that :

$$V(\rho, \tau) = -\tfrac{1}{2}\mathrm{Ei}\left(-\frac{\rho^2}{4\tau}\right) - X_1 \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where the correction term X_1 is small when τ is large. In equation (12), Ei denotes the exponential integral * used by Theis in his non-equilibrium formula for pumping under artesian conditions. Thus, the exponential integral is shown to apply to water-table conditions in an unconfined aquifer in the limiting case when the duration of pumping is sufficiently great.

The function V in equation (11), unlike the exponential integral, has not previously been available in tabular form. A few values which have been computed for this Paper are given in Table 3. Sufficient tabulation for all practical purposes would not be difficult, making equation (11) as simple to use as any other drawdown formula.

For small values of the time factor ($\tau < 0{\cdot}05$), the function V is closely given by the simple equation (22), derived in Appendix I, for the case when the permeable medium extends downwards indefinitely and there is no underlying impermeable stratum. Introducing the correction X_0, to be added to the right-hand side of equation (22) to obtain V :

$$V(\rho, \tau) = \sinh^{-1}\frac{1}{\rho} + \sinh^{-1}\frac{\tau}{\rho} - \sinh^{-1}\frac{1+\tau}{\rho} + X_0 \quad . \quad . \quad . \quad (13)$$

where X_0 is small when τ is small.

* C. E. Jacob and others have used the notation, $W(u)$, for $-\mathrm{Ei}(-u)$.

Numerical Evaluation of V

To determine numerical values of V, it is convenient to find by quadrature, values of the correction X_0 for the smaller values of τ, and values of X_1 for the larger values of τ. Values of V can then be found from equations (13) and (12), using the published tables of $\sinh^{-1} x$ and $\mathrm{Ei}(-x)$.

Tables 1 and 2 give some values of X_0 and X_1. Intermediate values may be found with sufficient accuracy by linear interpolation. The corresponding values of V are given in Table 3, which, however, is not so suitable for interpolation.

TABLE 1.—VALUES OF $X_0(\rho, \tau)$

τ	$\rho = 0{\cdot}0$	$\rho = 0{\cdot}2$	$\rho = 0{\cdot}4$	$\rho = 0{\cdot}6$	$\rho = 0{\cdot}8$	$\rho = 1{\cdot}0$	$\rho = 1{\cdot}5$
0·05	0·0150	0·0143	0·0125	0·0101	0·0077	0·0056	0·0021
0·20	0·0564	0·0541	0·0480	0·0398	0·0312	0·0234	0·0099

TABLE 2.—VALUES OF $X_1(\rho, \tau)$

τ	$\rho = 0{\cdot}0$	$\rho = 0{\cdot}2$	$\rho = 0{\cdot}4$	$\rho = 0{\cdot}6$	$\rho = 0{\cdot}8$	$\rho = 1{\cdot}0$	$\rho = 1{\cdot}5$
1·00	0·1810	0·1747	0·1575	0·1331	0·1057	0·0787	0·0250
5·00	0·0344	0·0343	0·0338	0·0330	0·0320	0·0306	0·0264

TABLE 3.—VALUES OF $V(\rho, \tau)$

τ	$\rho = 0{\cdot}2$	$\rho = 0{\cdot}4$	$\rho = 0{\cdot}6$	$\rho = 0{\cdot}8$	$\rho = 1{\cdot}0$	$\rho = 1{\cdot}5$
0·05	0·214	0·092	0·051	0·032	0·021	0·008
0·20	0·756	0·358	0·207	0·132	0·088	0·035
1·00	1·844	1·183	0·826	0·599	0·443	0·220
5·00	2·785	2·096	1·696	1·416	1·203	0·832

From the values of X_0 for $\tau = 0{\cdot}05$, the error in drawdown, due to neglecting X_0, does not exceed 0·2 foot when the constant $\dfrac{Q}{2\pi T} = 10$.

Comparing the values of X_1 and V for $\tau = 5$, it is found that the error involved in using the exponential integral instead of V does not exceed about 3 per cent. For $\tau = 1$, the corresponding error is about 18 per cent. Hence, when $\tau > 5$, the function V is given with sufficient accuracy for practical purposes by the exponential integral.

Accuracy and Limitations of the Drawdown Equation. Method of Correction

Three cases arise when considering the error in drawdown due to the approximations involved in deriving equation (11).

Case I ($\tau < 0{\cdot}05$).—For this small range of τ, consideration of a numerical example (not given here) indicates that the flow pattern only varies slightly from its initial form. Consequently, if s' denotes the drawdown at any radius r, using equations (6) at the well surface, and s the drawdown using the assumed equation (8), the ratio $\frac{s'}{s}$ may be replaced by the ratio of the initial rates of drawdown :

$$\sigma = \frac{[\partial\phi/\partial t]'}{\partial\phi/\partial t}\,; \quad t = 0\,; \quad z = h_e$$

the dash indicating the true value. From equation (4*a*), this ratio is also equal to :

$$\sigma = \frac{[\partial\phi/\partial z]'}{\partial\phi/\partial z}\,; \quad t = 0\,; \quad z = h_e$$

The ratio σ has been calculated from the Fourier–Bessel series * which are obtainable for $\frac{\partial\phi}{\partial z}$, for both boundary conditions under consideration, and values of $100(\sigma - 1)$ are shown plotted on a base of ρ, to a logarithmic scale, in *Fig. 1.*

Curves are shown for two values of the constant $\frac{\pi r_w}{2h_e}$ and four values of $\frac{Q}{kh_e^2}$. The corresponding values of $\frac{h_w}{h_e}$ are also given, h_w here denoting the initial depth of water in the well immediately after pumping is started. The vertical ordinate gives the percentage correction to be added to the drawdown, the correction being obtained by interpolating between the curves, if necessary. The variation in the correction is fairly small as $\frac{r_w}{h_e}$ varies over the range considered.

Case II ($0{\cdot}05 \leqslant \tau \leqslant 5$).—Considering the lower end of this range of τ, it is seen from curves A, B, and E (*Fig. 1*) that if :

$$\frac{Q}{kh_e^2} \leqslant 0{\cdot}18 \text{ when } \frac{r_w}{h_e} = 0{\cdot}0013\,;$$

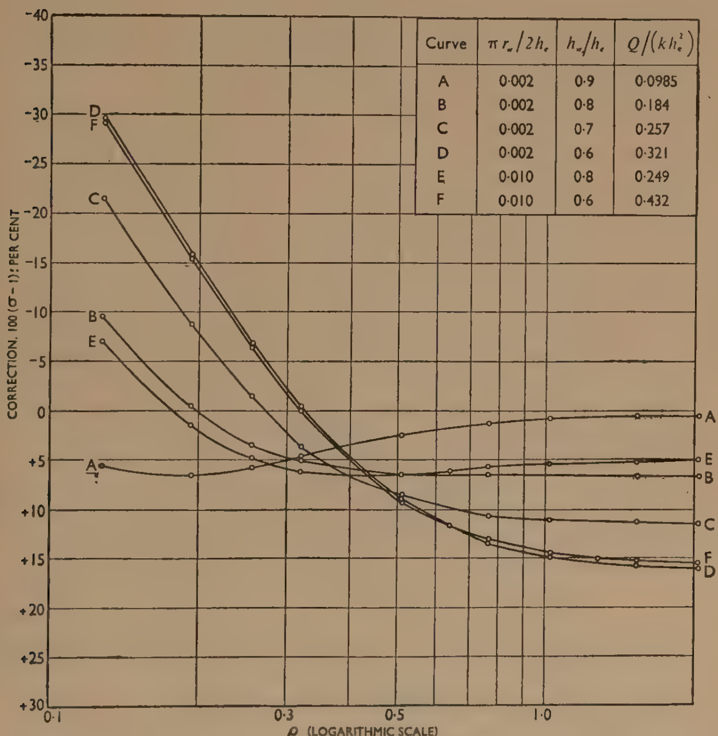
* The series are given in Appendix II.

$$\frac{Q}{kh_e^2} \leqslant 0{\cdot}25 \text{ when } \frac{r_w}{h_e} = 0{\cdot}0064\,;$$

and $$\rho \geqslant 0{\cdot}2,$$

then the drawdown correction does not exceed 6 per cent. Also, considering the upper end of the range of τ, it is seen from *Fig. 2* (Case III) that the drawdown correction does not exceed 6 per cent if $\rho \geqslant 0{\cdot}2$. Now it is reasonable to suppose that, for intermediate values of τ, the drawdown correction will lie between its values at the ends of the range

Fig. 1



Curves for correcting Drawdown of Free Surface when $\tau < 0{\cdot}05$

of τ, and consequently will be less than 6 per cent. Since corrections to the drawdown are not available when $0{\cdot}05 < \tau < 5$, drawdown calculations should in this case be restricted to the above ranges of Q and ρ.

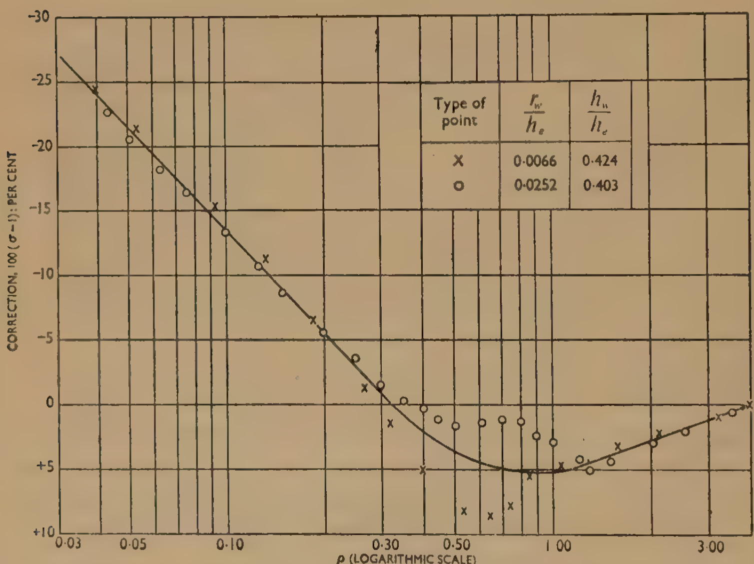
Case III ($\tau > 5$).—The restrictions imposed on the magnitudes of Q and ρ in Case II are unnecessary in this case and the following method

of correction enables the drawdown to be accurately obtained, even for a low pumping level and at a point quite near to the pumped well.

The expansion of the exponential integral, used by Jacob [10] and others, gives the expression for the drawdown :

$$s = \frac{Q}{4\pi T}\left(-0{\cdot}5772 - \log_e u + u - \frac{u^2}{4}\ldots\right) \quad . \quad . \quad (14)$$

Fig. 2



Curve for correcting Drawdown of Free Surface when $\tau > 5$

where $u = \frac{\rho^2}{4\tau}$, using the present notation. If u is small so that the power series on the right-hand side may be neglected,

$$s_c \simeq \frac{Q}{2\pi T}\log_e\frac{\rho_e}{\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where $\rho_e = 1{\cdot}5\sqrt{\tau}$.

Equation (15), which approximates to equation (14) with increasing accuracy as the well is approached, is the equation (except for the changed notation) proposed by Cambefort [11] to represent the drawdown of the free surface under conditions of steady flow. In a previous Paper,[12] the Author has shown that equation (15) accurately represents the drawdown

only when $\frac{r_w}{h_e} = 0{\cdot}11$.* For the much smaller values of this ratio which are usual, equation (15) requires a correction, the magnitude of which is proportional to the vertical distance of the appropriate curve in *Fig. 5* of reference 5 from the straight line representing equation (15).

Writing $\sigma = \frac{s}{s_c}$, where s_c denotes Cambefort's value of s, the required correction to the drawdown, as given by equation (12) or equation (14), is $100(\sigma - 1)$ per cent. This correction is shown by the ordinates in *Fig. 2*, on a base of ρ (to a logarithmic scale). It must be added to the calculated drawdown. The correction strictly applies only when $\rho_e = 4$. However, it is found that considerable departure from this value has little practical effect on the value of the correction. Also, the correction is practically independent of the ratio $\frac{h_w}{h_e}$

Different Horizontal and Vertical Permeability

If the coefficients of permeability in horizontal and vertical directions have different but constant values, k_r and k_z respectively, k is replaced by k_z in boundary equation (4a) and in the expression for τ. Equations (9) to (13), previously derived for the isotropic case, will then apply provided r (or ρ) in them is replaced by μr (or $\mu\rho$), the quantity μ denoting $\sqrt{\frac{k_z}{k_r}}$. This is the simple transformation [13] applicable to the case of steady flow, which also applies to the case of non-steady flow since the time does not appear in the differential equation (2). The boundary equation (4a) is clearly satisfied when μ is introduced. Also, boundary equation (8) is satisfied since $r \frac{\partial\phi}{\partial r}$ is unaltered by the transformation.

Equation for Recovery

If τ and τ' are the time factors, reckoned from the commencement of pumping and from the end, respectively, the drawdown of the free surface during recovery is found from equation (11) to be :

$$s = \frac{Q}{2\pi T}\left\{V(\rho, \tau) - V(\rho, \tau')\right\} \quad . \quad . \quad . \quad . \quad (16)$$

* Inaccurately shown as 0·13 in reference 5, p. 544.

Calculation of Pumping Level

Owing to the approximate nature of boundary equation (8), assumed in obtaining the drawdown equation, the depth h_w of water in the well does not appear explicitly and there is some uncertainty as to how it should be calculated. As an approximation it may be assumed that $h_w = \phi_0$, where ϕ_0 denotes the value of ϕ at the bottom of the well. Then, putting $z = 0$ and $\phi = h_w$ in equation (9), and changing the variables r, t and β as before:

$$h_e - h_w = \frac{Q}{2\pi T}\int_0^\infty \frac{J_0(\lambda\rho_w)}{\lambda}\left\{1 - \operatorname{sech}\lambda \exp(-\tau\lambda\tanh\lambda)\right\}d\lambda \quad . \quad . \quad (17)$$

which gives the depth of pumping level below rest level.

It has been estimated that h_w, as given by equation (17), is about 1 per cent too great, for small values of τ and for $\frac{h_w}{h_e} = 0{\cdot}8$.

The integral in equation (17) has been computed for values of the time factor assumed in Table 3. Writing:

$$h_e - h_w = (-\log_e \rho_w + m)\,\frac{Q}{2\pi T} \quad . \quad . \quad . \quad (18)$$

the values of m, defined by equation (18), are given in Table 4.

Table 4

τ	m
0·05	− 0·043
0·20	+ 0·087
1·00	+ 0·512
5·00	+ 1·228

When the time factor $\tau > 5$, equation (14) is applicable and there is then a simple method for calculating h_w, which is accurate even when the drawdown of the pumping level is large. This method is based on the Dupuit equation:[14]

$$h_w^2 = h_e^2 - \frac{Q}{\pi k}\log_e\frac{\rho_e}{\rho_w} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

which is known to give an accurate relationship between h_w and Q for steady flow conditions. The depth h_w of water in the well is easily calculated from equation (19), using the previously determined relation:

$$\rho_e = 1{\cdot}5\sqrt{\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Examples of Drawdown Calculation

The following examples demonstrate the use of the foregoing equations :—

Example 1.—Data for thick and moderately permeable aquifer

Assume : $r_w = 1$ foot, $h_e = 800$ feet, $k = 2 \times 10^{-5}$ foot per second, $S = 0{\cdot}15$, $Q = 1$ cubic foot per second, $t = 3{\cdot}47$ days $= 3 \times 10^5$ seconds. Then time factor $\tau = \dfrac{kt}{Sh_e} = 0{\cdot}05$; $\dfrac{Q}{2\pi T} = 9{\cdot}95$.

From Table 3, $V(0{\cdot}2, 0{\cdot}05) = 0{\cdot}214$. Hence, from equation (11), for $\rho = 0{\cdot}2$, drawdown $s = 9{\cdot}95 \times 0{\cdot}214 = 2{\cdot}13$ feet.

The constant $\dfrac{\pi r_w}{2h_e} = 0{\cdot}002$ and $\dfrac{Q}{kh_e^2} = 0{\cdot}078$. Hence from curve A, *Fig. 1*, the correction to the drawdown is about 6 per cent, making the corrected drawdown 2·26 feet.

The drawdown of the water level in the well is found from equation (18) to be : $h_e - h_w = 9{\cdot}95(\log_e 800 - 0{\cdot}043) = 66{\cdot}1$ feet.

Example 2

Consider the well in Example 1, after prolonged pumping for 69·4 days. Then $\tau = 1$. From Table 3, for $\rho = 0{\cdot}2$, $V = 1{\cdot}844$. Therefore $s = 18{\cdot}35$ feet.

From equation (18), $h_e - h_w = 71{\cdot}6$ feet.

Example 3.—Data for thin and very permeable aquifer

Assume : $r_w = 1$ foot, $h_e = 100$ feet, $k = 10^{-3}$ foot per second, $S = 0{\cdot}2$, $Q = 4$ cubic feet per second, $t = 27{\cdot}8$ hours $= 10^5$ seconds. Then

$\tau = 5{\cdot}0$, $T = kh_e = 0{\cdot}1$, $\dfrac{Q}{2\pi T} \doteq 6{\cdot}366$.

From Table 3, for $\rho = 0{\cdot}2$, $V = 2{\cdot}785$.

Therefore $s = 6{\cdot}366 \times 2{\cdot}785 = 17{\cdot}73$ feet.

From *Fig. 2*, the correction to be subtracted from s, for $\rho = 0{\cdot}2$, is 5·5 per cent. Thus, the corrected value of s is 16·76 feet. Also, from equation (20), $\rho_e = 1{\cdot}5 \times \sqrt{5} = 3{\cdot}354$; and since $\dfrac{Q}{\pi k} = 1{,}273{\cdot}2$ square feet, equation (19) gives $h_w = 51$ feet.

Conclusions

Equation (11) gives the drawdown of the water-table for all values of the time factor τ.

For $\tau < 0{\cdot}05$ the drawdown should be corrected using the curves in *Fig. 1.*

For $0{\cdot}05 \leqslant \tau \leqslant 5$, ρ and Q in equation (11) are limited to the ranges stated under Case II (p. 572). Some values of the function V are given in Table 3.

For $\tau > 5$, the exponential integral (equation (12)) gives the drawdown for any value of Q (that is for all pumping levels) provided a correction is applied using the curve in *Fig. 2.*

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9. See reference 1, p. 519.
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12. See reference 5, p. 544.
13. Morris Muskat, " The Flow of Homogeneous Fluids through Porous Media." McGraw-Hill, 1937, p. 226.
14. See reference 5, p. 536.

The Paper is accompanied by two sheets of diagrams, from which the Figures in the text have been prepared, and by the following two Appendices.

Appendix I

The formula corresponding to equation (11), when the permeable medium extends downwards indefinitely below the bottom of the well, is derived as follows :—

Consider water being removed at a constant rate q from a small spherical cavity on the well-axis, at a depth η below the rest level of the water-table. It is convenient to employ the symbol y for the depth below the initial water-table of any point. Then after an interval of time t from the instant when water is first removed :

$$\phi = \frac{q}{4\pi k}\left[\frac{1}{\sqrt{\{r^2 + (y - \eta)^2\}}} + \frac{1}{\sqrt{\{r^2 + (y + \eta)^2\}}} - \frac{2}{\sqrt{\{r^2 + (y + \eta + kt/S)^2\}}}\right] \quad . \quad . \quad . \quad (21)$$

It can easily be verified that equation (21) satisfies boundary equation (4*a*) when $y = 0$.

The solution for a well of length l below the initial water-table may now be obtained by integration. Thus replacing q by $\frac{Q}{l} \, . \, d\eta$ and integrating (21) between the limits of 0 and l gives:

$$\phi = \frac{Q}{4\pi kl}\left(\sinh^{-1}\frac{l-y}{r} + \sinh^{-1}\frac{l+y}{r} - 2\sinh^{-1}\frac{l+y+kt/S}{r} + 2\sinh^{-1}\frac{y+kt/S}{r}\right)$$

Putting $y = 0$ and writing $\rho = \frac{r}{l}$, $\tau = \frac{kt}{Sl}$, the drawdown of the free surface is given approximately by:

$$s = \frac{Q}{2\pi kl}\left(\sinh^{-1}\frac{1}{\rho} - \sinh^{-1}\frac{1+\tau}{\rho} + \sinh^{-1}\frac{\tau}{\rho}\right) \quad . \quad . \quad . \quad (22)$$

APPENDIX II

The series referred to in this Paper, satisfying the correct boundary equations (6) at the well, are:

$$\frac{\partial\phi}{\partial z}_{z=h_e} = \frac{4}{\pi}\sum_{n=1,\,3,\,5\,\ldots}^{\infty}(-1)^{\frac{1}{2}(n-1)}\frac{1}{n}\cos\frac{n\pi h_w}{2h_e}\frac{K_0\left(\frac{n\pi r}{2h_e}\right)}{K_0\left(\frac{n\pi r_w}{2h_e}\right)} \ldots (n \text{ odd})$$

$$Q = \frac{16}{\pi}kr_w h_e\sum_{n=1,\,3,\,5\,\ldots}^{\infty}(-1)^{\frac{1}{2}(n-1)}\frac{1}{n^2}\cos\frac{n\pi h_w}{2h_e}\frac{K_1\left(\frac{n\pi r_w}{2h_e}\right)}{K_0\left(\frac{n\pi r_w}{2h_e}\right)} \ldots (n \text{ odd})$$

The series satisfying the approximate boundary equation (8) at the well is

$$\frac{\partial\phi}{\partial z}_{z=h_e} = \frac{Q}{\pi k h_e^2}\sum_{n=1,\,3\,5\,\ldots}^{\infty}K_0\left(\frac{n\pi r}{2h_e}\right) \ldots (n \text{ odd})$$

CORRESPONDENCE

on two Papers published in Proceedings Part III, April 1954

Hydraulics Paper No. 2

" Measurement and Utilization of the Water Resources of the Nile Basin " †

by

Harold Edwin Hurst, C.M.G., M.A., D.Sc.

Correspondence

Dr Serge Leliavsky, of Cairo, thought that Dr Hurst had underestimated his splendid work, for a casual reader studying his valuable Paper might possibly be misled into believing that the practical importance of the widespread net of Dr Hurst's measurements related chiefly to future works to be carried out on the Upper Nile. Dr Leliavsky wished, therefore, to place on record, and to emphasize, that in the two decades he had been in charge of the Designing Service of the Reservoirs and Nile Barrages Department of the Egyptian Ministry of Public Works, he had had frequent opportunities to use with advantage the precisely recorded data on the Nile regimen, supplied by the Physical Department, organized and directed by Dr Hurst. That information had been of inestimable help to Dr Leliavsky in preparing the basic outlines of several major works on the Nile ; they were, the remodelling of Assiut Barrage, the construction of the Mohammad Aly Barrages, the remodelling of Esna Barrage, but especially, the second heightening of the Aswan Dam.

The latter work called for particular mention, for, in that case, Dr Hurst's discharge observations had been employed by Dr Leliavsky to solve one of the most important problems of modern river-control technique, that was, abstracting from the natural river the maximum possible volume of water for filling a storage reservoir without causing the latter to be obliterated with silt. That problem was of acute importance at the present time, for troubles with sediment deposits had lately been reported in connexion with several of the American storage schemes, which had recently been completed. Since the earliest time when the Aswan Dam had first been conceived, the importance of that problem had been fully realized, and, when the second heightening scheme had been put forward

† Proc. Instn Civ. Engrs, Part III, vol. 3, p. 1 (April 1954).

in the middle nineteen-twenties, much apprehension had existed as to whether the larger reservoir could indeed be filled without dangerous effects on its capacity by deposited silt. Dr Leliavsky's solution of that problem, in collaboration with the Director General of Reservoirs, had subsequently been published by the Egyptian Government,[26] and since it had been vindicated by two decades of practical application, its original points might possibly deserve quotation as enhancing the value of the information supplied by the Physical Department.

The first question was, whether a "statistical" year might be substituted, in the study of the filling problem, for a number of actual years, which would have rendered the solution materially simpler. In fact, from Dr Hurst's accurate discharge observations the percentage probability of the 10-day mean discharges for the complete water-year had been known (the water-year in Egypt started and ended on July 15 when the reservoir was empty). A reasonably severe set of equal-probability discharges for the entire cycle of operation of the storage scheme could, therefore, have been easily calculated, and conclusions deduced therefrom.

A 95-per-cent-dry year had actually been considered, but not used on further examination, chiefly on account of the criterion for the date on which the filling operation should start. That operation, of course, could not begin until the Nile water became clear of silt after flooding; and therefore, the rule, which had been followed in the period previous to the investigations, had been that filling began when the water level on a certain gauge, 15 kilometres ($9\frac{1}{2}$ miles) downstream of the dam, fell to the level R.L. 88.00.*

The effect of the rule had been twofold. In the first place, it prevented the use of statistical years, in general, because it made the amount of water available for storage a function of the rate of fall of the discharge, the probability of which was beyond the usual statistical analysis; and secondly, it stood in the way of a second heightening scheme, for, with so late a date for its beginning, the filling operation of the larger reservoir could not be completed in time.

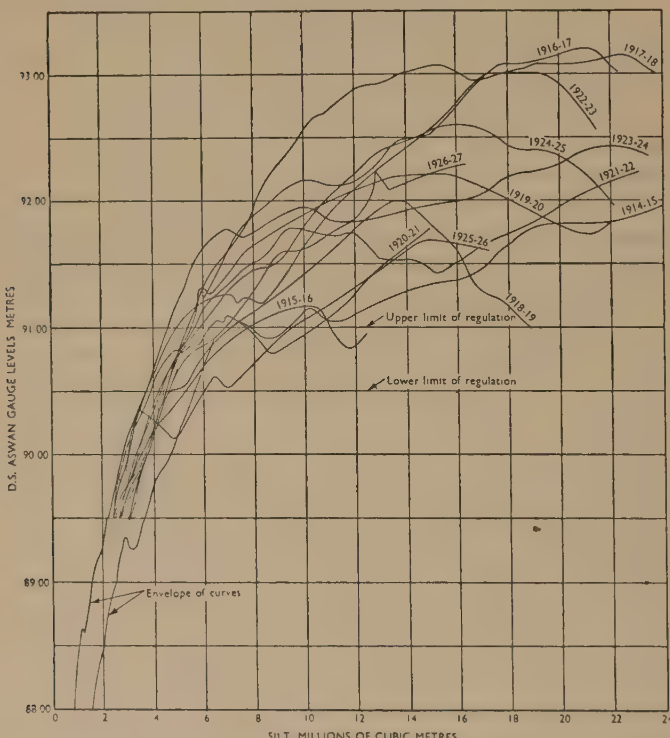
The R.L. 88.00 rule had been, therefore, the main objective of the investigation. No one knew, when, and for what reason, it had first been put forward, but since it had become part of the established routine and since anything even remotely connected with the Aswan Dam was taboo, it was not easy to find arguments against it which were convincing in the right quarters. One of these arguments is reproduced in *Fig. 13*, which appeared in Dr Leliavsky's publication [26] and was reproduced, again, in the printed report of the resident engineer. Unless its principle was fully explained its argument might be difficult to grasp. Although the ordinates

* The reduced levels were supposed to refer to a conventional datum which at one time was believed to be the mean sea level at Alexandria.

[26] References 26 to 41 are given on pp. 593 and 594.

were levels, they should be interpreted as criteria for the dates of the beginning of the yearly storage operations, and the corresponding abscissae were then the volumes of silt carried by the river, after the dates in question, in each individual year investigated. The characteristic point of the diagram was, that up to a certain level, all the curves fell close to each other and rose evenly (the " envelope " region of the diagram), but above that level they dispersed. *Fig. 13* was, therefore, interpreted to mean

Fig. 13



Cumulative Amount of Silt referred to Water Levels

that beneath a certain critical limit the river level was symptomatic of the safety (or otherwise) of the filling operation, and could, consequently, be used as a criterion for the date on which the filling should start. Since the limit in question was R.L. 90.50, or even R.L. 91.00, but not R.L. 88.00, the antiquated rule had been modified accordingly, and Dr Hurst's reference to the absence of deposits confirmed the correctness in judgement in taking that step.

Dr Leliavsky concluded that the described incident was believed to substantiate the following points :—

(1) The operational programmes of proposed storage schemes should not be calculated from statistical years (whether mean years or otherwise), but from a representative series of actual past years.

(2) To ensure safety against sedimentation, storage programmes should be based on hydraulic criteria, and not on fixed dates of the year.

(3) The significance of river levels, as such criteria, could best be ascertained by diagrams such as *Fig. 13*.

Mr R. P. Black, of Cairo, stated that a measurement of discharge consisted of sounding and velocity observations at a number of equally spaced points across the stream. That was at best a long and tedious process especially under flood conditions when, to the other difficulties was added the management of a launch in a strong-flowing stream.

In order to reduce the time required to reasonable limits, it was necessary to reduce the number of observations to a minimum. The computation of a discharge reduced basically to the determination of an area bounded by a curve, its end ordinates, and the axis of co-ordinates, hence the accuracy depended on the number of observations whatever the width of the stream might be and it was highly important that the end points should be so close to the banks that the unmeasured portions at the banks were negligible. On the Nile, under ideal conditions, it had been found by trial that twenty-five equally spaced observation points, the first and last being as close as possible to the banks, were sufficient. Current-meter observations were made only at half depth, and a factor of 0·96 was used to reduce the half-depth velocity to the mean velocity in the same vertical. The meter was assumed to measure the velocity at a point in the section, and that was sensibly true for a stream of any size, where the velocity was practically constant over the area of the rotor. That was the condition under which the meter was rated and set a limit to the smallness of the channel in which it could be used.

The 0·96 coefficient had been determined at Khannaq, which was the discharge site some 40 kilometres (25 miles) below the Aswan Dam, where the depth of water varied from 4 to 12 metres at the different stages of the river. Some 500 vertical-velocity curves had been drawn and although individual curves might depart considerably from the parabolic form, the mean of a fair number of them was very approximately parabolic and no systematic departure from that form was detectable at depths of less than 4 metres. The curves also showed that the mean of the velocities at 0·2, 0·6, and 0·8 depth was 0·99 multiplied by mean velocity, but as the three-point method took so much longer, the half depth was standard on the Nile.

The ratio mean velocity half depth velocity had been carefully determined at a number of points and nowhere had a departure of as much as 5 per cent from the 0·96 value been found.

Mr Black quoted the following examples of departures :—

(1) In the Blue Nile, 6 kilometres (approx. 4 miles) from Lake Tana where the mean depth was $3\frac{1}{2}$ metres and the bed was of very rough basalt boulders, the factor was 0·92.

(2) In a new canal, of pitched side slopes and smooth concrete bed, at Atf in Lower Egypt, where the depth was 2 metres, the factors were 0·955 over the smooth bed, and 0·915 over the pitched sides.

(3) In a flume, 4 metres wide with vertical sides, and a depth of 2·70 metres the factor was 0·935.

To anyone who had had experience of flood-discharge measurement on the Nile, where every source of error tended to exaggerate the discharge, Dr Hurst's Table 2, showing the close agreement of sluices and current meters was a remarkable tribute to the reliability of the Gurley current-meter when used at a well-chosen discharge site.

As a further vindication of the reliability of the current meter, Mr Black thought it might be of interest to quote a comparison with a Venturi-meter survey which he had carried out at the Delta Barrage. The sites, just downstream of a regulator, had been far from ideal, but had been the only ones available.

TABLE 4.—COMPARISON OF CURRENT METER AND VENTURI-METER AT THE DELTA BARRAGE

Site	Mean of four current discharges : cubic metres per sec.	Venturi-meter discharge : cubic metres per sec.
No. 1	3·68	3·80
No. 2	3·82	3·80
No. 3	3·66	3·80
No. 4	3·98	3·80
Mean	3·78	

From an experience of hundreds of current meters and many thousands of discharges, Mr Black could list only two objections to the Gurley meter : (*a*) the rotor turned with any up-and-down motion of the meter, and (*b*) the meter was not sensitive enough below 25 centimetres per second. The results of those objections were that, (*a*) caused the flood discharges to be exaggerated, and (*b*) demanded a site not easily found on the Nile, where the low-stage velocity did not fall below 25 centimetres per second. His S-rotor meter eliminated both those faults and would give reliable results at velocities as low as 10 centimetres per second.

With reference to current meter rating Dr Hurst had described the

Delta Barrage Rating Station, and it was only necessary to add that the rating was entirely automatic and quite independent of the skill of an observer. The chronograph sheets, which contained the whole of the observations and computations, were filed and thus a record was kept of every instrument throughout its whole period of service. Anyone interested could check a rating Table against the original observations at any time.

In the past, objections had been raised against the rating of a meter by towing it in still water, the contention being that the instrument should be rated in flowing water. He had never heard, however, of a meter rated in flowing water having any claim to the accuracy obtained by a meter rated in still water. If the flume used was small, it was not possible to get in the required number of measuring points, nor was it possible to get rid of the boundary effect, nor was the rotor in any sense a point in the water-section. Not one of the three conditions for the accurate measurement of a discharge mentioned above could be satisfied. If the flume was large enough for those conditions to be satisfied, then the volume of water to be supplied was prohibitive. If the volume of water to be measured did not constitute a difficulty as at Aswan, to get the required range of velocities would require half a flood season and the method was quite impracticable.

A few years previously Mr Black had experimented with a small flume 30 centimetres wide and 15 centimetres deep, and had secured what had looked like a steady uniform flow with a velocity of about 10 centimetres per second, the discharge being measured directly into a tank. When a small Gurley meter had been moved about in the flume, variations of velocity of 10 per cent had been recorded, and later when the flume had been explored with his miniature S-rotor meter, velocity variations of 50 per cent had been recorded, or just about the same as were found in a section of the Main Nile at low stage. As a matter of interest, the miniature meter had a rotor diameter of 12 millimetres, gave 47 revolutions per minute for a velocity of 10 centimetres per second, and could be carried in the waistcoat pocket. It was very useful for measuring low velocities in a model where it would record satisfactorily in a depth of water of only 2 centimetres.

Fortunately the problem of rating current meters in flowing water was no longer of practical importance. Dr Hurst's Table 2 where current meter measurements at Aswan were compared with what amounted to absolute measurements into a tank, should convince anyone that the method of rating by towing in still water, which was in general use to-day, and which could be simply and expeditiously carried out with great accuracy, was sound.

Mr R. B. Bulman thought that one significant aspect had been neglected in the Paper and also appeared to have been neglected in more detailed works, such as "The Nile Basin." [5] That aspect was what

happened to the water after it had been delivered into the field water-courses?

On several occasions recently attention had been drawn to the much greater dividends which would result from a small increase in agricultural productivity rather than bringing large areas of additional land into cultivation.

By analogy, would not a small increase in the efficiency of use of irrigation water, at present available, pay better dividends than the expensive schemes of storage and control now being proposed?

If efficiency of use of irrigation water was defined as the proportion of the whole actually transpired by useful crops, what efficiency was experienced in Egypt? In other words, could the Author say how much of the water available at the field water-course was transpired by crops and how much was wasted in direct evaporation, etc.?

In the traditionally irrigated countries there appeared to have been a gap in research and a lack of co-operation between the irrigation engineer and the agriculturist. Each had been afraid of trespassing on the domains of the other. The irrigation engineer had considered his work finished when an adequate supply of water arrived at the field water-courses. The agriculturist had taken for granted the soil moisture made available for his crops.

An interesting field of research existed in the gap between the provinces of the irrigation engineer and the agriculturist. It could make a valuable contribution to world food problems.

Mr William Allard in connexion with half-depth velocity measurements, drew attention to a Paper published by the Institution.[27] In the discharge measurement examined, incidentally, the coefficient was 0·91, not 0·96, whilst R. F. Wileman (p. 293, in reference 4) thought 0·92 to be a possible value for a certain British river.

He wondered why, at present, whenever a single-point method was used, 0·6 depth was (except in Egypt) largely preferred to mid-depth. Was it because some arithmetical labour was obviously avoided in subsequent computations by the use of a coefficient of unity rather than one of 0·96. W. A. Liddell,[28] some years previously, had quoted, without himself supporting, a claim that a greater constancy existed for mid-depth velocities than for those at 0·6 depth. Whichever of the two single-point methods was used, it seemed probable that there would be, from gauging station to gauging station some variation in the value of the standard coefficient, whether that was 0·96 or unity.

What exercised the minds of some river gaugers more was the relationship between results obtained arithmetically by the full-scale method, that was, multiple points in each vertical, and those secured by the short-cut methods, that was, one, two, or three points in the vertical. Mr Allard's impression was that in the past those who favoured the full-scale method, including most gaugers on the Continent of Europe, were inclined to dismiss

the short-cut methods without actually testing their adequacy, as several of the short-cut school had done. The reason usually given was that the validity of all short-cut methods depended upon a parabolic distribution of the velocities in a vertical; that form of distribution was, in practice, rarely found in vertical velocity curves, "owing to pulsations and surges"[29] and other causes. The possibility of such irregulatities cancelling one another largely or entirely did not seem to have been examined by such critics.

During the last few years, however, the matter had been discussed helpfully by British river gaugers such as R. F. Wileman,[4, 30] R. H. MacDonald[31] and J. J. O'Kelly.[31] A notable investigation of the matter had now been reported from the Antipodes, where modern hydrometric practice had already been described in Papers by Australian[32] and New Zealand[33] engineers. One of these engineers, K. D. Green, had said[34] that the 0·2/0·8 method had not been adopted in the State of Victoria as general practice but added—"However, data accumulated from plotting the results of many gaugings indicates some cases in which the method may be used, but velocity pulsations must then be considered. . . ."

The investigation reported recently was made by A. C. Hopkins, of the staff of the Soil Conservation and Rivers Control Council of New Zealand.[35] He compared with the discharge calculated by graphical means, from what he called the plotted mass curve, the discharges obtained purely arithmetically by the following methods:—

(1) One-point, one measurement at 0·6 depth.
(2) Two-point, the mean of measurements at 0·2 and 0·8 depth.
(3) Three-point, the arithmetic mean of measurements at 0·2, 0·6, and 0·8 depth.
(4) Two-and-One (2/1), the mean of the two values obtained by (1) and (2) above, respectively.
(5) Multiple point, the arithmetic mean of measurements at four or more points in each vertical.

From 52 gaugings related to (1), (2) and (4); 59 to (3); and 45 to (5) he found that the differences from the discharges calculated from the plotted mass curve were as indicated in Table 5.

Mr Hopkins had stated further that the greatest percentage differences related to the smaller flows, particularly in wide shallow streams. He considered that in general for best results, when time was limited for a particular gauging, velocities at 0·2, 0·6, and 0·8 depth should be observed and the two-and-one method used for computation of discharge. Elsewhere in the report he remarked: "Unless a particular gauging site has been the subject of an analysis of several gaugings which decrees otherwise, the two-point and one-point methods are best avoided if at all possible."

Mr Allard recommended that the combinations of measuring points

Table 5

Method	Range of differences: per cent	Number of deviations		Overall average of differences, neglecting plus and minus signs: per cent
		Neg.	Pos.	
(1)	− 7·4 to + 9·3	33	19	3·0
(2)	− 6·6 + 10·2	33	19	2·6
(3)	− 6·6 + 6·3	38	21	2·4
(4)	− 6·8 + 5·1	35	17	2·3
(5)	− 4·2 + 5·5	16	29	1·4

shown in Table 6, should be adopted, when any current meter of the type and size of a "Price" meter was being used.

Mr Allard suggested that Mr Hopkins's thorough quantitative study and other Australasian information had done much to help define the situation more clearly and establish a balance between the views of the opposed schools.

Mr Allard asked whether Dr Hurst felt that seasonal rates of loss and gain in transit could be calculated for lengthy reaches of a river like the Nile, or whether the possible error in any individual discharge measurement rendered it questionable to set an upstream station's results against those of another downstream? Dr Hurst would doubtless be acquainted with what was said on the subject in the Handbook of Egyptian Irrigation.[36]

Table 6

Depth of vertical: feet	Points in total depth at which to measure
1–2	0·6
2–4	0·2, 0·6, and 0·8
4–5	0·1, 0·2, 0·4, 0·6, 0·8, and 0·9
5–6	0·1, 0·2, 0·3, 0·5, 0·6, 0·7, 0·8, and 0·9
6–10	Nine points at 0·1 intervals
Over 10	0·04, 0·20, 0·24, 0·42, 0·50, 0·58, 0·76, 0·80, and 0·96

He asked also whether Dr Hurst felt able to express any further views upon A. B. Buckley's Paper [37] on the influence of silt on the velocity of water flowing in open channels, which was based upon data obtained in Egypt. It was thought that there had been some recent work in the U.S.A. on the subject.

He assumed that, had space sufficed, Dr Hurst would under "Other Measurements" have included a reference to the notable use made in Egypt of sluice openings in river barrages and canal regulators for the

measuring of discharges, the openings being calibrated by current-meter. Details were, however, obtainable elsewhere.[38, 39]

The Author, in reply, stated that the question of the most efficient use of irrigation water, mentioned by Mr Bulman, was a matter of great importance. Apart from technical considerations, which were difficult enough, it involved in Egypt the whole system of life of the country, and would make a large chapter in a treatise on irrigation. It was not possible therefore to deal with it in a short Paper into which many subjects mainly of a technical nature had been compressed. Volume 9 of "The Nile Basin" was still in preparation. The work dealt principally with hydrology and its connexion with major projects, so that irrigation matters were introduced only to make clear the objects of the work.

Work had of course been done on the question of plant requirements, but Mr Bulman was right in emphasizing the need for the closest collaboration between the agriculturist and the irrigation engineer. However, whatever improvements were made in the distribution and use of water on the land, the rapidly increasing population of the Nile Basin would still make necessary the major projects designed to render available the maximum amount of Nile water.

Mr Allard had asked about losses over long stretches of rivers like the Nile. It was the regular practice in Egypt to use average losses derived from many measurements made in the past of the discharges at the head and tail of the various reaches of the river and its tributaries, from Roseires on the Blue Nile and Lake Albert on the White Nile to the Delta Barrage. Those entered into all forecasts of the water supply reaching Egypt, as well as into the distribution of water for irrigation, where, for example, the supply sent down from Aswan must be sometimes larger and sometimes smaller than the amount which was required at the canal heads. As Mr Allard had suggested, no reliance could be placed on a loss of the order 5 or 10 per cent based on two or three pairs of discharges. However, where a good deal of information was available about the past, together with regular measurements in the time immediately preceding the regulation, very useful information could be obtained as to probable existing losses. Losses as used were dependent also on rates of travel of changes of discharge.

The influence of silt on the velocity of water in open channels had been discussed in a Paper by Vanoni[40] and in a forthcoming book on fluvial hydraulics by Dr Serge Leliavsky.

Several contributors had given valuable information on the vertical distribution of velocity in a stream, which covered a wide range of conditions, from small rapid streams with stony beds to large rivers in alluvial plains like the Nile and Indus. More information was contained in the valuable references given by Mr Allard. As he had remarked the single observation at half-depth was not much used outside the Nile Basin. That was strange considering its obvious advantages over the point at 0·6

depth, which had been previously mentioned. The very valuable data from New Zealand [35] did not specifically mention half-depth observations, though it was clear that information on that point could be extracted from some of the individual discharge records. It was recommended in various publications that when measuring point velocities on a vertical the readings at 0·2, 0·6, and 0·8 depth should wherever possible be included. It would add greatly to the value of such observations if 0·5 depth could also be added. It was better to have direct measurement at those points rather than deduce values from a curve.

In assessing the value of short-cut methods of measuring the mean velocity it was important to distinguish between accidental and systematic errors, since accidental errors would be made of small importance by increasing the number of observations, whilst the systematic errors remain unaffected, and in the long run were often of much more importance. In a steadily flowing river at a good discharge site, where the discharge and level remained constant over several hours, the velocity at individual points was continually varying, irregularly with the time to a greater or less extent, and also from point to point over the cross-section. At the same time the average velocity over the whole section would vary irregularly, but with a much smaller amplitude. Excluding very shallow or very turbulent streams and those of very steep slope which perhaps carried much matter in suspension, about which there was not much available information, if velocities were measured at say ten points on a vertical for periods of 2 or 3 minutes at each, the velocity-depth curves approximated to parabolae. Experience on many rivers showed that the mean velocity on a vertical occurred at about 0·6 depth below the surface; that the mean of velocities at 0·2 and 0·8 depth was approximately the mean for the vertical; and that the velocity at half-depth was approximately 1·04 times the mean. (The Author drew attention to contributions by Brigadier Hawes (p. 31), Mr Newhouse (p. 41), Mr Black (p. 583), and to references 28, 29, 31 and 35 on p. 593). It should be noted that in any particular case the average values of those relations could not be established with precision except by a number of discharges giving something of the order of 10 or 20 times as many vertical velocity curves each based on 6 to 10 individual observations. It was probable that at least 100 to 200 vertical velocity curves (5 to 10 discharges at a station) were required to establish fairly precisely the factor required to reduce the velocities for a given combination of 1, 2, or 3 observation points to the mean velocity on the vertical. With that amount of evidence the probability was that such factors would not depart by more than 1 or 2 per cent from the values given above. A point to be noted was that a reasonable length of time must be adopted for each velocity observation. The practice in Egypt, when observing at half-depth, was to take three runs of 1 minute each on about twenty verticals. That reduced the effect of the accidental irregularities which occurred. As an example of those, from a

discharge of the Nile in flood at Aswan, three runs of a minute each were taken at half-depth on twenty-two verticals. The average range of a set of three was 6 per cent, and there were five ranges of 10 per cent or more, with a maximum of 13 per cent. The range of the means of revolutions for the 1st, 2nd, and 3rd minutes was 0·5 per cent. Thus to reduce the effect of those variations to a reasonable amount three observations at each point was not excessive, and it was very probable that on any ordinary river three such observations at half-depth using a reduction factor of 0·96 gave a more accurate result and took less time than two observations occupying 1 minute each at 0·2 and 0·8 depth. The practice of observing velocity at a point over 1 minute only was common (see references 28, 29 and 35), and it seemed probable that the shortness of the total time during which velocity was under observation was largely responsible for the irregularities observed.

To illustrate some of the above remarks, Table 4 was taken from a Table in Mr Hopkins's Paper,[35] in which he had attributed anomalous variations to the small number of observations.

TABLE 4.—FIVE DISCHARGES MEASURED ON DIFFERENT NEW ZEALAND RIVERS.

Characteristics of site			No. of verticals	Total No. of point velocities	Percentage differences from mean of vertical velocity curves			
					Depths			Multiple point
Width: feet	Depth: feet	Mean vel. ft/sec.			0·6	0·2, 0·8	0·2, 0·6, 0·8	
105	5	3·5	14	56	− 4·1	+ 4·7	− 0·6	+ 5·4
35	8*	1·3	7	34	+ 4·9	+ 2·3	+ 3·9	+ 1·0
45	9*	1·5	8	62	+ 0·5	− 3·9	− 2·6	+ 0·8
40	3·5	3·4	7	28	+ 3·8	− 0·5	+ 0·7	+ 2·9
168	6	2·5	14	93	− 1·0	+ 0·3	− 0·2	+ 0·2
				Means	+ 0·8	+ 0·6	+ 0·2	+ 2·1
Total number of point velocities					50	96	144	273
Mean number of point velocities					10	19	29	55

Duration of point velocity observation: 40 to 60 secs.

*Depth at centre of river

The Author's (Dr Hurst's) comments on Table 4 should be considered as supplementary to Mr Hopkins' Paper,[35] which was devoted entirely to discussion of irregular variations of point velocity observations, and not

to derivation of reduction factors for various velocity combinations. The facts which appeared in Table 4 were :—

(*a*) The average number of velocities measured at a single depth in a discharge was ten, covering about 10 minutes.

Judging by experience on the Nile, to reduce the effect of irregular time-variations of velocity to the order of 1 per cent, about 6 times that length of observation time was required.

(*b*) Assuming the variations in Table 4 to be mainly random, if equal numbers of observations were made for each combination there would be little to choose between 0·6 ; 0·2 and 0·8 ; and 0·2, 0·6, and 0·8 depths. The ranges of 8·7, 8·6, and 6·5 per cent for those combinations tended to confirm that.

(*c*) The multiple point method with an average of 5½ points to a vertical had a range of 8·3 per cent and differed from the result obtained by drawing vertical velocity curves through the points by more than any of the previous combinations. That was probably because of the inclusion, in 29 out of 50 verticals, of observations at 0·4 depth, where normally the velocity was about 8 per cent more than the mean, thus throwing out the balance of observations.

(*d*) In determining a reduction factor for a combination of observations at various depths at any station, it was important to have a total time of velocity observations long enough to render unimportant irregular velocity fluctuations of short period, and to make the observations on a number of different occasions. The reduction factor as determined was no doubt a quantity subject to slight fluctuations about an average value, due partly to errors of observation and partly to natural variations. The Author suggested that for observations at a single point, for example, 0·5 depth, ten discharges, consisting of twenty verticals each and 3 minutes of velocity observation at each point should be measured. For comparison with that, in order to draw reasonably accurate vertical velocity curves, observations should, whenever possible, be made at ten points on a vertical with a duration of one minute at each. In shallow streams it might not be possible to put in as many as ten points to a vertical and in that case the same precision (percentage) could not be expected.

Mr C. L. Berg [41] had described a measurement of the discharge of the Victoria Nile, which consisted of multiple points on nineteen verticals, including observations at half-depth. From them he had deduced a reduction factor of 0·91 to reduce the velocity at half-depth to the mean

velocity. In view of the experience already described it would seem advisable to wait for further observations before deciding to adopt that value instead of the value of 0·96 previously used.

The process of plotting velocities and drawing vertical velocity curves was costly in time, and depended a little on the judgement of the draughtsman. The work could be done much more quickly and without any personal bias by an arithmetical method, and of course could be equally quickly checked.

If $v_1\ v_2\ v_3 \ldots v_n$ were a series of velocities on a vertical, observed at depths of $x_1\ x_2\ x_3 \ldots x_n$, and x_{n+1} was the depth of the river, then, following the ordinary process of numerical integration:

$$\begin{aligned}\int v.dx &= v_1[x_1 + \tfrac{1}{2}(x_2 - x_1)] + v_2[\tfrac{1}{2}(x_2 - x_1) + \tfrac{1}{2}(x_3 - x_2)] + \\ &\quad \ldots + v_n[\tfrac{1}{2}(x_n - x_{n-1}) + (x_{n+1} - x_n)] \\ &= \tfrac{1}{2}v_1(x_1 + x_2) + \tfrac{1}{2}v_2(x_3 - x_1) + \\ &\quad \ldots + \tfrac{1}{2}v_{n-1}(x_n - x_{n-2}) + v_n(x_n{}^{+}{}_1 - \tfrac{1}{2}x_n - \tfrac{1}{2}x_{n-1})\end{aligned}$$

If the first interval was a and the last c and all the intermediate ones were equal to b then:

$$\int v.dx = av_1 + b(\tfrac{1}{2}v_1 + v_2 + \ldots + v_{n-1} + \tfrac{1}{2}v_n) + cv_n$$

and the mean velocity was $\dfrac{1}{x_{n+1}}\displaystyle\int v.dx.$

In conclusion the Author thanked the Institution for accepting the Paper, and the contributors to the discussion for the valuable information which they had supplied.

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[26] Ibrahim Rizk and Serge Leliavsky, "Report on the Filling of the Larger Reservoir," Ministry of Public Works, Government Press, Cairo, 1929.

[27] C. L. Berg, "Detailed Analysis of a Discharge Measurement on the Victoria Nile." Proc. Instn Civ. Engrs, vol. 2, Pt. III, p. 609 (Dec. 1953).

[28] W. A. Liddell, "Stream Gaging." McGraw-Hill Co., New York, 1927. p. 37.

[29] "Stream Gaging Procedure." United States Geological Survey Water-Supply Paper No. 888 (Washington, D.C., 1945).

[30] S. M. Dixon, G. FitzGibbon, and M. A. Hogan, "The Flow of the River Severn, 1921–36." J. Instn Civ. Engrs, vol. 6, p. 81 (June 1937). *Discussion by* R. F. Wileman, p. 148.

[31] G. N. Croker, "Records of Flows in the River Wye System, as Determined by Current Meter Measurements, with a Note on Flood Warning Arrangements." J. Instn Wat. Engrs, vol. 5, p. 39 (Feb. 1951). *Discussion by* R. H. MacDonald and J. J. O'Kelly, pp. 84–88.

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Structural Paper No. 36

"A Moment Distribution Method for the Analysis and Design of Structures by the Plastic Theory" †

by

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Correspondence

Mr A. Fruchtlander observed that the special moment distribution method for structures by the plastic theory, introduced by the Author, was very interesting but it might be noticed that it could only be applied to structural members which were actually in the collapse mode. It was obvious that, for a partial-collapse mode, the application to members which were still in a fully elastic state might lead to incorrect results as would be shown by the following example.

In *Example 1*, on pp. 53–55, the final improved design for a two-span continuous beam gave :—

Point	A	D	B	E	C
Moment : (inch-tons) .	108	108	108	54	72

The moments at points E and C were not correct and should be 63 inch-tons at E and 54 inch-tons at C. When the plastic hinge had developed at the support B under the load $P = 4$ tons, the short span BC was still in the elastic state, since at no point throughout the span had the plastic moment been reached ; the member BC acted therefore at that stage as a beam hinged at B and fixed at C. The distribution of the unbalanced

† Proc. Instn Civ. Engrs, Part III, vol. 3, p. 51 (April 1954).

moment of 36 inch-tons at B had then to be carried over for a beam in the elastic state, as shown in *Fig. 31.*

That gave at C a positive moment of + 18 inch-tons,
and at E a negative moment of — 9 inch-tons.

Fig. 31

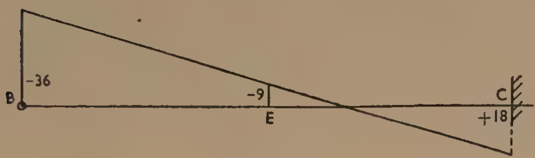


Fig. 32

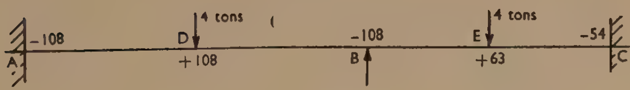


Fig. 33

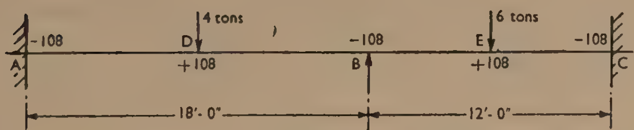
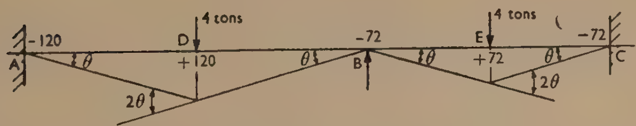


Fig. 34



Those combined with the moments before the distribution became :

$$\text{at C,} \; -72 + 18 = -54 \text{ inch-tons}$$
$$\text{and at E,} \; +72 - 9 = +63 \text{ inch-tons.}$$

Thus the final moments for that case would be as shown in *Fig. 32.*

The long span AB had now reached its partial collapse mode, whereas the short span BC was still in the elastic state and able to take up more load. By increasing the load P from 4 to 6 tons, additional moments were obtained :

$$\text{at the support C,} \; \Delta M_c = -\frac{3}{16} \times 2 \times 12 \times 12 = -54 \text{ inch-tons,}$$

$$\text{and at E,} \; \Delta M_E = 2 \times 12 \times 12\left(\frac{1}{4} - \frac{3}{32}\right) = +45 \text{ inch-tons.}$$

Those combined with the former moments gave :

$$\text{at C, } M_c = -54 - 54 = -108 \text{ inch-tons.}$$

$$\text{at E, } M_E = +63 + 45 = +108 \text{ inch-tons.}$$

Thus the total collapse loads of both spans were shown by *Fig. 33.*

By applying the principle of virtual work for the case of a uniform sectional modulus, for both spans, the relations between the collapse loads and the plastic hinge moments were obtained immediately :—

For the span AB

$$4M_p\theta = \frac{P_1L_1}{2}\theta$$

For the short span BC

$$4M_p\theta = \frac{P_2L_2}{2}\theta$$

hence for $P_1 = 4$ tons,

$$M_p = 4 \times \frac{18}{8} \times 12 = 108 \text{ inch-tons.}$$

$$\text{and } P_2 = \frac{8 \times 108}{12 \times 12} = 6 \text{ tons.}$$

For the case where the two collapse loads were given with 4 tons for each span, the sectional moduli might be different for both spans :

$$M_{p2} = \frac{4 \times 12}{8} \times 12 = 72 \text{ inch-tons.}$$

The required M_{p1} for the span AB was found as follows :—

The virtual work of the plastic hinges was :

$$(3M_{p1} + M_{p2})\theta$$

Assuming that $M_{p1} = nM_{p2}$,

$$\text{then, } M_{p2}(3n + 1)\theta = \frac{P_1L_1}{2}\theta$$

$$\therefore 72(3n + 1) = \frac{4 \times 18}{2} \times 12$$

$$\therefore n = \frac{5}{3} \quad \text{or, } M_{p1} = \frac{5}{3} \times 72 = 120 \text{ inch-tons.}$$

The total collapse loads of both spans were shown by *Fig. 34.*

Thus, for that case, the results obtained by the Author were correct because the distribution had been applied to both spans in the collapse mode.

The Author, in reply, agreed that the use of the distribution coefficients in Table 1 (p. 56) would not lead to a correct derivation of bending moments for those parts of a structure which did not collapse. The plastic moment distribution process did, however, establish without ambiguity the ability of a structure to support a given set of loads, and the derivation of the

exact bending moments throughout the structure at collapse was largely without interest. Approximate values of those bending moments could be obtained, if desired, by assuming plastic deformation to be localized at plastic-hinge positions, when elastic moment distribution (or some other method of elastic analysis) could be applied to those parts of the structure which did not collapse. That was the procedure which Mr Fruchtlander had applied to Example 1. The bending moments he had derived in *Fig. 32*, although almost correct, were not exact, since the span BC was not entirely elastic. Plastic zones would spread from the hinge at B into the span BC. The calculation of the exact bending moments would be very tedious, and it would be difficult to conceive a situation in which the labour entailed would be worth while.

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CORRIGENDA

Proceedings, Part III, April 1954

pp. 67 and 74, the diagrams numbered *Fig. 10* and *Fig. 19* should be interchanged.

p. 99, line 2, *for* " 10 November, 1952 " *read* " 10 November, 1953 "

p. 114, line 6 from bottom, *for* " approved appliances " *read* " improved appliances "

Plate 2, facing p. 275, Figs 14, on scale No. 8 the words " oil " and " water " should be interchanged.

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